CHAPTER III.—THE TEETH OF GEAR-WHEELS (*continued*).

THE revolving cutters employed in gear-cutting machines, gear-cutters, or cutting engines (as the machines for cutting the teeth of gear-wheels to shape are promiscuously termed), are of the form shown in Fig. 107, which represents what is known as a Brown and Sharpe patent cutter, whose peculiarities will be explained presently. This class of cutters is made as follows :—

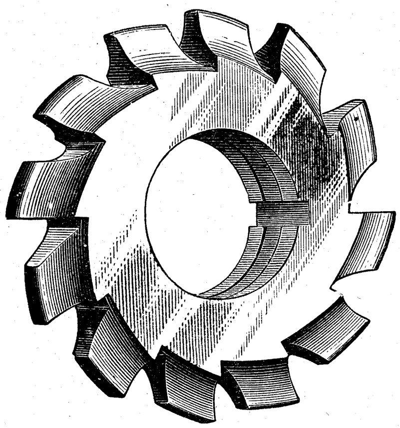

Fig. 107.

A cast steel disk is turned in the lathe to the required form and outline. After turning, its circumference is serrated as shown, so as to provide protuberances, or teeth, on the face of which the cutting edges may be formed. To produce a cutting edge it is necessary that the metal behind that edge should slope or slant away leaving the cutting edge to project. Two methods of accomplishing this are employed: in the first, which is that embodied in the Brown and Sharpe system, each tooth has the curved outline, forming what may be termed its circumferential outline, of the same curvature and shape from end to end, and from front to back, as it may more properly be termed, the clearance being given by the back of the tooth approaching the centre of the cutter, so that if a line be traced along the circumference of a tooth, from the cutting edge to the back, it will approach the centre of the cutter as the back is approached, but the form of the tooth will be the same at every point in the line. It follows then that the radial faces of the teeth may be ground away to sharpen the teeth without affecting the shape of the tooth, which being made correct will remain correct.

This not only saves a great deal of labor in sharpening the teeth, but also saves the softening and rehardening process, otherwise necessary at each resharpening.

The ordinary method of producing the cutting edges after turning the cutter and serrating it, is to cut away the metal with a file or rotary cutter of some kind forming the cutting edge to correct shape, but paying no regard to the shape of the back of the tooth more than to give it the necessary amount of clearance. In this case the cutter must be softened and reset to sharpen it. To bring the cutting edge up to a sharp edge all around its profile, while still preserving the shape to which it was turned, the pantagraphic engine, shown in Fig. 108, has been made by the Pratt and Whitney Company. Figs. 109 and 110 show some details of its construction.* "The milling cutter N is driven by a flexible train acting upon the wheel O, whose spindle is carried by the bracket B, which can slide from right to left upon the piece A, and this again is free to slide in the frame F. These two motions are in horizontal planes, and perpendicular to each other.

"The upper end of the long lever P C is formed into a ball, working in a socket which is fixed to B. Over the cylindrical upper part of this lever slides an accurately fitted sleeve D, partly spherical externally, and working in a socket which can be clamped at any height on the frame F. The lower end P of this lever being accurately turned, corresponds to the roller P in Fig. 106, and is moved along the edge of the template T, which is fastened in the frame in an invariable position.

"By clamping D at various heights, the ratio of the lever arms P D, D C, may be varied at will, and the axis of N made to travel in a path similar to that of the axis of P, but as many times smaller as we choose; and the diameter of N must be made less than that of P in the same proportion.

"The template being on the left of the roller, the cutter to be shaped is placed on the right of N, as shown in the plan view at Z, because the lever reverses the movement.

"This arrangement is not mathematically perfect, by reason of the angular vibration of the lever. This is, however, very small, owing to the length of the lever; it might have been compensated

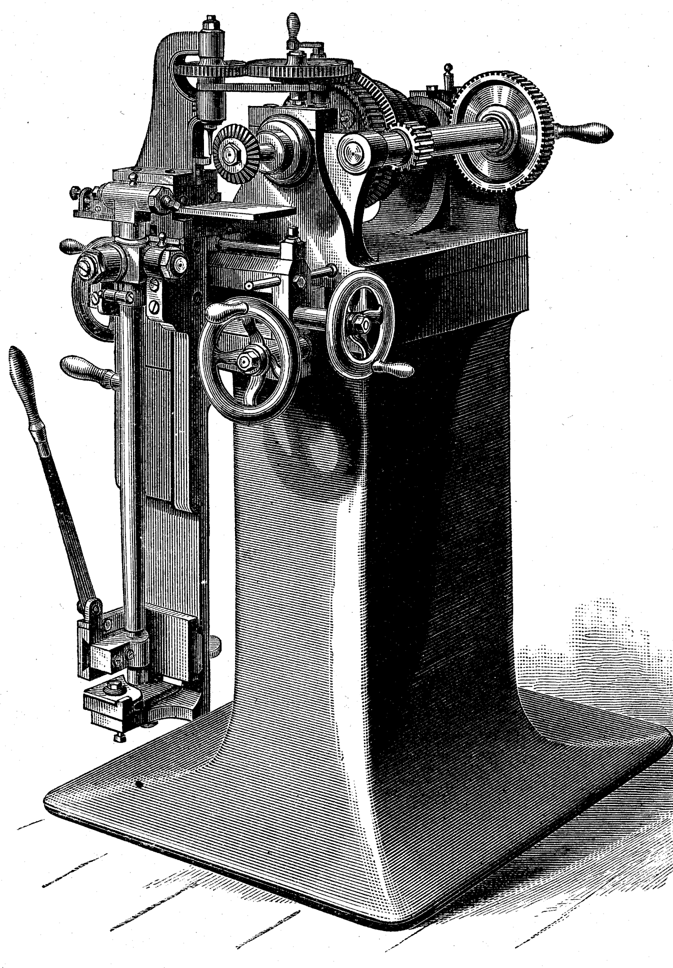

Fig. 108.

for by the introduction of another universal joint, which would practically have introduced an error greater than the one to be obviated, and it has, with good judgment, been omitted.

"The gear-cutter is turned nearly to the required form, the notches are cut in it, and the duty of the pantagraphic engine is merely to give the finishing touch to each cutting edge, and give

* From "The Teeth of Spur Wheels," by Professor McCord.

it the correct outline. It is obvious that this machine is in no way connected with, or dependent upon, the epicycloidal engine; but by the use of proper templates it will make cutters for any desired form of tooth; and by its aid exact duplicates may be made in any numbers with the greatest facility.

"It forms no part of our plan to represent as perfect that which is not so, and there are one or two facts, which at first thought might seem serious objections to the adoption of the epicycloidal system. These are:

"1. It is physically impossible to mill out a *concave* cycloid, by any means whatever, because at the pitch line its radius of curvature is zero, and a milling cutter must have a sensible diameter.

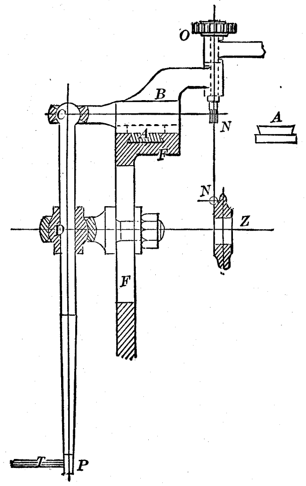

Fig. 109.

"2. It is impossible to mill out even a *convex* cycloid or epicycloid, by the means and in the manner above described.

"This is on account of a hitherto unnoticed peculiarity of the curve at a constant normal distance from the cycloid. In order to show this clearly, we have, in Fig. 110, enormously exaggerated the radius C D, of the milling cutter (M of Figs. 105 and 106). The outer curve H L, evidently, could be milled out by the cutter, whose centre travels in the cycloid C A; it resembles the cycloid somewhat in form, and presents no remarkable features. But the inner one is quite different; it starts at D, and at first goes down, *inside the circle whose radius is* C D, forms a cusp at E, then begins to rise, crossing this circle at G, and the base line at F. It will be seen, then, that if the centre of the cutter travel in the cycloid A C, its edge will cut away the part G E D, leaving the template of the form O G I. Now if a roller of the same radius C D, be rolled along this edge, its centre will travel in the cycloid from A, to the point P, where a normal from G, cuts it; then the roller will turn upon G as a fulcrum, and its centre will travel from P to C, in a circular arc whose radius G P = C D.

"That is to say even a roller of the same size as the original milling cutter, will not retrace completely the cycloidal path in which the cutter travelled.

"Now in making a rack template, the cutter, after reaching C, travels in the reversed cycloid C R, its left-hand edge, therefore, milling out a curve D K, similar to H L. This curve lies wholly *outside* the circle D I, and therefore cuts O G at a point between F and G, but very near to G. This point of intersection is marked S in Fig. 110, where the actual form of the template O S K is shown. The roller which is run along this template is *larger*, as has been explained, than the milling cutter. When the point of contact reaches S (which so nearly corresponds to G that they practically coincide), this roller cannot now swing about S through an angle so great as P G C of Fig. 110; because at the root D, the radius of curvature of

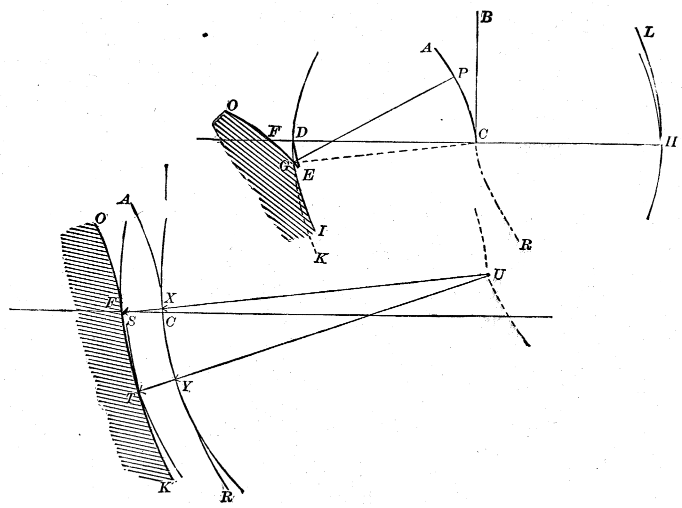

Fig. 110.

D K is only equal to that of the cutter, and G and S are so near the root that the curvature of S K, near the latter point, is greater than that of the roller. Consequently there must be some point U in the path of the centre of the roller, such, that when the centre reaches it, the circumference will pass through S, and be also tangent to S K. Let T be the point of tangency; draw S U and T U, cutting the cycloidal path A R in X and Y. Then, U Y being the radius of the new milling cutter (corresponding to N of Fig. 109), it is clear that in the outline of the gear cutter shaped by it, the circular arc X Y will be substituted for the true cycloid.

The System Practically Perfect.

"The above defects undeniably exist; now, what do they amount to? The diagram is drawn purposely with these sources of error greatly exaggerated, in order to make their nature apparent and their existence sensible. The diameters used in practice, as previously stated, are: describing circle, $7\frac{1}{2}$ inches; cutter for shaping template, $\frac{1}{8}$ of an inch; roller used against edge of template, $1\frac{1}{8}$ inches; cutter for shaping a 1-pitch gear cutter, 1 inch.

"With these data the writer has found that the *total length* of the arc X Y of Fig. 110, which appears instead of the cycloid in the outline of a cutter for a 1-pitch rack, is less than 0·0175 inch; the real *deviation* from the true form, obviously, must be much less than that. It need hardly be stated that the effect upon the velocity ratio of an error so minute, and in that part of the contour, is so extremely small as to defy detection. And the best proof of the practical perfection of this system of making epicycloidal teeth

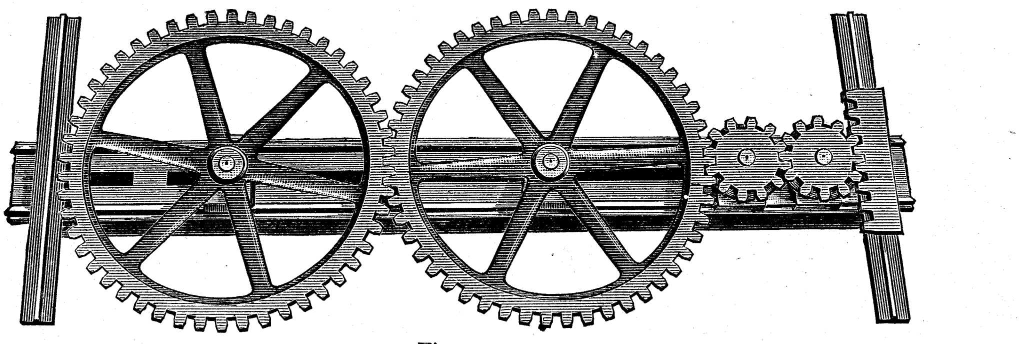

Fig. 111.

is found in the smoothness and precision with which the wheels run; a set of them is shown in gear in Fig. 111, the rack gearing as accurately with the largest as with the smallest. To which is to be added, finally, that objection taken, on whatever grounds, to the epicycloidal form of tooth, has no bearing upon the method above described of producing duplicate cutters for teeth of any form, which the pantagraphic engine will make with the same facility and exactness, if furnished with the proper templates.

"The front faces of the teeth of rotary cutters for gear-cutting are usually radial lines, and are ground square across so as to stand parallel with the axis of the cutter driving spindle, so that to whatever depth the cutter may have entered the wheel, the whole of the cutting edge within the wheel will meet the cut simultaneously. If this is not the case the pressure of the cut will spring the cutter, and also the arbor driving it, to one side. Suppose, for example, that the tooth faces not being square across, one side of the teeth meets the work first, then there will be as each tooth meets its cut an endeavour to crowd away from the cut until such time as the other side of the tooth also takes its cut."

It is obvious that rotating cutters of this class cannot be used to cut teeth having the width of the space wider below than it is at the pitch line. Hence, if such cutters are required to be used upon epicycloidal teeth, the curves to be theoretically correct must be such as are due to a generating circle that will give at least parallel flanks. From this it becomes apparent that involute teeth being always thicker at the root than at the pitch line, and the spaces being, therefore, narrower at the root, may be cut with these cutters, no matter what the diameter of the base circle of the involute.

To produce with revolving cutters teeth of absolutely correct theoretical curvature of face and flank, it is essential that the cutter teeth be made of the exact curvature due to the diameter of pitch circle and generating circle of the wheel to be cut; while to produce a tooth thickness and space width, also theoretically correct, the thickness of the cutter must also be made to exactly answer the requirements of the particular wheel to be cut; hence, for every different number of teeth in wheels of an equal pitch a separate cutter is necessary if theoretical correctness is to be attained.

This requirement of curvature is necessary because it has been shown that the curvatures of the epicycloid and hypocycloid, as also of the involute, vary with every different diameter of base circle, even though, in the case of epicycloidal teeth, the diameter of the generating circle remain the same. The requirement of thickness is necessary because the difference between the arc and the chord pitch is greater in proportion as the diameter of the base or pitch circle is decreased.

But the difference in the curvature on the short portions of the curves used for the teeth of fine pitches (and therefore of but little height) due to a slight variation in the diameter of the base circle is so minute, that it is found in practice that no sensible error is produced if a cutter be used within certain limits upon wheels having a different number of teeth than that for which the cutter is theoretically correct.

The range of these limits, however, must (to avoid sensible error) be more confined as the diameter of the base circle (or what is the same thing, the number of the teeth in the wheel) is decreased, because the error of curvature referred to increases as the diameters of either the base or the generating circles decrease. Thus the difference in the curve struck on a base circle of 20 inches diameter, and one of 40 inches diameter, using the same diameter of generating circle, would be very much less than that between the curves produced by the same diameter of generating circle on base circles respectively 10 and 5 inches diameter.

For these reasons the cutters are limited to fewer wheels according as the number of teeth decreases, or, per contra, are allowed to be used over a greater range of wheels as the number of teeth in the wheels is increased.

Thus in the Brown and Sharpe system for involute teeth there are 8 cutters numbered numerically (for convenience in ordering) from 1 to 8, and in the following table the range of the respective cutters is shown, and the number of teeth for which the cutter is theoretically correct is also given.

BROWN AND SHARPE SYSTEM.

No. of cutter.	Involute teeth.	Teeth.
1	Used upon all wheels having from 135 teeth to a rack correct for	200
2	,, ,, ,, ,, ,, 55 ,, to 134 teeth,	68
3	,, ,, ,, ,, ,, 35 ,, to 54 ,,	40
4	,, ,, ,, ,, ,, 26 ,, to 34 ,,	29
5	,, ,, ,, ,, ,, 21 ,, to 25 ,,	22
6	,, ,, ,, ,, ,, 17 ,, to 20 ,,	18
7	,, ,, ,, ,, ,, 14 ,, to 16 ,,	16
8	,, ,, ,, ,, ,, 12 ,, to 14 ,,	13

Suppose that it was required that of a pair of wheels one make twice the revolutions of the other; then, knowing the particular number of teeth for which the cutters are made correct, we may obtain the nearest theoretically true results as follows: If we select cutters Nos. 8 and 4 and cut wheels having respectively 13 and 26 teeth, the 13 wheel will be theoretically correct, and the 26 will contain the minute error due to the fact that the cutter is used upon a wheel having three less teeth than the number it is theoretically correct for. But we may select the cutters that are correct for 16 and 29 teeth respectively, the 16th tooth being theoretically correct, and the 29th cutter (or cutter No. 4 in the table) being used to cut 32 teeth, this wheel will contain the error due to cutting 3 more teeth than the cutter was made correct for. This will be nearer correct, because the error is in a larger wheel, and, therefore, less in actual amount. The pitch of teeth may be selected so that with the given number of teeth the diameters of the wheels will be that required.

We may now examine the effect of the variation of curvature in combination with that of the thickness, upon a wheel having less and upon one having more teeth than the number in the wheel for which the cutter is correct.

First, then, suppose a cutter to be used upon a wheel having less teeth and it will cut the spaces too wide, because of the variation of thickness, and the curves too straight or insufficiently curved because of the error of curvature. Upon a wheel having more teeth it will cut the spaces too narrow, and the curvature of the teeth too great; but, as before stated, the number of wheels assigned to each cutter may be so apportioned that the error will be confined to practically unappreciable limits.

If, however, the teeth are epicycloidal, it is apparent that the spaces of one wheel must be wide enough to admit the teeth of the other to a depth sufficient to permit the pitch lines to coincide on the line of centres; hence it is necessary in small diameters, in which there is a sensible difference between the arc and the chord pitches, to confine the use of a cutter to the special wheel for which it is designed, that is, having the same number of teeth as the cutter is designed for.

Thus the Pratt and Whitney arrangement of cutters for epicycloidal teeth is as follows:—

PRATT AND WHITNEY SYSTEM.

EPICYCLOIDAL TEETH.

[All wheels having from 12 to 21 teeth have a special cutter for each number of teeth.]*

Cutter correct for No. of teeth.	
23.	Used on wheels having from 22 to 24 teeth.
25	,, ,, ,, ,, 25 to 26 ,,
27	,, ,, ,, ,, 26 to 29 ,,
30	,, ,, ,, ,, 29 to 32 ,,
34	,, ,, ,, ,, 32 to 36 ,,
38	,, ,, ,, ,, 36 to 40 ,,
43	,, ,, ,, ,, 40 to 46 ,,
50	,, ,, ,, ,, 46 to 55 ,,
60	,, ,, ,, ,, 55 to 67 ,,
76	,, ,, ,, ,, 67 to 87 ,,
100	,, ,, ,, ,, 87 to 123 ,,
150	,, ,, ,, ,, 123 to 200 ,,
300	,, ,, ,, ,, 200 to 600 ,,
Rack	,, ,, ,, ,, 600 to rack.

Here it will be observed that by a judicious selection of pitch and cutters, almost theoretically perfect results may be obtained

* For wheels having less than 12 teeth the Pratt and Whitney Co. use involute cutters.

for almost any conditions, while at the same time the cutters are so numerous that there is no necessity for making any selection with a view to taking into consideration for what particular number of teeth the cutter is made correct.

For epicycloidal cutters made on the Brown and Sharpe system so as to enable the grinding of the face of the tooth to sharpen it, the Brown and Sharpe company make a separate cutter for wheels from 12 to 20 teeth, as is shown in the accompanying table, in which the cutters are for convenience of designation denoted by an alphabetical letter.

24 CUTTERS IN EACH SET.

Letter A	cuts	12 teeth.	Letter M	cuts 27 to 29 teeth.
B	,,	13 ,,	N	,, 30 ,, 33 ,,
C	,,	14 ,,	O	,, 34 ,, 37 ,,
D	,,	15 ,,	P	,, 38 ,, 42 ,,
E	,,	16 ,,	Q	,, 43 ,, 49 ,,
F	,,	17 ,,	R	,, 50 ,, 59 ,,
G	,,	18 ,,	S	,, 60 ,, 74 ,,
H	,,	19 ,,	T	,, 75 ,, 99 ,,
I	,,	20 ,,	U	,, 100 ,, 149 ,,
J	,,	21 to 22 ,,	V	,, 150 ,, 249 ,,
K	,,	23 ,, 24 ,,	W	,, 250 ,, Rack.
L	,,	25 ,, 26 ,,	X	,, Rack.

In these cutters a shoulder having no clearance is placed on each side of the cutter, so that when the cutter has entered the wheel until the shoulder meets the circumference of the wheel, the tooth is of the correct depth to make the pitch circles coincide.

In both the Brown and Sharpe and Pratt and Whitney systems, no side clearance is given other than that quite sufficient to prevent the teeth of one wheel from jambing into the spaces of the other. Pratt and Whitney allow $\frac{1}{8}$ of the pitch for top and bottom clearance, while Brown and Sharpe allow $\frac{1}{10}$ of the thickness of the tooth for top and bottom clearance.

It may be explained now, why the thickness of the cutter if employed upon a wheel having more teeth than the cutter is correct for, interferes with theoretrical exactitude.

First, then, with regard to the thickness of tooth and width of space. Suppose, then, Fig. 112 to represent a section of a wheel having 12 teeth, then the pitch circle of the cutter will be represented by line A, and there will be the same difference between

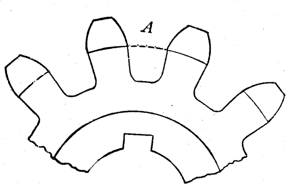

Fig. 112.

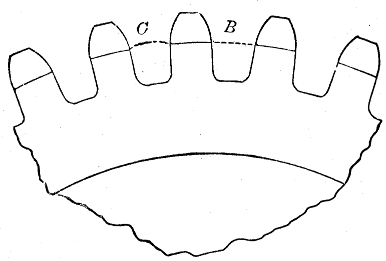

Fig. 113.

the arc and chord pitch on the cutter as there is on the wheel; but suppose that this same cutter be used on a wheel having 24 teeth, as in Fig. 113, then the pitch circle on the cutter will be more curved than that on the wheel as denoted at C, and there will be more difference between the arc and chord pitches on the cutter than there is on the wheel, and as a result the cutter will cut a groove too narrow.

The amount of error thus induced diminishes as the diameter of the pitch circle of the cutter is increased.

But to illustrate the amount. Suppose that a cutter is made to be theoretically correct in thickness at the pitch line for a wheel to contain 12 teeth, and having a pitch circle diameter of 8 inches, then we have

3·1416 = ratio of circumference to diameter.
8 = diameter.
Number of teeth = 12)25·1328 = circumference.
2·0944 = arc pitch of wheel.

If now we subtract the chord pitch from the arc pitch, we shall obtain the difference between the arc and the chord pitches of the wheel; here

2·0944 = arc pitch.
2·0706 = chord pitch.
·0238 = difference between the arc and the chord pitch.

Now suppose this cutter to be used upon a wheel having the same pitch, but containing 18 teeth; then we have

2·0944 = arc pitch.
2·0836 = chord pitch.
·0108 = difference between the arc and the chord pitch.

Then

·0238 = difference on wheel with 12 teeth.
·0108 = ,, ,, ,, 18 ,,
·0130 = variation between the differences.

And the thickness of the tooth equalling the width of the space, it becomes obvious that the thickness of the cutter at the pitch line being correct for the 12 teeth, is one half of ·013 of an inch too thin for the 18 teeth, making the spaces too narrow and the teeth too thick by that amount.

Now let us suppose that a cutter is made correct for a wheel having 96 teeth of 2·0944 arc pitch, and that it be used upon a wheel having 144 teeth. The proportion of the wheels one to the other remains as before (for 97 bears the proportion to 144 as 12 does to 18).

Then we have for the 96 teeth

2·0944 = arc pitch.
2·0934 = chord pitch.
·0010 = difference.

For the 144 teeth we have

2·0944 = arc pitch.
2·0937 = chord pitch.
·0007 = difference.

We find, then, that the variation decreases as the size of the wheels increases, and is so small as to be of no practical consequence.

If our examples were to be put into practice, and it were actually required to make one cutter serve for wheels having, say, from 12 to 18 teeth, a greater degree of correctness would be obtained if the cutter were made to some other wheel than the smallest. But it should be made for a wheel having less than the mean diameter (within the range of 12 and 18), that is, having less than 15 teeth; because the difference between the arc and chord pitch increases as the diameter of the pitch circle increases, as already shown.

A rule for calculating the number of wheels to be cut by each cutter when the number of cutters in the set and the number of teeth in the smallest and largest wheel in the train are given is as follows:—

Rule.—Multiply the number of teeth in the smallest wheel of the train by the number of cutters it is proposed to have in the set, and divide the amount so obtained by a sum obtained as follows:—

From the number of cutters in the set subtract the number of the cutter, and to the remainder add the sum obtained by multiplying the number of the teeth in the smallest wheel of the set or train by the number of the cutter and dividing the product by the number of teeth in the largest wheel of the set or train.

Example.—I require to find how many wheels each cutter should cut, there being 8 cutters and the smallest wheel having 12 teeth, while the largest has 300.

Number of teeth in smallest wheel.		Number of cutters in the set.		
12	×	8	=	96

Then

Number of cutters in set.		Number of cutter.		
8	−	7	=	1

Then

Number of teeth in smallest wheel.		The number of the cutter.		The number of the teeth in largest wheel.
12	×	8	÷	300

12
8
300) 960 (0·32
900
600
600

Now add the 1 to the ·32 and we have 1·32, which we must divide into the 96 first obtained.

Thus

1·32) 96·00 (72
924
360
264
96

Hence No. 8 cutter may be used for all wheels that have between 72 teeth and 300 teeth.

To find the range of wheels to be cut by the next cutter, which we will call No. 7, proceed again as before, but using 7 instead of 8 as the number of the cutter.

Thus

Number of teeth in smallest wheel.		Number of cutters in the set.		
12	×	8	=	96

Then

Number of cutters in the set.		Number of cutters.		
8	—	6	=	2

And

Number of teeth in smallest wheel.		The number of the cutter.		The number of teeth in the largest wheel.
12	×	8	÷	300

Here

12
8
300) 960 (0·32
900
600
600

Add the 2 to the ·32 and we have 2·32 to divide into the 96.

Thus

2·32) 96·00 (41
928
320
232
88

Hence this cutter will cut all wheels having not less than the 41 teeth, and up to the 72 teeth where the other cutter begins. For the range of the next cutter proceed the same, using 6 as the number of the cutter, and so on.

By this rule we obtain the lowest number of teeth in a wheel for which the cutter should be used, and it follows that its range will continue upwards to the smallest wheel cut by the cutter above it.

Having by this means found the range of wheels for each cutter, it remains to find for what particular number of teeth within that range the cutter teeth should be made correct, in order to have whatever error there may be equal in amount on the largest and smallest wheel of its range. This is done by using precisely the same rule, but supposing there to be twice as many cutters as there actually are, and then taking the intermediate numbers as those to be used.

Applying this plan to the first of the two previous examples we have—

Number of teeth in the smallest wheel.		Number of cutters in the set.		
12	×	16	=	192

Then

Number of cutters in the set.		Number of the cutter.		
16	—	15	=	1

And

Number of teeth in smallest wheel.		The number of the cutter.		The number of teeth in the largest wheel.
12	×	15	÷	300

12
15
60
12
300) 180·0 (0·6
1800

Then add the 1 to the ·6 = 1·6, and this divided into 192 = 120.

By continuing this process for each of the 16 cutters we obtain the following table:—

Number of Cutter.	Number of Teeth.	Number of Cutter.	Number of Teeth.
1	12	9	26
*2	13	*10	30
3	14	11	35
*4	15	*12	42
5	17	13	54
*6	18	*14	75
7	20·61	15	120
*8	23	*16	300

Suppose now we take for our 8 cutters those marked by an asterisk, and use cutter 2 for all wheels having either 12, 13, or 14 teeth, then the next cutter would be that numbered 4, cutting 14, 15, or 16 toothed wheels, and so on.

A similar table in which 8 cutters are required, but 16 are used in the calculation, the largest wheel having 200 teeth in the set, is given below.

Number of Cutter.	Number of Teeth.	Number of Cutter.	Number of Teeth.
1	12·7	9	26·5
2	13·5	10	29
3	14·5	11	35
4	15·6	12	40.6
5	16·9	13	52·9
6	18	14	67·6
7	21	15	101
8	23·5	16	200

To assist in the selections as to what wheels in a given set the determined number of cutters should be made correct for, so as to obtain the least limit of error, Professor Willis has calculated the following table, by means of which cutters may be selected that will give the same difference of form between any two consecutive numbers, and this table he terms the table of equidistant value of cutters.

TABLE OF EQUIDISTANT VALUE OF CUTTERS.

Number of Teeth.

Rack— 300, 150, 100, 76, 60, 50, 43, 38, 34, 30, 27, 25, 23, 21, 20, 19, 17, 16, 15, 14, 13, 12.

The method of using the table is as follows:—Suppose it is required to make a set of wheels, the smallest of which is to contain 50 teeth and the largest 150, and it is determined to use but one cutter, then that cutter should be made correct for a wheel containing 76; because in the table 76 is midway between 50 and 150.

But suppose it were determined to employ two cutters, then one of them should be made correct for a wheel having 60 teeth, and used on all the wheels having between 50 and 76 teeth, while the other should be made correct for a wheel containing 100 teeth, and used on all wheels containing between 76 and 150 teeth.

In the following table, also arranged by Professor Willis, the most desirable selection of cutters for different circumstances is given, it being supposed that the set of wheels contains from 12 teeth to a rack.

Number of cutters in the set.	Number of Teeth in Wheel for which the Cutter is to be made correct.																							
2	50	16																						
3	75	25	15																					
4	100	34	20	14																				
6	150	50	30	21	16	13																		
8	200	67	40	29	22	18	15	13																
10	200	77	50	35	27	22	19	16	14	13														
12	300	100	60	43	34	27	23	20	17	15	14	13												
18	300	150	100	70	50	40	30	26	24	22	20	18	16	15	14	13	12							
24	Rack	300	150	100	76	60	50	43	38	34	30	27	25	23	21	20	19	18	17	16	15	14	13	12

Suppose now we take the cutters, of a given pitch, necessary to cut all the wheels from 12 teeth to a rack, then the thickness of the teeth at the pitch line will for the purposes of designation be the thickness of the teeth of all the wheels, which thickness may be a certain proportion of the pitch.

But in involute teeth while the depth of tooth on the cutter may be taken as the standard for all the wheels in the range, and the actual depth for the wheel for which the cutter is correct, yet the depth of the teeth in the other wheels in the range may be varied sufficiently on each wheel to make the thickness of the teeth equal the width of the spaces (notwithstanding the variation between the arc and chord pitches), so that by a variation in the tooth depth the error induced by that variation may be corrected. The following table gives the proportions in the Brown and Sharpe system.

Arc Pitch.	Depth of Tooth.	Depth in terms of the arc pitch.
inches.	inches.	inches.
1·570	1·078	·686
1·394	·958	·687
1·256	·863	·686
1·140	·784	·697
1·046	·719	·687
·896	·616	·686
·786	·539	·685
·628	·431	·686
·524	·359	·685
·448	·307	·685
·392	·270	·686
·350	·240	·686
·314	·216	·687

To avoid the trouble of measuring, and to assist in obtaining accuracy of depth, a gauge is employed to mark on the wheel face a line denoting the depth to which the cutter should be entered.

Suppose now that it be required to make a set of cutters for a certain range of wheels, and it be determined that the cutters be so constructed that the greatest permissible amount of error in any wheel of the set be $\frac{1}{100}$ inch. Then the curves for the smallest wheel, and those for the largest in the set, and the amount of difference between them ascertained, and assuming this difference to amount to $\frac{1}{16}$ inch, which is about $\frac{6}{100}$, then it is evident that 6 cutters must be employed for the set.

It has been shown that on bevel-wheels the tooth curves vary at every point in the tooth breadth; hence it is obvious that the cutter being of a fixed curve will make the tooth to that curve. Again, the thickness of the teeth and breadth of the spaces vary at every point in the breadth, while with a cutter of fixed thickness the space cut will be parallel from end to end. To overcome these difficulties it is usual to give to the cutter a curve corresponding to the curve required at the middle of the wheel face and a thickness equal to the required width of space at its smallest end, which is at the smallest face diameter.

The cutter thus formed produces, when passed through the wheel once, and to the required depth, a tooth of one curve from end to end, having its thickness and width of space correct at the smaller face diameter only, the teeth being too thick and the spaces too narrow as the outer diameter of the wheel is approached. But the position and line of traverse of the cutter may be altered so as to take a second cut, widening the space and reducing the tooth thickness at the outer diameter.

By moving the cutter's position two or three times the points of contact between the teeth may be made to occur at two or three points across the breadth of the teeth and their points of contact; the wear will soon spread out so that the teeth bear all the way across.

Another plan is to employ two or three cutters, one having the correct curve for the inner diameter, and of the correct thickness for that diameter, another having the correct curve for the pitch circle, and another having the correct curve at the largest diameter of the teeth.

The thickness of the first and second cutters must not exceed the required width of space at the small end, while that for the third may be the same as the others, or equal to the thickness of the smallest space breadth that it will encounter in its traverse along the teeth.

The second cutter must be so set that it will leave the inner end of the teeth intact, but cut the space to the required width in the middle of the wheel face. The third cutter must be so set as to leave the middle of the tooth breadth intact, and cut the teeth to the required thickness at the outer or largest diameter.

Cutting Worm-Wheels.

The most correct method of cutting the teeth of a worm-wheel is by means of a worm-cutter, which is a worm of the pitch and form of tooth that the working worm is intended to be, but of hardened steel, and having grooves cut lengthways of the worm so as to provide cutting edges similar to those on the cutter shown in Fig. 107.

The wheel is mounted on an arbor or mandril free to rotate on its axis and at a right angle to the cutter worm, which is rotated and brought to bear upon the perimeter of the worm-wheel in the same manner as the working worm-wheel when in action. The worm-cutter will thus cut out the spaces in the wheel, and must therefore be of a thickness equal to those spaces. The cutter worm acting as a screw causes the worm-wheel to rotate upon its axis, and therefore to feed to the cutter.

In wheels of fine pitch and small diameter this mode of procedure is a simple matter, especially if the form of tooth be such that it is thicker, as the root of the tooth is approached from the pitch line, because in that case the cutter worm may be entered a part of the depth in the worm-wheel and a cut be taken around the wheel. The cutter may then be moved farther into the wheel and a second cut taken around the wheel, so that by continuing the process until the pitch line of the cutter worm coincides with that of the worm-cutter, the worm-wheel may be cut with a number of light cuts, instead of at one heavy cut.

But in the case of large wheels the strain due to such a long line of cutting edge as is possessed by the cutter worm-teeth springs or bends the worm-wheel, and on account of the circular form of the breadth of the teeth this bending or spring causes that part of the tooth arc above the centre of the wheel thickness to lock against the cutter.

To prevent this, several means may be employed. Thus the grooves forming the cutting edges of the worm-cutter may wind spirally along instead of being parallel to the axis of the cutter.

The distance apart of these grooves may be greater than the breadth of tooth a width of worm-wheel face, in which case the cutting edge of one tooth only will meet the work at one time. In addition to this two stationary supports may be placed beneath the worm-wheel (one on each side of the cutter). But on coarse pitches with their corresponding depth of tooth, the difficulty presents itself, that the arbor driving the worm-cutter will spring, causing the cutter to lift and lock as before; hence it is necessary to operate on part of the space at a time, and shape it out to so nearly the correct form that the finishing cut may be a very light one indeed, in which case the worm-cutter will answer for the final cut.

The removal of the surplus metal preparatory to the introduction of the worm-cutter to finish, may be made with a cutter-worm that will cut out a narrow groove being of the thickness equal to the bottom of the tooth space and cutting on its circumference only. This cutter may be fed into the wheel to the permissible depth of cut, and after the cut is taken all around the wheel, it may be entered deeper and a second cut taken, and so on until it has entered the wheel to the necessary depth of tooth. A second cutter-worm may then be used, it being so shaped as to cut the face curve only of the teeth. A third may cut the flank curve only, and finally a worm-cutter of correct form may take a finishing cut over both the faces and the flanks. In this manner teeth of any pitch and depth may be cut. Another method is to use a revolving cutter such as shown in Fig. 107, and to set it at the required angle to the wheel, and then take a succession of cuts around the wheel, the first cut forming a certain part of the tooth depth, the second increasing this depth, and so on until the final cut forms the tooth to the requisite depth. In this case the cutter operates on each space separately, or on one space only at a time, and the angle at which to set the cutter may be obtained as follows in Fig. 114. Let the length of the line A A equal the diameter of the worm at the pitch circle, and B B (a line at a right angle to A A) represent the axial line of the worm. Let the distance C equal the pitch of the teeth, and the angle of the

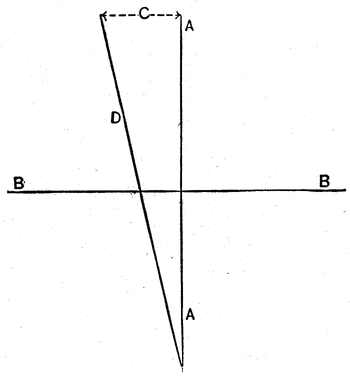

Fig. 114.

line D with A A or B B according to circumstances, will be that to which the cutter must be set with reference to the tooth.

If then a piece of sheet metal be cut to the lines A, D, and the cutter so set that with the edge D of the piece held against the side face of the cutter (which must be flat or straight across), the edge A will stand truly vertical, and the cutter will be at the correct angle supposing the wheel to be horizontal.

In making patterns wherefrom gear-wheels may be cast in a mould, the true curves are frequently represented by arcs of circles struck from the requisite centres and of the most desirable

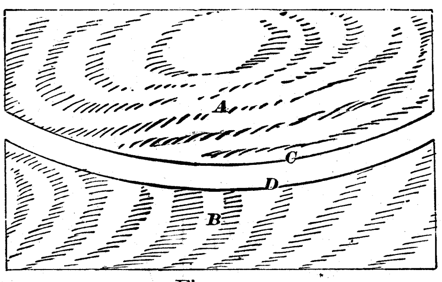

Fig. 115.

radius with compasses, and this will be treated after explaining the pattern maker's method of obtaining true curves by rolling segments by hand. If, then, the wheels are of small diameter, as say, less than 12 inches in diameter, and precision is required, it is best to turn in the lathe wooden disks representing in their diameters the base and generating circles. But otherwise, wooden segments to answer the same purpose may be made as from a piece of soft wood, such as pine or cedar, about three-

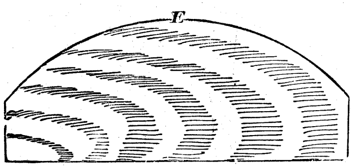

Fig. 116.

eighths inch thick, make two pieces A and B, in Fig. 115, and trim the edges C and D to the circle of the pitch line of the required wheel. If the diameter of the pitch circle is marked on a drawing, the pieces may be laid on the drawing and sighted for curvature by the eye. In the absence of a drawing, strike a portion of the pitch circle with a pair of sharp-pointed compasses on a piece of zinc, which will show a very fine line quite clear. After the pieces are filed to the circle, try them together by laying them flat on a piece of board, bringing the curves in contact and sweeping A against B, and the places of contact will plainly show, and may be filed until continuous contact along the curves is obtained. Take another similar piece of wood and form it as shown in Fig. 116, the edge E representing a portion of the rolling circle. In preparing these segments it is an excellent plan to file the convex edges, as shown in Fig. 117, in which P is a piece of iron or wood

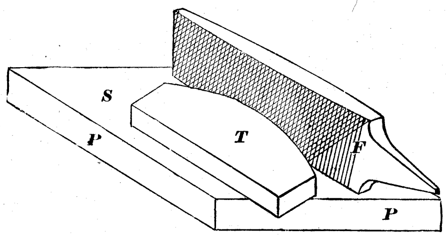

Fig. 117.

having its surface S trued; F is a file held firmly to S, while its surface stands vertical, and T is the template laid flat on S, while swept against the file. This insures that the edge shall be square across or at least at the same angle all around, which is all that is absolutely necessary. It is better, however, that the edges be square. So likewise in fitting A and B (Fig. 115) together, they should be laid flat on a piece of board. This will insure that they will have contact clear across the edge, which will give more grip and make slip less likely when using the segments. Now take a piece of stiff drawing paper or of sheet

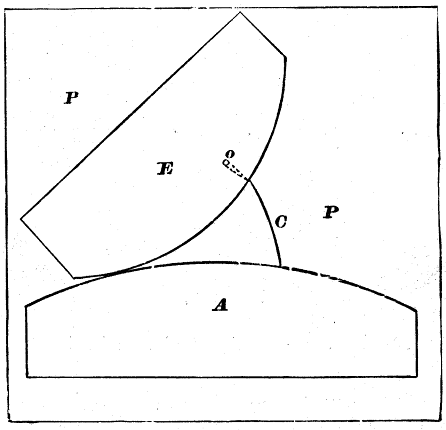

Fig. 118.

zinc, lay segment A upon it, and mark a line coincident with the curved edge. Place the segment representing the generating circle flat on the paper or zinc, hold its edge against segment A, and roll it around a sufficient distance to give as much of the curve as may be required; the operation being illustrated in Fig. 118, in which A is the segment representing the pitch or base circle, E is the segment representing the generating circle, P is the paper, C the curve struck by the tracing point or pencil O.

This tracing point should be, if paper be used to trace on, a piece of the *hardest* pencil obtainable, and should be filed so

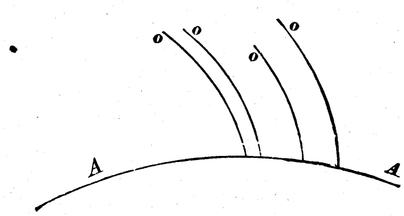

Fig. 119.

that its edge, if flat, shall stand as near as may be in the line of motion when rolled, thus marking a fine line. If sheet zinc be used instead of paper a needle makes an excellent tracing point. Several of the curves, C, should be struck, moving the position of the generating segment a little each time.

On removing the segments from the paper, there will appear the lines shown in Fig. 119; A representing the pitch circle, and O O O the curves struck by the tracing point.

Cut out a piece of sheet zinc so that its edge will coincide with the curve A and the epicycloid O, trying it with all four of the epicycloids to see that no slip has occurred when marking them; shape a template as shown in Fig. 120. Cutting the notches at *a* *b*, acts to let the file clear well when filing the template, and to allow the scriber to go clear into the corner. Now take the segment A in Fig. 118, and use it as a guide to carry the pitch circle across the template as at P, in Fig. 120. A zinc template for the flank curve is made after the same manner, using the rolling segment in conjunction with the segment B in Fig. 115.

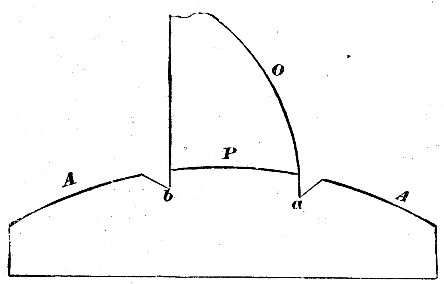

Fig. 120.

But the form of template for the flank should be such as shown in Fig. 121, the curve P representing, and being of the same radius as the pitch circle, and the curve F being that of the hypocycloid. Both these templates are set to the pitch circles and to coincide with the marks made on the wheel teeth to denote the thickness, and with a hardened steel point a line is traced on the tooth showing the correct curve for the same.

An experienced hand will find no difficulty in producing true templates by this method, but to avoid all possibility of the segments slipping on coarse pitches, and with large segments,

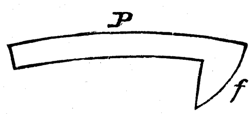

Fig. 121.

the segments may be connected, as shown in Fig. 122, in which O represents a strip of steel fastened at one end into one segment and at the other end to the other segment. Sometimes, indeed, where great accuracy is requisite, two pieces of steel are thus employed, the second one being shown at P P, in the figure. The surfaces of these pieces should exactly coincide with the edge of the segments.

The curve templates thus produced being shaped to apply to the pitch circle may be correctly applied to that circle independently of its concentricity to the wheel axis or of the points of the

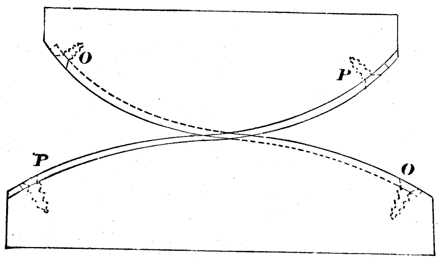

Fig. 122.

teeth, but if the points of the teeth are turned in the lathe so as to be true (that is, concentric to the wheel axis) the form of the template may be such as shown in Fig. 123, the radius of the arc A A equalling that of the addendum circle or circumference at the points of the teeth, and the width at B (the pitch circle) equaling the width of a space instead of the thickness of a tooth. The curves on each side of the template may in this case be filed for the full side of a tooth on each side of the template so that it will completely fill the finished space, or the sides of two contiguous teeth may be marked at one operation. This template may be set to the marks made on the teeth at the pitch circle to denote their requisite thickness, or for greater accuracy, a similar template made double so as to fill two finished tooth spaces may be employed, the advantage being that in this case the template

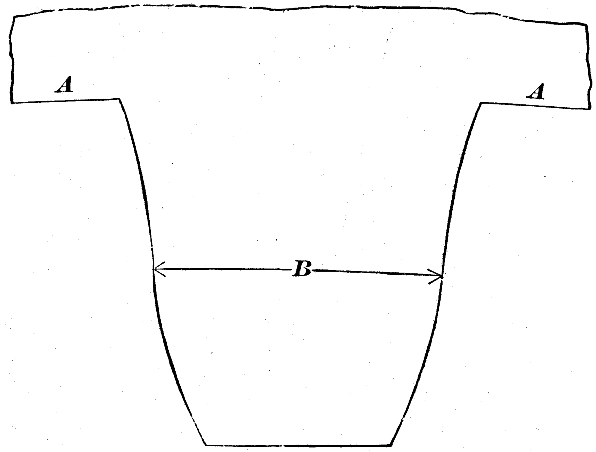

Fig. 123.

also serves to mark or test the thickness of the teeth. Since, however, a double template is difficult to make, a more simple method is to provide for the thickness of a tooth, the template

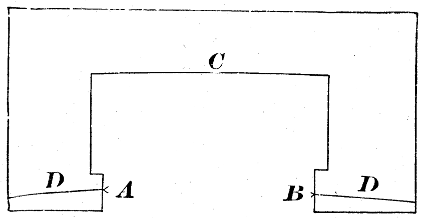

Fig. 124.

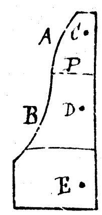

Fig. 125.

shown in Fig. 124, the width from A to B being either the thickness of tooth required or twice the thickness of a tooth plus the width of a space, so that it may be applied to the outsides of two

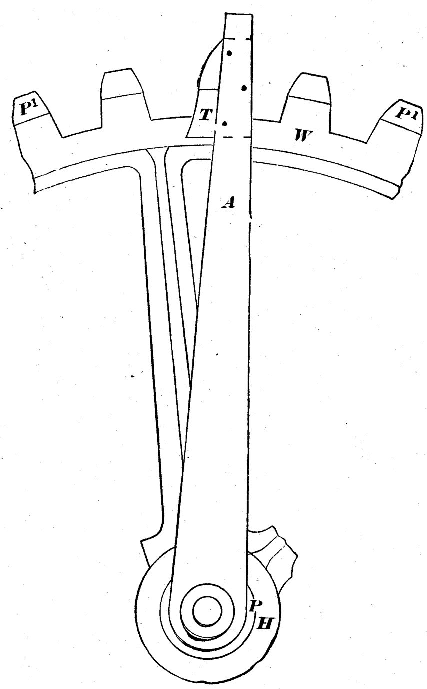

Fig. 126.

contiguous teeth. The arc C may be made both in its radius and distance from the pitch circle D D to equal that of the addendum circle, so as to serve as a gauge for the tooth points, if the latter are not turned true in the lathe, or to rest on the addendum circle (if the teeth points are turned true), and adjust the pitch circle D D to the pitch circle on the wheel.

The curves for the template must be very carefully filed to the lines produced by the rolling segments, because any error in the template is copied on every tooth marked from it. Furthermore, instead of drawing the pitch circle only, the addendum circle and circle for the roots of the teeth or spaces should also be drawn, so that the template may be first filed to them, and then adjusted to them while filing the edges to the curves.

Another form of template much used is shown in Fig. 125. The curves A and B are filed to the curve produced by rolling segments as before, and the holes C,D,E, are for fastening the template to an arm, such as shown in Fig. 126, which represents a section of a wheel W, with a plug P, fitting tightly into the hub H of the wheel. This plug carries at its centre a cylindrical pin on which pivots the arm A. The template T is fastened to the arm by screws, and set so that its pitch circle coincides with the pitch circle P on the wheel, when the curves for one side of all the teeth may be marked. The template must then be turned over to mark the other side of the teeth.

The objection to this form of template is that the length of arc representing the pitch circle is too short, for it is absolutely essential that the pitch line on the template (or line representing the arc of the addendum if that be used) be greater than the width of a single tooth, because an error of the thickness of a line (in the thickness of a tooth), in the coincidence of the pitch line of the template with that of the tooth, would throw the tooth curves out to an extent altogether inadmissible where true work is essential.

To overcome this objection the template may be made to equal half the thickness of a tooth and its edge filed to represent a radial line on the wheel. But there are other objections, as, for example, that the template can only be applied to the wheel when adjusted on the arm shown in Fig. 126, unless, indeed, a radial

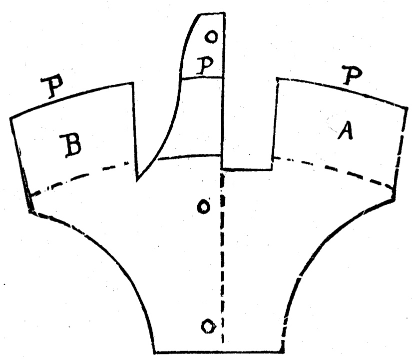

Fig. 127.

line be struck on every tooth of the wheel. Again, to produce the template a radial line representing the radius of the wheel must be produced, which is difficult where segments only are used to produce the curves. It is better, therefore, to form the template as shown in Fig. 127, the projections at A B having their edges filed to coincide with the pitch circle P, so that they may be applied to a length of one arc of pitch circle at least equal to the pitch of the teeth.

The templates for the tooth curves being obtained, the wheel must be divided off on the pitch circle for the thickness of the teeth and the width of the spaces, and the templates applied to the marks or points of division to serve as guides to mark the tooth curves. Since, however, as already stated, the tooth curves are as often struck by arcs of circles as by templates, the application of such arcs and their suitability may be discussed.

Marking the Curves by Hand.

In the employment of arcs of circles several methods of finding the necessary radius are found in practice.

In the best practice the true curve is marked by the rolling segments already described, and the compass points are set by trial to that radius which gives an arc nearest approaching to the true face and flank curves respectively. The degree of curve error thus induced is sufficient that the form of tooth produced cannot with propriety be termed epicycloidal teeth, except in the case of fine pitches in which the arc of a circle may be employed to so nearly approach the true curve as to be permissible as a

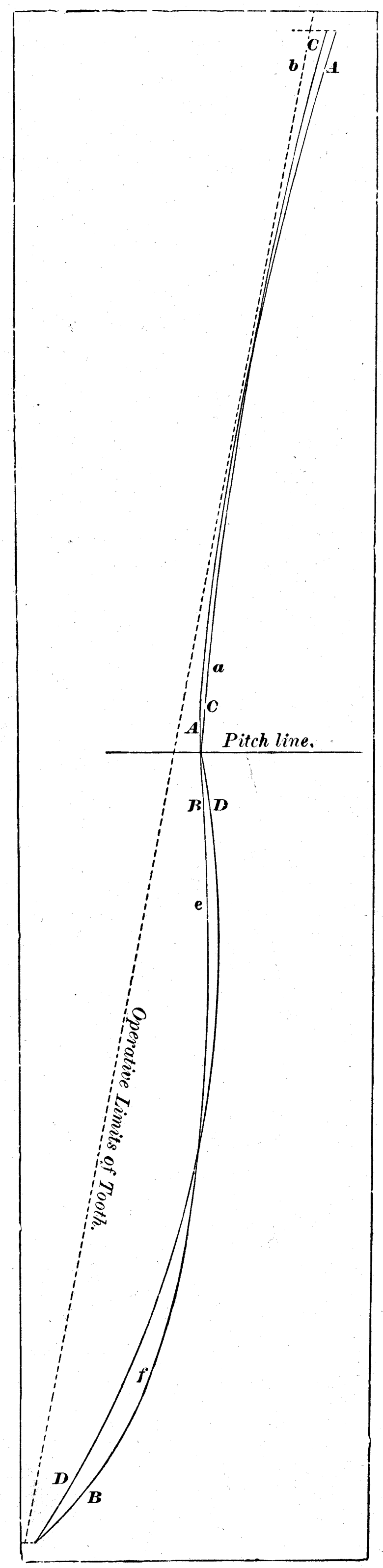

Fig. 128.

substitute. But in coarse pitches the error is of much importance. Thus in Fig. 128 is shown the curve of the *former* or *template* attachment used on the celebrated Corliss Bevel Gear Cutting Machine, to cut the teeth on the bevel-wheels employed upon the line shafting at the Centennial Exhibition. These gears, it may be remarked, were marvels of smooth and noiseless running, and attracted wide attention both at home and abroad. The engraving is made from a drawing marked direct from the *former* itself, and kindly furnished me by Mr. George H. Corliss. A A is the face and B B the flank of the tooth, C C is the arc of a circle

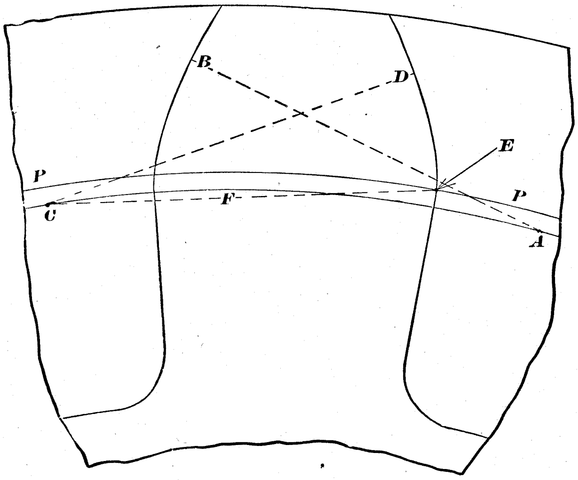

Fig. 129.

nearest approaching to the face curve, and D D the arc of a circle nearest approaching the flank curve. In the face curve, there are but two points where the circle coincides with the true curve, while in the flank there are three such points; a circle of smaller radius than C C would increase the error at *b*, but decrease it at *a*; one of a greater radius would decrease it at *b*, and increase it at *a*. Again, a circle larger in radius than D D would decrease the error at *e* and increase it at *f*; while one smaller would increase it at *e* and decrease it at *f*. Only the working part of the tooth is given in the illustration, and it will be noted that the error is greatest in the flank, although the circle has three points of coincidence.

In this case the depth of the *former* tooth is about three and three-quarter times greater than the depth of tooth cut on the bevel-wheels; hence, in the figure the actual error is magnified three and three-quarter times. It demonstrates, however, the impropriety of calling coarsely pitched teeth that are found by arcs of circles "epicycloidal" teeth.

When, however, the pitches of the teeth are fine as, say an inch or less, the coincidence of an arc of a circle with the true curve is sufficiently near for nearly all practical purposes, and in the case of cast gear the amount of variation in a pitch of 2 inches would be practically inappreciable.

To obtain the necessary set of the compasses to mark the curves, the following methods may be employed.

First by rolling the true curves with segments as already described, and the setting the compass points (by trial) to that radius which gives an arc nearest approaching the true curves. In this operation it is not found that the location for the centre from which the curve must be struck always falls on the pitch circle, and since that location will for every tooth curve lie at the same radius from the wheel centre it is obvious that after the proper location for one of the curves, as for the first tooth face or tooth flank as the case may be, is found, a circle may be struck denoting the radius of the location for all the teeth. In Fig. 129, for example, P P represents the pitch circle, A B the radius that will produce an arc nearest approaching the true curve produced by rolling segments, and A the location of the centre from which the face arc B should be struck. The point A being found by trial with the compasses applied to the curve B, the circle A C may be struck, and the location for the centres from which the face arcs of each tooth must be struck will also fall on this circle, and all that is necessary is to rest one point of the compasses on the side of the tooth as, say at E, and mark on the second circle A C the point C, which is the location wherefrom to mark the face arc D.

If the teeth flanks are not radial, the locations of the centre wherefrom to strike the flank curves are found in like manner by trial of the compasses with the true curves, and a third circle, as I in Fig. 130, is struck to intersect the first point found, as at G in the figure. Thus there will be upon the wheel face three circles, P P the pitch circle, J J wherefrom to mark the face curves, and I wherefrom to mark the flank curves.

When this method is pursued a little time may be saved, when dividing off the wheel, by dividing it into as many divisions as there are teeth in the wheel, and then find the locations for the curves as in Fig. 131, in which 1, 2, 3 are points of divisions on the pitch circle P P, while A, B, struck from point 2, are centres wherefrom to strike the arcs E, F; C, D, struck also from point 2 are centres wherefrom to strike the flank curves G, H.

It will be noted that all the points serving as centres for the face curves, in Fig. 130, fall within a space; hence if the teeth were

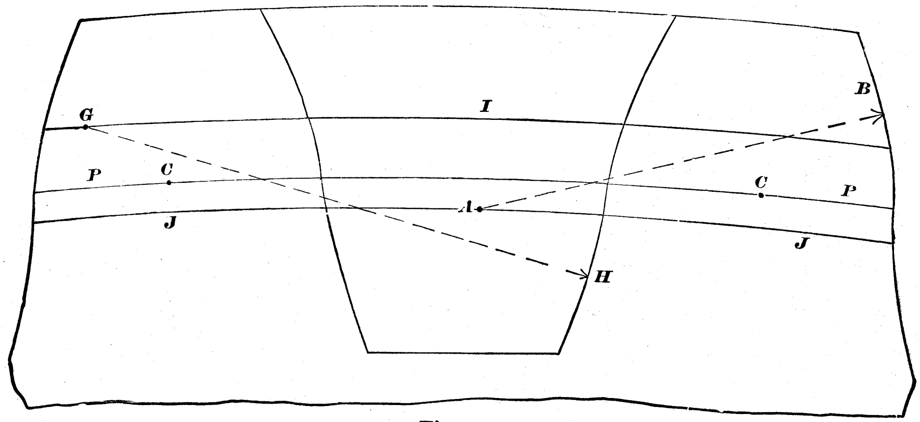

Fig. 130.

rudely cast in the wheel, and were to be subsequently cut or trimmed to the lines, some provision would have to be made to receive the compass points.

To obviate the necessity of finding the necessary radius from rolling segments various forms of construction are sometimes employed.

Thus Rankine gives that shown in Fig. 132, which is obtained

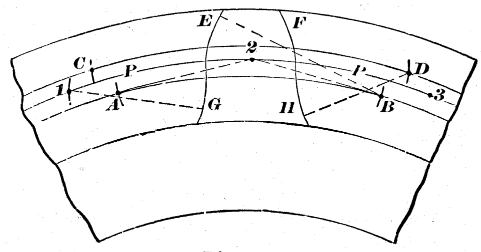

Fig. 131.

as follows. Draw the generating circle D, and A D the line of centres. From the point of contact at C, mark on circle D, a point distance from C one-half the amount of the pitch, as at P, and draw the line P C of indefinite length beyond C. Draw a line from P, passing through the line of centres at E, which is equidistant between C and A. Then multiply the length from P to C by the distance from A to D, and divide by the distance between D and E. Take the length and radius so found, and mark it upon P C, as at F, and the latter will be the location of centre for compasses to strike the face curve.

Another method of finding the face curve, with compasses, is as follows: In Fig. 133, let P P represent the pitch circle of the wheel to be marked, and B C the path of the centre of the generating or describing circle as it rolls outside of P P. Let the point B represent the centre of the generating circle when that circle is in contact with the pitch circle at A. Then from B, mark off on B C any number of equidistant points, as D, E, F, G, H, and from

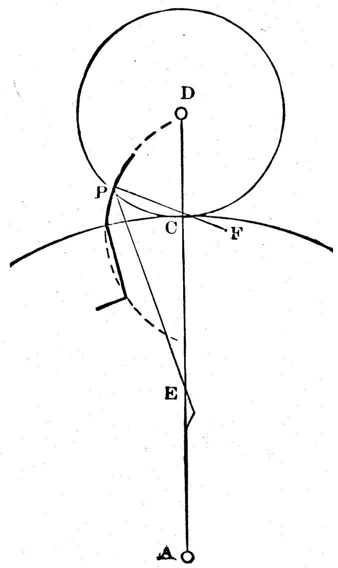

Fig. 132.

A, mark on the pitch circle, points of division, as 1, 2, 3, 4, 5, at the intersection of radial lines from D, E, F, G, and H. With the radius of the generating circle, that is, A B, from B, as a centre, mark the arc I, from D the arc J, from E the arc K, &c., to M, marking as many arcs as there are points of division on B C. With the compasses set to the radius of divisions 1, 2, step off on arc M the five divisions, N, O, S, T, V, and V will be a point in the epicycloidal curves. From point of division 4, step off on L four points of division, as *a*, *b*, *c*, *d*, and *d* will be another point in the epicycloidal curve. From point 3 set off three divisions on K, from point 2 two dimensions on L, and so on, and through the

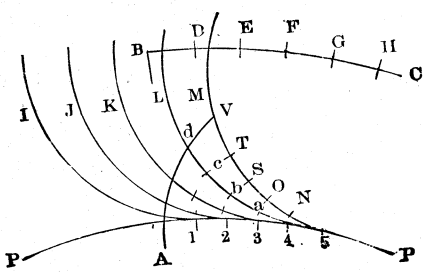

Fig. 133.

points so obtained, draw by hand or with a scroll the curve represented in the cut by curve A V.

Hypocycloids for the flanks of the teeth may be traced in a similar manner. Thus in Fig. 134 P P is the pitch circle, and B C the line of motion of the centre of the generating circle to be rolled within P P, and R a radial line. From 1 to 6 are points of equal division on the pitch circle, and D to I are arc locations for the centre of the generating circle. Starting from A, which represents the supposed location for the centre of the generating circle, the point of contact between the generating and base circles will be at B. Then from 1 to 6 are points of equal division on the pitch circle, and from D to I are the corresponding locations for the centres of the generating circle. From these centres the arcs J, K, L, M, N, O, are struck. From 6 mark the six points of division from *a* to *f*, and *f* is a point in the curve. Five divisions on N, four on M, and so on, give respectively points in the curve which is marked in the figure from A to *f*.

There is this, however, to be noted concerning the constructions of the last two figures. Since the circle described by the centre of the generating circle is of different arc or curve to that of the pitch circle, the chord of an arc having an equal length on each will be different. The amount is so small as to be practically correct. The direction of the error is to give to the curves a less curvature, as though they had been produced by a generating circle of larger diameter. Suppose, for example, that the difference between the arc N 5 (Fig. 133) and its chord is ·1, and that the difference between the arc 4 5, and its chord is ·01, then the error in one step is ·09, and, as the point V is formed in 5 steps, it will contain this error multiplied five times. Point *d* would contain it multiplied four times, because it has 4 steps, and so on.

The error will increase in proportion as the diameter of the generating is less than that of the pitch circle, and though in large wheels, working with large wheels (so that the difference between the radius of the generating circle and that of the smallest wheel is not excessive), it is so small as to be practically inappreciable, yet in small wheels, working with large ones, it may form a sensible error.

An instrument much employed in the best practice to find the radius which will strike an arc of a circle approximating the true

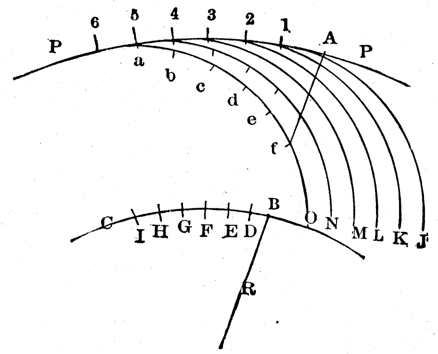

Fig. 134.

epicycloidal curve, *and for finding at the same time* the location of the centre wherefrom that curve should be struck, is found in the Willis' odontograph. This is, in reality, a scale of centres or radii for different and various diameters of wheels and generating circles. It consists of a scale, shown in Fig. 135, and is formed of a piece of sheet metal, one edge of which is marked or graduated in divisions of one-twentieth of an inch. The edge meeting the graduated edge at O is at angle of 75° to the graduated edge.

On one side of the odontograph is a table (as shown in the cut), for the flanks of the teeth, while on the other is the following table for the faces of the teeth:

TABLE SHOWING THE PLACE OF THE CENTRES UPON THE SCALE.

CENTRES FOR THE FACES OF THE TEETH.

Pitch in Inches and Parts.

No. of Teeth	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	3	$3\frac{1}{2}$
12	1	2	2	3	4	5	6	7	9	10	11	12	15	17
15	..	..	3	..	..	..	7	8	10	11	12	14	17	19
20	2	..	..	4	5	6	8	9	11	12	14	15	18	21
30	..	3	4	..	..	7	9	10	12	14	16	18	21	25
40	..	..	..	..	6	8	..	11	13	15	17	19	23	26
60	..	..	..	5	..	..	10	12	14	16	18	20	25	29
80	..	..	..	..	..	9	11	13	15	17	19	21	26	30
100	..	..	..	..	7	..	..	..	..	18	20	22	..	31
150	..	..	5	6	..	..	..	14	16	19	21	23	27	32
Rack.	..	4	..	..	..	10	12	15	17	20	22	25	30	34

The method of using the instrument is as follows: In Fig. 136, let C represent the centre, and P the pitch circle of a wheel to contain 30 teeth of 3 inch arc pitch. Draw the radial line L, meeting the pitch circle at A. From A mark on the pitch circle, as at B, a radius equal to the pitch of the teeth, and the thickness of the tooth as A k. Draw from B to C the radial line E. Then for the flanks place the slant edge of the odontograph coincident and parallel with E, and let its corners coincide with the pitch circle as shown. In the table headed *centres for the flanks of the teeth*, look down the column of 3 inch pitch, and opposite to the 30 in the column of numbers of teeth, will be found the number 49, which indicates that the centre from which to draw an arc for the flank is at 49 on the graduated edge of the odontograph, as denoted in the cut by r. Thus from r to the side k of the tooth is the radius for the compasses, and at r, or 49, is the location for the centre to strike the flank curve f. For the face curve set the slant edge of the odontograph coincident with the radial line L, and in the table of centres for the faces of teeth, look down the column of 3-inch pitch, and opposite to 30 in the number of teeth column will be found the number 21, indicating that at 21 on the graduated edge of the odontograph, is the location of the centre wherefrom to strike the curve d for the face of the tooth, this location being denoted in the cut at R.

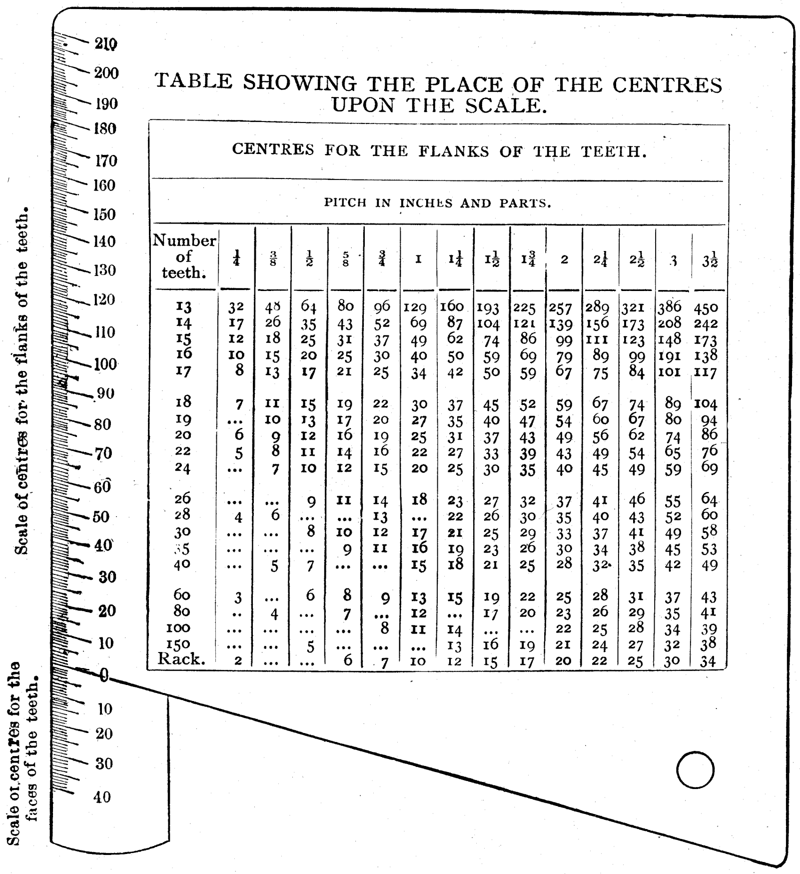

TABLE SHOWING THE PLACE OF THE CENTRES UPON THE SCALE.

CENTRES FOR THE FLANKS OF THE TEETH.

PITCH IN INCHES AND PARTS.

Number of teeth.	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	3	$3\frac{1}{2}$
13	32	48	64	80	96	129	160	193	225	257	289	321	386	450
14	17	26	35	43	52	69	87	104	121	139	156	173	208	242
15	12	18	25	31	37	49	62	74	86	99	111	123	148	173
16	10	15	20	25	30	40	50	59	69	79	89	99	191	138
17	8	13	17	21	25	34	42	50	59	67	75	84	101	117
18	7	11	15	19	22	30	37	45	52	59	67	74	89	104
19	...	10	13	17	20	27	35	40	47	54	60	67	80	94
20	6	9	12	16	19	25	31	37	43	49	56	62	74	86
22	5	8	11	14	16	22	27	33	39	43	49	54	65	76
24	...	7	10	12	15	20	25	30	35	40	45	49	59	69
26	...	...	9	11	14	18	23	27	32	37	41	46	55	64
28	4	6	...	...	13	...	22	26	30	35	40	43	52	60
30	...	...	8	10	12	17	21	25	29	33	37	41	49	58
35	...	...	...	9	11	16	19	23	26	30	34	38	45	53
40	...	5	7	...	...	15	18	21	25	28	32	35	42	49
60	3	...	6	8	9	13	15	19	22	25	28	31	37	43
80	..	4	...	7	...	12	...	17	20	23	26	29	35	41
100	...	...	...	...	8	11	14	...	...	22	25	28	34	39
150	...	...	5	...	...	...	13	16	19	21	24	27	32	38
Rack.	2	...	...	6	7	10	12	15	17	20	22	25	30	34

Fig. 135.

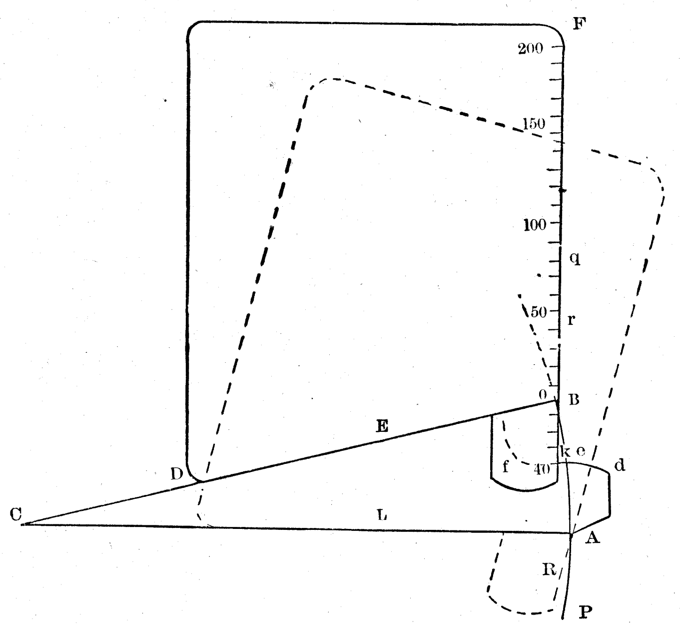

Fig. 136.

The requisite number on the graduated edge for pitches beyond $3\frac{1}{2}$ (the greatest given in the tables), may be obtained by direct proportion from those given in the tables. Thus for 4 inch pitch, by doubling the numbers given for a 2 inch pitch, containing the same number of teeth, for $4\frac{1}{2}$ inch pitch by doubling the numbers given for a $2\frac{1}{4}$ inch pitch. If the pitch be a fraction that cannot be so obtained, no serious error will be induced if the nearest number marked be taken.

An improved form of template odontograph, designed by Professor Robinson of the Illinois School of Industry, is shown in Fig. 137.

In this instrument the curved edge, having graduated lines, approaches more nearly to the curves produced by rolling circles than can be obtained from any system in which an arc of a circle is taken to represent the curve; hence, that edge is applied direct to the teeth and used as a template wherefrom to mark the curve. The curve is a logarithmic spiral, and the use of the instrument involves no other labor than that of setting it in position. The applicability of this curve, for the purpose, arises from two of its properties: first, that the involute of the logarithmic spiral is another like spiral with poles in common; and, second, that the obliquity or angle between a normal and radius sector is constant, the latter property being possessed by this curve only. By the first property it is known that a line, lying tangent to the curve C E H, will be normal or perpendicular to the curve C D B; so that when the line D E F is tangent to the pitch line, the curve A D B will coincide very closely with the true epicycloidal curve, or, rather, with that portion of it which is applied to the tooth curve of the wheel. By the second quality, all sectors of the spiral, with given angle at the poles, are similar figures which admit of the same degree of coincidence for all while rolling segments and the making of templates are entirely dispensed with, and the degree of accuracy is greater than is obtainable by means of the employment of arcs of circles.

The tables wherefrom to find the number or mark on the graduated edge, which is to be placed coincident with the tangent line in each case, are as follows :—

TABLE OF TABULAR VALUES WHICH, MULTIPLIED BY THE ARC PITCH OF THE TEETH, GIVES THE SETTING NUMBER ON THE GRADUATED EDGE OF THE INSTRUMENT.

	Number of Teeth in Wheel Sought; or, Wheel for Which Teeth are Sought.																
	8	12	16	20	30	40	50	60	70	80	90	100	120	150	200	300	500
RATIOS.*	*For Faces : Flanks Radial or Curved.* Draw Setting Tangent at Middle of Tooth.—Epicycloidal Spur or Bevel Gearing.																
$\frac{1}{12}$ = ·083	·32	·39	·46	·51													
$\frac{1}{4}$ = ·250	·31	·37	·44	·49	·61	·70	·78	·85	·92	·99	1·05	1·11	1·22	1·36	1·55	1·94	2·54
$\frac{1}{2}$ = ·500	·28	·34	·41	·46	·57	·66	·73	·80	·87	·93	1·00	1·06	1·15	1·29	1·50	1·86	2·41
$\frac{2}{3}$ = ·667	·27	·32	·38	·43	·54	·62	·70	·77	·83	·89	·95	1·01	1·11	1·24	1·45	1·79	2·32
1 . . .	·23	·28	·34	·39	·49	·58	·65	·72	·78	·83	·89	·94	1·03	1·15	1·36	1·65	2·10
$\frac{3}{2}$ = 1·50	·19	·25	·29	·34	·44	·51	·58	·64	·69	·74	·79	·84	·93	1·05	1·25	1·53	1·94
2 . . .	·17	·22	·26	·30	·38	·46	·53	·59	·63	·68	·72	·76	·84	·95	1·13	1·40	1·81
3 . . .		·16	·19	·23	·31	·38	·44	·49	·53	·57	·60	·63	·71	·82	·97	1·23	1·60
4 . . .		·14	·17	·20	·26	·33	·38	·42	·46	·49	·53	·56	·63	·73	·87	1·08	1·42
6 . . .					·22	·26	·30	·34	·37	·41	·44	·47	·53	·61	·71	·90	1·20
12 . . .						·20	·23	·25	·28	·30	·32	·34	·37	·42	·49	·60	·82
24 . . .											·19	·21	·23	·26	·31	·40	·57
	For Flanks, when Curved. Draw Setting Tangent at Side of Tooth.—Epicycloidal Spur and Bevel Gearing. Faces of Internal, and Flanks of Pinion Teeth.																
Degree of Flank Curvature. 1·5 slight.	·77	·98	1·18	1·36	1·75	2·05	2·31	2·56	2·75	2·92	3·08	3·24	3·52	3·87	4·51	5·50	7·20
2 good.	·44	·54	·63	·72	·92	1·09	1·24	1·38	1·49	1·59	1·79	1·79	1·98	2·23	2·67	3·22	4·50
3 more.	·20	·28	·35	·40	·54	·65	·76	·86	·95	1·02	1·10	1·18	1·31	1·46	1·67	2·08	2·76
4 much.		·20	·23	·25	·34	·42	·51	·59	·66	·71	·77	·82	·92	1·06	1·25	1·64	2·15
6 . . .			·16	·17	·26	·32	·38	·43	·48	·52	·56	·60	·66	·76	·93	1·20	1·54
12 . . .					·19	·24	·28	·31	·34	·36	·38	·40	·45	·52	·63	·80	·98
24 . . .												·22	·25	·28	·33	·47	·60
For Faces of Racks, and of Pinions for Racks and Internal Gears ; for Flanks of Internal and Sides of Involute Teeth. Draw Setting Tangent at Middle of Tooth, regarding Space as Tooth in Internal Teeth. For Rack use Number of Teeth in Pinion.																	
Pinion.	·31	·39	·48	·57	·73	·88	1·00	1·10	1·20	1·30	1·40	1·48	1·65	1·85	2·15	2·65	3·50
Rack.	·32	·38	·44	·50	·62	·72	·80	·87	·93	·99	1·03	1·08	1·16	1·27	1·49	1·86	2·44

* These ratios are obtained by dividing the radius of the wheel sought by the diameter of the generating circle.

similar epicycloids, whether great or small, and nearly the same for epicycloids in general; thus enabling the application of the instrument to epicycloids in general.

To set the instrument in position for drawing a tooth face a table which accompanies the instrument is used. From this table a numerical value is taken, which value depends upon the diameters of the wheels, and the number of teeth in the wheel for which the curve is sought. This tabular value, when multiplied by the pitch of the teeth, is to be found on the graduated edge on the instrument A D B in Fig. 137. This done, draw the line D E F tangent to the pitch line at the middle of the tooth, and mark off the half thickness of the tooth, as E, D, either on the tangent line or the pitch line. Then place the graduated edge of the odontograph at D, and in such a position that the number and division found as already stated shall come precisely on the tangent line at D, and at the same time so set the curved edge H F C so that it shall be tangent to the tangent line, that is to say, the curved edge C H must just meet the tangent line at some one point, as at F in the figure. A line drawn coincident with the graduated edge will then mark the face curve required, and the odontograph may be turned over, and the face on the other side of the tooth marked from a similar setting and process.

For the flanks of the teeth setting numbers are obtained from a separate table, and the instrument is turned upside down, and the tangent line D F, Fig. 137, is drawn from the side of the tooth (instead of from the centre), as shown in Fig. 138.

It is obvious that this odontograph may be set upon a radial arm and used as a template, as shown in Fig. 126, in which case the instrument would require but four settings for the whole wheel,

From these tables may be found a tabular value which, multiplied by the pitch of the wheel to be marked (as stated at the head of the table), will give the setting number on the graduated edge of the instrument, the procedure being as follows :—

For the teeth of a pair of wheels intended to gear together only (and not with other wheels having a different number of teeth).

For the face of such teeth where the flanks are to be radial lines.

Rule.—Divide the pitch circle radius of the wheel to have its teeth marked by the pitch circle radius of the wheel with which it is to gear: or, what is the same thing, divide the number of teeth in the wheel to have its teeth marked by the number of teeth in the wheel with which it is to gear, and the quotient is the "ratio." In the ratio column find this number, and look along that line, and in the column at the head of which is the number of teeth contained in the wheel to be marked, is a number termed the tabular value, which, multiplied by the arc pitch of the teeth, will give the number on the graduated edge by which to set the instrument to the tangent line.

Example.—What is the setting number for the face curves of a wheel to contain 12 teeth, of 3-inch arc pitch, and to gear with a wheel having 24 teeth?

Here number of teeth in wheel to be marked = 12, divided by the number of teeth (24) with which it gears; 12 ÷ 24 = ·5. Now in column of ratios may be found $\frac{1}{2}$ = 500 (which is the same thing as ·5), and along the same horizontal line in the table, and in the column headed 12 (the number of teeth in the wheel) is found ·34. This is the tabular value, which, multiplied by 3 (the arc pitch of the teeth), gives 1·02, which is the setting number on the graduated edge. It will be noted, however, that the graduated edge is marked 1, 2, 3, &c., and that between each consecutive division are ten subdivisions; hence, for the decimal ·02 an allowance may be made by setting the line 1 a proportionate amount below the tangent line marked on the wheel to set the instrument by.

It is to be noted here that the pinion, having radial lines, the other wheel must have curved flanks; the rule for which is as follows:—

CURVED FLANKS FOR A PAIR OF WHEELS.

Note.—When the flanks are desired to be curved instead of radial, it is necessary to the use of the instrument to select and

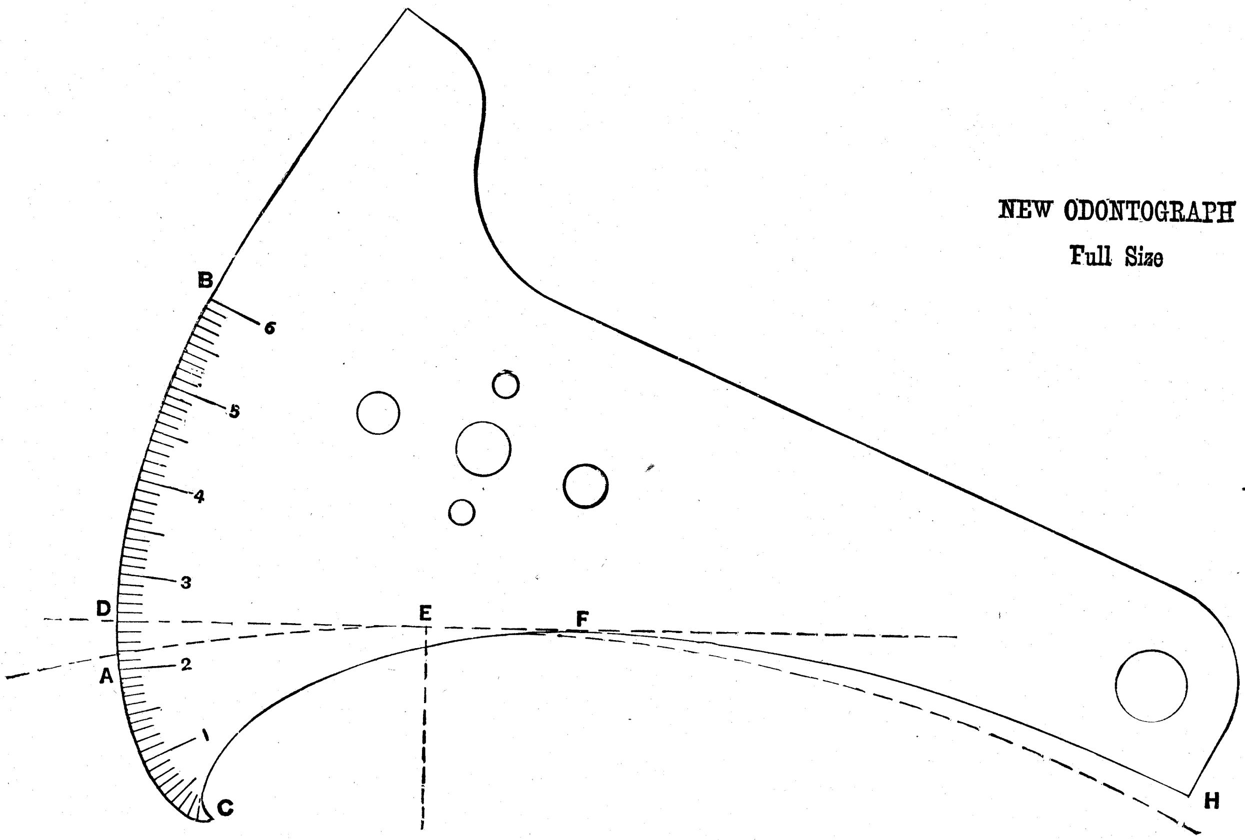

Fig. 137.

Required now the setting number for the wheel to have the 24 teeth.

Here number of teeth on the wheel = 24, divided by the number of teeth (12) on the wheel with which it gears; 24÷12=2. Now, there is no column in the "number of teeth sought" for 24 teeth; but we may find the necessary tabular value from the columns given for 20 teeth and 30 teeth, thus:—opposite ratio 2, and under 20 teeth is given ·30, and under 30 teeth is given ·38—the difference between the two being ·08. Now the difference between

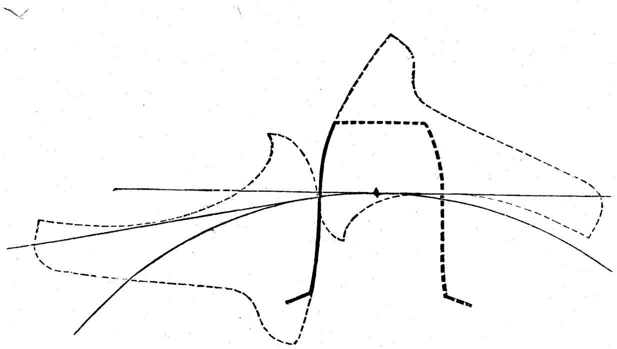

Fig. 138.

20 teeth and 24 teeth is $\frac{4}{10}$; hence, we take $\frac{4}{10}$ of the ·08 and add it to the tabular value given for 20 teeth, thus: ·08 × 4 ÷ 10 = ·032, and this added to ·30 (the tabular value given for 20 teeth = ·33, which is the tabular value for 24 teeth The ·33 multiplied by arc pitch (3) gives ·99. This, therefore, is the setting number for the instrument, being sufficiently near to the 1 on the graduated edge to allow that 1 to be used instead of ·99.

assume a value for the degree of curve, as is done in the table in the column marked "Degree for flank curving;" in which

1·5 slight—a slight curvature of flank.
2 good—an increased curvature of flank.
3 more—a degree of pronounced spread at root.
4 much—spread at root is a distinguishing feature of tooth form.
6—still increased spread in cases where the strength at root of pinion is of much importance to give strength.
12—as above, under aggravated conditions.
24—undesirable (unless requirement of strength compels this degree), because of excessive strain on pinion.

Rule.—For faces of teeth to have curved flanks.

Divide the number of teeth in the wheel to be marked by the number of teeth in the wheel with which it gears, and multiply by the degree of flank curve selected for the wheel with which that to be marked is to gear, and this will give the ratio. Find this number in ratio column, and the tabular number under the column of number of teeth of wheel to be marked; multiply tabular number so found by arc pitch of wheel to be marked, and the product will be the setting number for the instrument.

Example.—What is the setting number on the graduated edge of the odontograph for the faces of a wheel (of a pair) to contain 12 teeth of 2-inch arc pitch, and to gear with a wheel having 24 teeth and a flank curvature represented by 3 in "Degree of flank curving" column?

Here teeth in wheel to be marked (12) divided by number of teeth in the wheel it is to gear with (24), 12÷24=·5, which multiplied by 3 (degree of curvature selected for flanks of 24-teeth wheel), ·5 × 3 = 1·5. In column of ratio numbers find 1·5, and in 12-teeth column is ·25, which multiplied by pitch (2) gives ·5 as the setting number for the instrument; this being the fifth line on the instrument, and half way between the end and mark 1.

For Curved Flanks.

Rule.—Assume the degree of curve desired for the flanks to be marked, select the corresponding value in the column of "Degrees of flank curving," and find the tabular value under the number of teeth column.

Multiply tabular value so found by the arc pitch of the teeth, and the product is the setting number on the instrument.

Example.—What is the setting number on the odontograph for the flanks of a wheel to contain 12 teeth and gear with one having 24 teeth, the degree of curvature for the flanks being represented by 4 in the column of "Degree of flank curvature ?"

Here in column of degrees of flank curvature on the 3 line and under 12 teeth is ·20, which multiplied by pitch of teeth (2) is ·20 × 2 =40, or $\frac{4}{10}$; hence, the fourth line of division on the curved corner is the setting line, it representing $\frac{4}{10}$ of 1.

For Interchangeable Gearing (that is, a Train of Gears any one of which will work correctly with any other of the same set).

Rule—both for the faces and for the flanks. For each respective wheel divide the number of teeth in that wheel by some one number not greater than the number of teeth in the smallest wheel in the set, which gives the ratio number for the wheel to be marked. On that line of ratio numbers, and in the column of numbers of teeth, find the tabular value number; multiply this by the arc pitch of the wheel to be marked, and the product is the setting number of the instrument.

Example.—A set of wheels is to contain 10 wheels; the smallest is to contain 12 teeth; the arc pitch of the wheels is four inches. What is the setting number for the smallest wheel ?

Here number of teeth in smallest wheel of set is 10; divide this by any number smaller than itself (as say 5), 10 ÷ 5 = 2 = the ratio number on ratio line for 2; and under column for 12 is ·17, which is the tabular value, which multiplied by pitch (4) is ·17 × 4 = 68, or $\frac{6}{10}$ and $\frac{8}{100}$; hence, the instrument must be set with its seventh line of division just above the tangent line marked on the wheel. It will be noted that, if the seventh line were used as the setting, the adjustment would be only the $\frac{2}{100}$ of a division out, an amount scarcely practically appreciable.

Both for the faces and flanks, the second number is obtained in *precisely* the same manner for every wheel in the set, except that instead of 10 the number of teeth in each wheel must be substituted.

Rack and Pinion.—*For radial flanks* use for faces the two lower lines of table. *For curved flanks* find tabular value for pinion faces in lowest line. For flanks of pinion choose degree of curving, and find tabular value under "flanks," as for other wheels. For faces of rack divide number of teeth in pinion by degree of curving, which take for number of teeth in looking opposite "rack." Flanks of rack are still parallel, but may be arbitrarily curved beyond half way below pitch line.

Internal Gears.—For tooth curves within the pitch lines, divide radius of each wheel by any number not greater than radius of pinion, and look in the table under "flanks." For curves outside pitch line use lower line of table; or, divide radii by any number and look under "faces." In applying instrument draw tangents at middle and side of *space*, for internal teeth.

Involute Teeth.—For tabular values look opposite "Pinion," under proper number of teeth, for each wheel. Draw setting tangent from "base circle" of involute, at middle of tooth. For this the instrument gives the whole side of tooth at once.

In all cases multiply the tabular value by the pitch in inches.

Bevel-Wheels.—Apply above rules, using the developed normal cone bases as pitch lines. For right-angled axes this is done by using in place of the actual ratio of radii, or of teeth numbers, the square of that ratio; and for number of teeth, the actual number multiplied by the square root of one plus square of ratio or radii; the numerator of ratio, and number of teeth, belonging to wheel sought.

When the first column ratio and teeth numbers fall between those given in the table, the tabular values are found by interpolating as seen in the following examples :

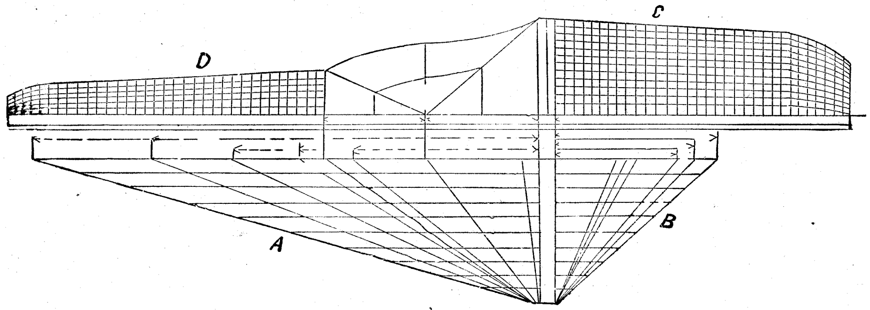

Fig. 139.

EXAMPLES OF TABULAR VALUES AND SETTING NUMBERS.

Take a pair of 16 *and* 56 *teeth ; radii* 5·09 *and* 17·82 *inches respectively ; and* 2 *inches pitch.*

Kind of Gearing.	Number of Teeth.	Kind of Flank.	Ratio Radii.	First Column Ratio.		Tab. Val.	
				Flank.	Face.	Flank.	Face.
Epicycloidal, Radial Flanks	Small	Radial	·29	Radial	·29	..	·44
	Large	Radial	3·5	Radial	3·5	..	·44
Epicycloidal. Curved Flanks.	Small	Curved 2 deg.	·29	2	·87	·63	·36
	Large	Curved 3 deg.	3·5	3	7·	·82	·30
Epicycloidal, Interchange'bl.	Small	"Sets," Divide Radii by 2.55	2·	2	2·	·63	·26
	Large		7·	7	7·	·40	·30
Epicycloidal, Internal.	Pinion	Curved 2 deg.		2	Pinion	·63	·44
	Wheel	Int. face 7 deg.	3·5	Pinion	7*	·84	·39
Epicycloidal, Rack & Pinion.	Pinion	Curved 2 deg.		2	Pinion	·63	·44
	Rack	Parallel		Parallel	Rack	..	·31
Involute Gearing.	Small	Face and Flank		Pinion.		·44	
	Large	One Curve		Pinion.		·84	

* The face being here internal, the tabular value is to be found under "flanks."
If bevels, use ratio radii ·082 and 12·25 ; and teeth numbers 16·6 and 203·8 respectively.

Walker's Patent Wheel Scale.—This scale is used in many manufactories in the United States to mark off the teeth for patterns, wherefrom to mould cast gears, and consists of a diagram from which the compasses may be set to the required radius to strike the curves of the teeth.

The general form of this diagram is shown in Fig. 139. From the portion A the length of the teeth, according to the pitch, is obtained. From the portion B half the thickness of the tooth at the pitch line is obtained. From the part C half the thickness at the root is obtained, and from the part D half the thickness at the point is obtained.

Each of these parts is marked with the number of teeth the wheel is to contain, and with the pitch of the teeth as shown in Fig. 140, which represents part C full size. Now suppose it is required to find the thickness at the root, for a tooth of a wheel having 60 teeth of one inch pitch, the circles from the point A,

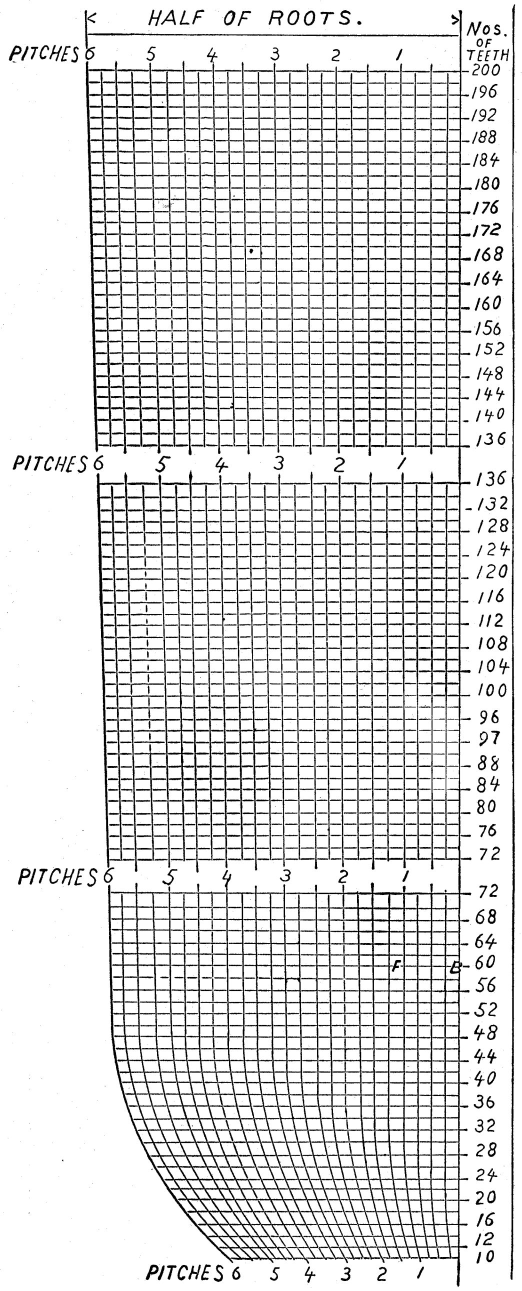

Fig. 140.

pitch line B and root C being drawn, and a radial line representing the middle of the tooth being marked, as is shown in Fig. 142, the compass points are set to the distance F B, Fig. 140—F being at the junction of line I with line 60; the compasses are then rested at G, and the points H I are marked. Then, from the portion B, Fig. 139 of the diagram, which is shown full-size in Fig. 141, the compasses may be set to half the thickness at the pitch circle, as in this case (for ordinary teeth) from E to E, and the points J K, Fig. 142, are marked. By a reference to the portion D of the diagram, half the thickness of the tooth at the point is obtained, and marked as at L M in Fig. 142. It now remains to set compasses to the radius for the face and that for the flank curves, both of which may be obtained from the part A of the diagram. The locations of the centres, wherefrom to strike these curves, are obtained as in Fig. 142. The compasses set for the face curve are rested at H, and the arc N is struck; they are then rested at J and the arc O struck; and from the intersection of N O, as a centre, the face curve H J is marked. By a similar process, reference to the portion D of the diagram, half the thickness of

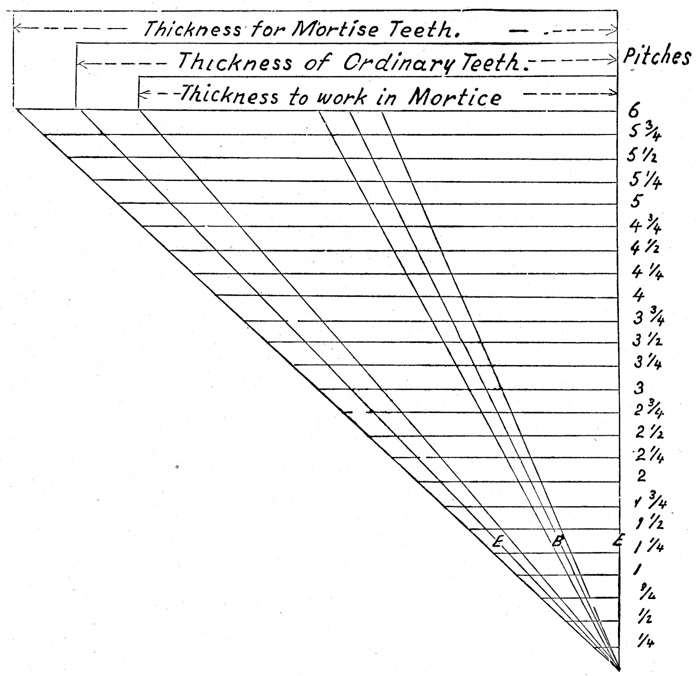

Fig. 141.

the tooth at the point is obtained, and marked as at L M in Fig. 142. It now remains to set the compasses to the radius to strike the respective face and flank curves, and for this purpose the operator turns to the portion A, Fig. 139, of the diagram or scale, and sets the compasses from the marks on that portion to the required radii.

It now remains to find the proper location from which to strike the curves.

The face curve on the other side of the tooth is struck. The compasses set to the flank radius is then rested at M, and the arc P is marked and rested at K to mark the arc Q; and from the intersection of P Q, as a centre, the flank curve K M is marked: that on the other side of the tooth being marked in a similar manner.

Additional scales or diagrams, not shown in Fig. 139, give

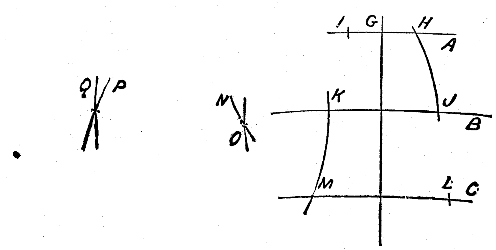

Fig. 142.

similar distances to set the compasses for the teeth of internal wheels and racks.

It now remains to explain the method whereby the author of the scale has obtained the various radii, which is as follows: A wheel of 200 teeth was given the form of tooth curve that would be obtained by rolling it upon another wheel, containing 200 teeth of the same pitch. It was next given the form of tooth that would be obtained by rolling upon it a wheel having 10 teeth of the same pitch, and a line intermediate between the two curves was taken as representing the proper curve for the large wheel.

The wheel having 10 teeth was then given the form of tooth that would be obtained by rolling upon it another wheel of the same diameter of pitch circle and pitch of teeth. It was next given the form of tooth that would be given by rolling upon it a wheel having 200 teeth, and a curve intermediate between the two curves thus obtained was taken as representing the proper curve for the pinion of 10 teeth. By this means the inventor does not claim to produce wheels having an exactly equal velocity ratio, but he claims that he obtains a curve that is the nearest approximation to the proper epicycloidal curve. The radii for the curves for all other numbers of teeth (between 10 and 200) are obtained in precisely the same manner, the pinion for each pitch being supposed to contain 10 teeth. Thus the scale is intended for interchangeable cast gears.

The nature of the scale renders it necessary to assume a constant height of tooth for all wheels of the same pitch, and this Mr. Walker has assumed as ·40 of the pitch, from the pitch line to the base, and ·35 from the pitch line to the point.

The curves for the faces obtained by this method have rather more curvature than would be due to the true epicycloid, which causes the points to begin and leave contact more easily than would otherwise be the case.

For a pair of wheels Mr. Walker strikes the face curve by a point on the pitch rolling circle, and the flanks by a point on the addendum circle, fastening a piece of wood to the pitch circle to carry the tracing point. The flank of each wheel is struck with a tracing point, thus attached to the pitch circle of the other wheel.

The proportions of teeth and of the spaces between them are usually given in turns of the pitch, so that all teeth of a given pitch shall have an equal thickness, height, and breadth, with an equal addendum and flank, and the same amount of clearance.

The term "clearance" as applied to gear-wheel teeth means the amount of space left between the teeth of one wheel, and the spaces in the other, or, in other words, the difference between the width of the teeth and that of the spaces between the teeth.

This clearance exists at the sides of the teeth, as in Fig. 143, at A, and between the tops of the teeth and the bottoms or roots of the spaces as at B. When, however, the simple term clearance is employed it implies the side clearance as at A, the clearance at B being usually designated as *top and bottom clearance.* Clearance is necessary for two purposes; first, in teeth cut in a machine to accurate form and dimensions, to prevent the teeth of one wheel from binding in the spaces of the other, and second, in cast teeth, to allow for the imperfections in the teeth which are incidental to casting in a founder's mould. In machine-cut teeth the amount of clearance is a minimum.

In wheels which are cast with their teeth complete and on the pattern, the amount of clearance must be a maximum, because, in the first place, the teeth on the pattern must be made taper to enable the extraction of the pattern from the mould without damage to the teeth in the mould, and the amount of this taper must be greater than in machine-moulded teeth, because the pattern cannot be lifted so truly vertical by hand as to avoid, in all cases, damage to the mould; in which case the moulder repairs the mould either with his moulding tools and by the aid of the eye, or else with a tooth and a space made on a piece of wood for the purpose. But even in this case the concentricity of the teeth is scarcely likely to be preserved.

It is obvious that by reason of this taper each wheel is larger in diameter on one side than on the other, hence to preserve the true curves to the teeth the pitch circle is made correspondingly smaller. But if in keying the wheels to their shafts the two large diameters of a pair of wheels be placed to work together, the teeth of the pair would have contact on that side of the wheel only, and to avoid this and give the teeth contact across their full breadth the wheels are so placed on their shafts that the large diameter of one shall work with the small one of the other, the amount of taper being the same in each wheel irrespective of their relative diameters. This also serves to keep the clearance equal in amount both top, and bottom, and sideways.

A second imperfection is that in order to loosen the pattern in the sand or mould, and enable its extraction by hand from the mould, the pattern requires to be *rapped* in the mould, the blows forcing back the sand of the mould and thus loosening the pattern. In ordinary practice the amount of this rapping is left entirely to the judgment of the moulder, who has nothing to guide him in securing an equal amount of pattern movement in each direction in the mould; hence, the finished mould may be of increased radius at the circumference in the direction in which the wheel moved most during the rapping. Again, the wood pattern is apt in time to shrink and become *out of round,* while even iron patterns are not entirely free from warping. Again, the cast metal is liable to contract in cooling more in one direction than in another. The amount of clearance usually allowed for pattern-moulded cast gearing is given by Professor Willis as follows:—Whole depth of tooth $\frac{7}{10}$, of the pitch working depth $\frac{6}{10}$; hence $\frac{1}{10}$ of the pitch is allowed for top and bottom clearance, and this is the amount shown at B in Fig. 143. The amount of side clearance given by Willis as that ordinarily found in practice is as follows:—"Thickness of tooth $\frac{5}{11}$ of the pitch; breadth of space $\frac{6}{11}$; hence, the side clearance equals $\frac{1}{11}$ of the pitch, which in a 3-inch pitch equals ·27 of an inch in each wheel. Calling this in round figures, which is near enough for our purpose, $\frac{1}{4}$ inch, we have thickness of tooth $1\frac{1}{4}$, width of space $1\frac{3}{4}$, or $\frac{1}{2}$ inch of clearance in a 3-inch

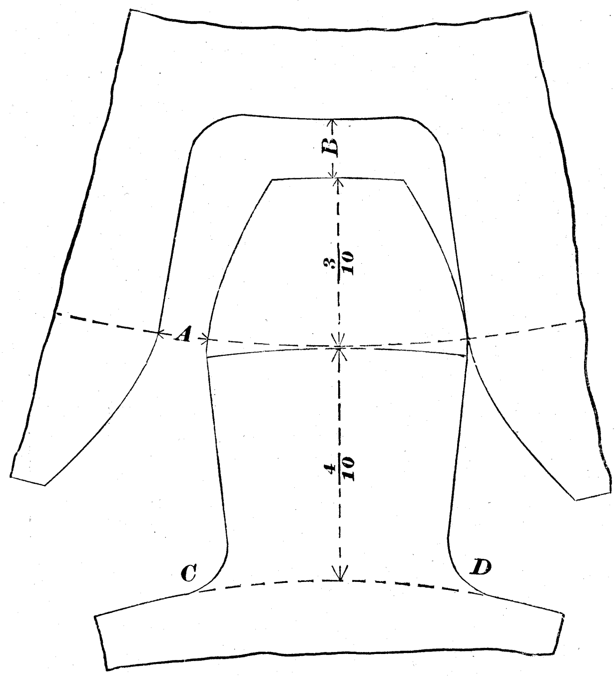

Fig. 143.

pitch, an amount which on wheels of coarse pitch is evidently more than that necessary in view of the accuracey of modern moulding, however suitable it may have been for the less perfect practice of Professor Willis's time. It is to be observed that the rapping of the pattern in the founder's mould reduces the thickness of the teeth and increases the width of the spaces somewhat, and to that extent augments the amount of side clearance allowed on the pattern, and the amount of clearance thus obtained would be nearly sufficient for a small wheel, as say of 2 inches diameter. It is further to be observed that the amount of rapping is not proportionate to the diameter of the wheel; thus, in a wheel of 2 inches diameter, the rapping would increase the size of the mould about $\frac{1}{32}$ inch. But in the proportion of $\frac{1}{32}$ inch to every 2 inches of diameter, the rapping on a 6-foot wheel would amount to $1\frac{1}{16}$ inches, whereas, in actual practice, a 6-foot wheel would not enlarge the mould more than at most $\frac{1}{8}$ inch from the rapping.

It is obvious, then, that it would be more in accordance with the requirements to proportion the amount of clearance to the diameter of the wheel, so as to keep the clearance as small as possible. This will possess the advantage that the teeth will be stronger, it being obvious that the teeth are weakened both from the loss of thickness and the increase of height due to the clearance.

It is usual in epicycloidal teeth to fill in the corner at the root of the tooth with a fillet, as at C, D, in Fig. 143, to strengthen it.

This is not requisite when the diameter of the generating circle is so small in proportion to the base circle as to produce teeth that are spread at the roots; but it is especially advantageous when the teeth have radial flanks, in which case the fillets may extend

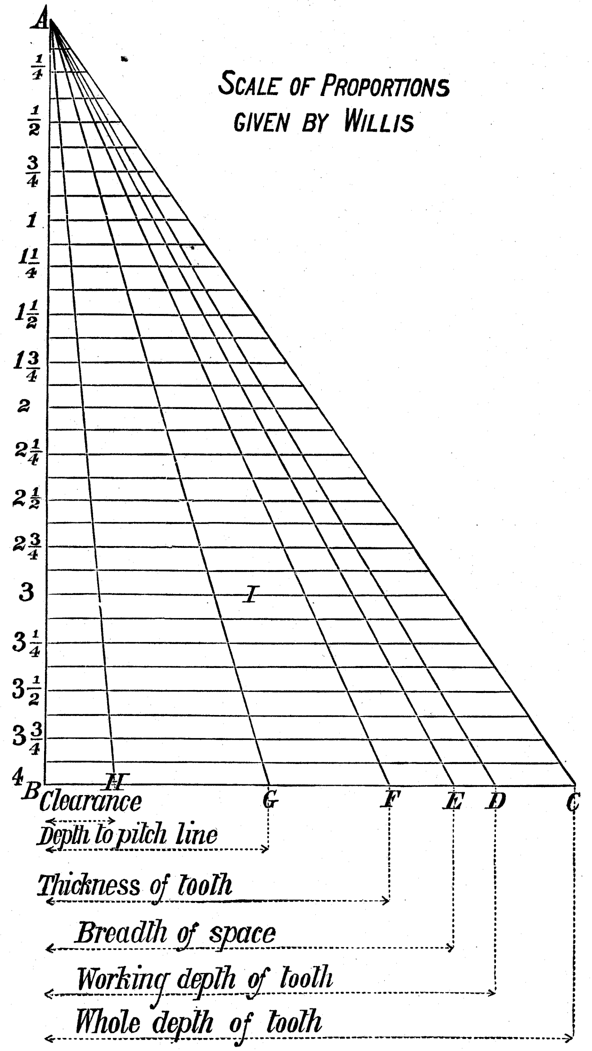

Fig. 144.

farther up the flanks than when they are spread; because, as shown in Fig. 47, the length of operative flank is a minimum in teeth having radial flanks, and as the smallest pinion in the set is that with radial flanks, and further as it has the least number of teeth in contact, it is the weakest, and requires all the strengthening that the fillets in the corners will give, and sometimes the addition of the flanges on the sides of the pinion, such gears being termed "shrouded."

The proportion of the teeth to the pitch as found in ordinary practice is given by Professor Willis as follows:—

Depth to pitch line . .	$\frac{3}{10}$ of the pitch.
Working depth . .	$\frac{6}{10}$ " "
Whole depth . . .	$\frac{7}{10}$ " "
Thickness of tooth . .	$\frac{5}{11}$ " "
Breadth of space . .	$\frac{6}{11}$ " "

The depth to pitch line is, of course, the same thing as the height of the addendum, and is measured through the centre of the tooth from the point to the pitch line in the direction of a radial line and not following the curve of tooth face.

Referring to the working depth, it was shown in Figs. 42 and 44 that the height of the addendum remaining constant, it varies with the diameter of the generating circle.

From these proportions or such others as may be selected, in which the proportions bear a fixed relation to the pitch, a scale may be made and used as a gauge, to set the compasses by, and in marking off the teeth for any pitch within the capacity of the scale. A vertical line A B in Fig. 144, is drawn and marked off in inches and parts of an inch, to represent the pitches of the teeth; at a right angle to A B, the line B C is drawn, its length equalling the whole depth of tooth, which since the coarsest pitch in the scale is 4 inches will be $\frac{7}{10}$ of 4 inches. From the end of line C we draw a diagonal line to A, and this gives us the whole depth of tooth for any pitch up to 4 inches: thus the whole depth for a 4-inch pitch is the full length of the horizontal line B C; the whole depth for a 3-inch pitch will be the length of the horizontal line running from the 3 on line A B, to line A C on the right hand of the figure; similarly for the full depth of tooth for a 2-inch pitch is the length of the horizontal line running from 2 to A C. The working depth of tooth being $\frac{6}{10}$ of the pitch a diagonal is drawn from A meeting line C at a distance from B of $\frac{6}{10}$ of 4 inches and we get the working depth for any other pitch by measuring (along the horizontal line corresponding to that pitch), from the line of pitches to the diagonal line for working depth of tooth. The thickness of tooth is $\frac{5}{11}$ of the pitch and its diagonal is distant $\frac{5}{11}$ of 4 (from B) on line B C, the thickness for other pitches being obtained on the horizontal line corresponding to those pitches as before.

The construction of a pattern wherefrom to make a foundry mould, in which to cast a spur gear-wheel, is as shown in section, and in plan of Fig. 145. The method of constructing these patterns depends somewhat on their size. Large patterns are constructed with the teeth separate, and the body of the wheel is built of separate pieces, forming the arms, the hub, the rim, and the teeth respectively. Pinion patterns, of six inches and less in diameter, are usually made out of a solid piece, in which case the grain of the wood must lie in the direction of the teeth height. The chuck or face plate of the lathe, for turning the piece, must be of smaller diameter than the pinion, so that it will permit access to a tool applied on both sides, so as to strike the pitch circle on both sides. A second circle is also struck for the roots

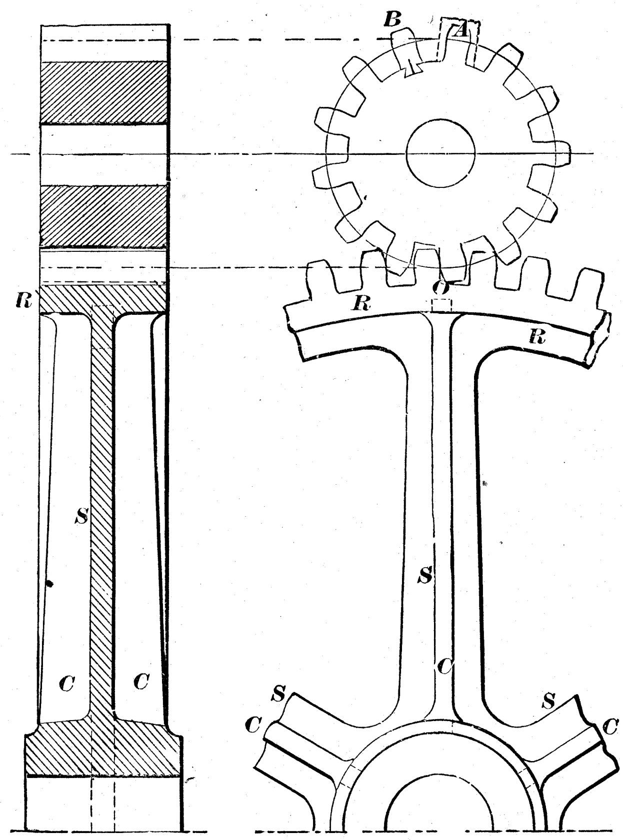

Fig. 145.

or depths of the teeth, and also, if required, an extra circle for striking the curves of the teeth with compasses, as was described in Fig. 130. All these circles are to be struck on both sides of the pattern, and as the pattern is to be left slightly taper, to permit of its leaving the mould easily, they must be made of smaller diameter on one side than on the other of the pattern; the reduction in diameter all being made on the same side of the pattern. The pinion body must then be divided off on the pitch

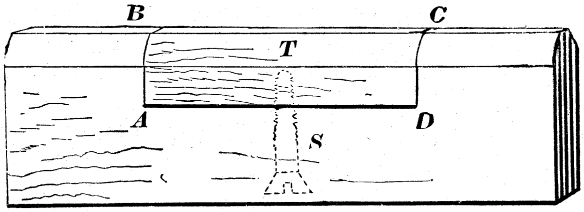

Fig. 146.

line into as many equal divisions as there are to be teeth in it; the curves of the teeth are then marked by some one of the methods described in the remarks on curves of gear-teeth. The top of the face curves are then marked along the points of the teeth by means of a square and scribe, and from these lines the curves are marked in on the other side of the pinion, and the spaces cut out, leaving the teeth projecting. For a larger pinion, without arms, the hub or body is built up of courses of quadrants, the joints of the second course *breaking joint* with those of the first.

The quadrants are glued together, and when the whole is formed and the glue dry, it is turned in the lathe to the diameter of the wheel at the roots of the teeth. Blocks of wood, to form the teeth, are then planed up, one face being a hollow curve to fit the circle of the wheel. The circumference of the wheel is divided, or pitched off, as it is termed, into as many points of equal division as there are to be teeth, and at these points lines are drawn, using a square, having its back held firmly against the radial face of the pinion, while the blade is brought coincidal with the point of division, so as to act as a guide in converting that point into a line running exactly true with the pinion. All the points of division being thus carried into lines, the blocks for the teeth are glued to the body of the pinion, as denoted by A, in Fig. 145. Another method is to dovetail the teeth into the pinion, as in Fig. 145 at B. After the teeth blocks are set, the process is, as already described, for a solid pinion.

The construction of a wheel, such as shown in Fig. 145, is as follows: The rim R must be built up in segments, but when the courses of segments are high enough to reach the flat sides of the arms they should be turned in the lathe to the diameter on the inside, and the arms should be let in, as shown in the figure at O. The rest of the courses of segments should then be added. The arms are then put in, and the inside of the segments last added may then be turned up, and the outside of the rim turned. The hub should then be added, one-half on each side of the arms, as in the figure. The ribs C of the arms are then added, and the body is completed (ready to receive the teeth), by filleting in the corners. An excellent method of getting out the teeth is as follows: Shape a piece of hard wood, as in Fig. 146, making it some five or six inches longer than the teeth, and about three inches deeper, the thickness being not less than the thickness of the required teeth at the pitch line. Parallel to the edge B C, mark the line A D, distant from B C to an amount equal to the required depth of tooth. Mark off, about midway of the piece, the lines A B and C D, distant from each other to an amount equal to the breadth of the wheel rim, and make two saw cuts to those lines. Take a piece of board an inch or two longer than the radius of the gear-wheel and insert a piece of wood (which is termed a box) tightly into the board, as shown in Fig. 147, E representing the box. Let the point F on the board represent the centre of the wheel, and draw a radial line R from F through the centre of the box. From the centre F, with a trammel, mark the addendum line G G, pitch line H I, and line J K for the depth of the teeth (and also a line wherefrom to strike the teeth curves, as shown in Fig. 129 if necessary). From the radial line R, as a centre, mark off on the pitch circle, points of division for several teeth, so as to be able to test the accuracy of the spacing across the several points, as well as from one point to the next, and mark the curves for the teeth on the end of the box, as shown. Turn the box end for end in the board, and mark out a tooth by the same method on the other end of the box. The box being removed from the board must now have its sides planed to the lines, when it will be ready to shape the teeth in. The teeth are got out for length, breadth, and thickness at the pitch line as follows: The lumber from which they are cut should be very straight grained, and should be first cut into strips of a width and thickness slightly greater than that of the teeth at the pitch line. These strips (which should be about two feet long) should then be planed down on the sides to very nearly the thickness of the tooth at the pitch line, and hollow on one edge to fit the curvature of the wheel rim. From these strips, pieces a trifle longer than the breadth of the wheel rim are cut, these forming the teeth. The pieces are then planed on the ends to the exact width of the

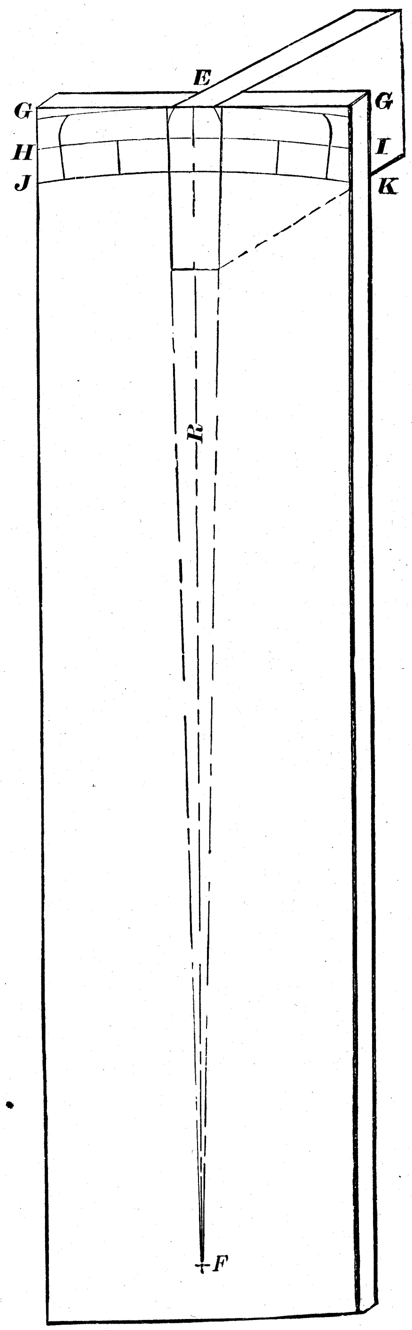

Fig. 147.

wheel rim. To facilitate this planing a number of the pieces or blank teeth may be set in a frame, as in Figs. 148 and 149, in which A is a piece having the blocks B B affixed to it. C is a clamp secured by the screws at S S, and 1, 2, 3, 4, 5, 6 are the ends of the blank teeth. The clamp need not be as wide as the teeth, as in Fig. 148, but it is well to let the pieces A and B B equal the breadth of the wheel rim, so that they will act as a template to plane the blank teeth ends to. The ends of B B may be blackleaded, so as to show plainly if the plane blade happens to shave them, and hence to prevent planing B B with the teeth. The blank teeth may now be separately placed in the box (Fig. 146) and secured by a screw, as shown in that figure, in which S is the screw, and T the blank tooth. The sides of the tooth must be carefully planed down equal and level with the surface of the box. The rim of the wheel, having been divided off into as many divisions as there are to be teeth in the wheel, as shown in Fig.

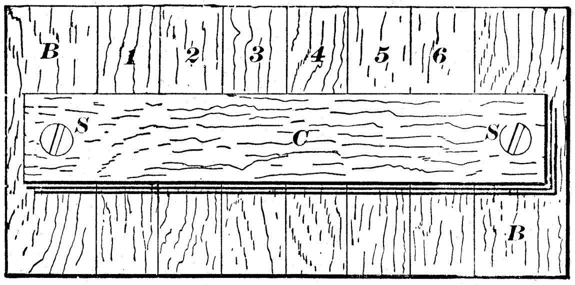

Fig. 148.

150, at *a,a,a,* &c., the finished teeth are glued so that the same respective side of each tooth exactly meets one of the lines *a*. Only a few spots of glue should be applied, and these at the middle of the root thickness, so that the glue shall not exude and hide the line *a*, which would make it difficult to set the teeth true to the line. When the teeth are all dry they must be additionally secured to the rim by nails. Wheels sufficiently large to incur difficulty of transportation are composed of a number of sections, each usually consisting of an arm, with an equal length of the rim arc on each side of it, so that the joint where the rim segments are bolted together will be midway between the two arms.

This, however, is not absolutely necessary so long as the joints are so arranged as to occur in the middle of tooth spaces, and not in the thickness of the tooth. This sometimes necessitates that the rim sections have an unequal length of arc, in which event the pattern is made for the longest segment, and when these are cast the teeth superfluous for the shorter segments are stopped off by the foundry moulder. This saves cutting or altering the pattern, which, therefore, remains good for other wheels when required.

When the teeth of wheels are to be cut in a gear-cutting machine the accurate spacing of the teeth is determined by the index plate and gearing of the machine itself; but when the teeth are to be cast upon the wheel and a pattern is to be made,

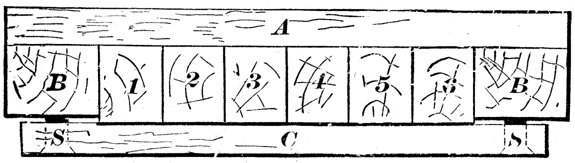

Fig. 149.

wherefrom to cast the wheel the points of division denoting the thickness of the teeth and the width of the spaces are usually marked by hand. This is often rendered necessary from the wheels being of too large a diameter to go into dividing machines of the sizes usually constructed.

To accurately divide off the pitch circle of a gear-wheel by hand, requires both patience and skilful manipulation, but it is time and trouble that well repays its cost, for in the accuracy of spaces lies the first requisite of a good gear-wheel.

It is a very difficult matter to set the compasses so that by commencing at any one point and stepping the compasses around the circle continuously in one direction, the compass point shall fall into the precise point from which it started, for if the compass point be set the 1-200th inch out, the last space will come an inch out in a circle having 200 points of divisions. It is, therefore, almost impossible and quite impracticable to accurately mark or divide off a circle having many points of division in this manner, not only on account of the fineness of the

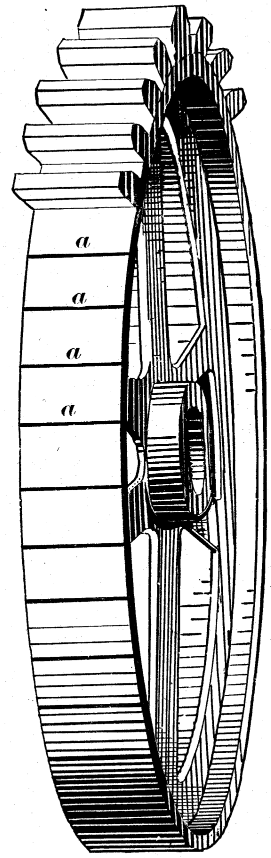

Fig. 150.

adjustment of the compass points, but because the frequent trials will leave so many marks upon the circle that the true ones will not be distinguishable from the false. Furthermore, the compass points are apt to spring and fall into the false marks when those marks come close to the true ones.

In Fig. 151 is shown a construction by means of which the compass points may be set more nearly than by dividing the circumference of the circle by the number of divisions it is required to be marked into and setting the compasses to the quotient, because such a calculation gives the length of the division measured around the arc of the circle, instead of the distance measured straight from point of division to point of division.

The construction of Fig. 151 is as follows: P P is a portion of the circle to be divided, and A B is a line at a tangent to the

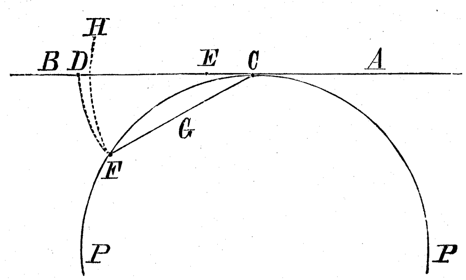

Fig. 151.

point C of the circle P P. The point D is set off distant from C, to an amount obtained by dividing the circumference of P P by the number of divisions it is to have. Take one-quarter of this distance C D, and mark it from C, giving the point E, set one point of the compass at E and the other at D, and draw the arc

D F, and the distance from F to C, as denoted by G, is the distance to which to set the compasses to divide the circle properly. The compasses being set to this distance G, we may rest one compass point at C, and mark the arc F H, and the distance between arc H and arc D, measured on the line A B, is the difference between the points C, F when measured around the circle P P, and straight across, as at G.

A pair of compasses set even by this construction will not, however, be entirely accurate, because there will be some degree of error, even though it be in placing the compass points on the

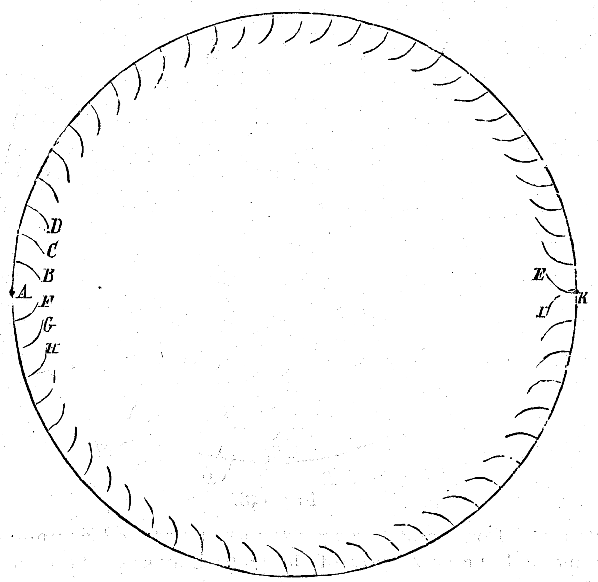

Fig. 152.

lines and on the points marked, hence it is necessary to step the compasses around the circle, and the best method off doing this is as follows: Commencing at A, Fig. 152, we mark off continuously one from the other, and taking care to be very exact to place the compass point exactly coincident with the line of the circle, the points B, C, D, &c., continuing until we have marked half as many divisions as the circle is to contain, and arriving at E, starting again at A, we mark off similar divisions (one half of the total number), F, G, H, arriving at I, and the centre K, between the two lines E, I, will be the true position of the point diametrally

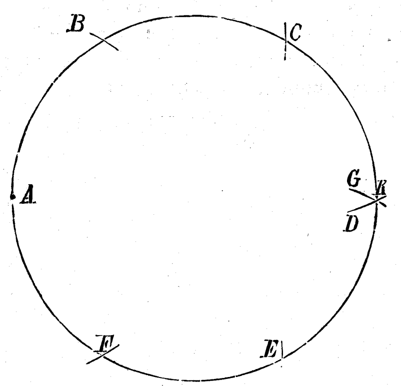

Fig. 153.

opposite to point A, whence we started. These points are all marked inside the circle to keep them distinct from those subsequently marked.

It will be, perhaps, observed by the reader that it would be more expeditious, and perhaps cause less variation, were we to set the compasses to the radius of the circle and mark off the point K, as shown in Fig. 153, commencing at the point A, and marking off on the one side the lines B, C, and D, and on the other side E, F, and G, the junction or centre, between G and D, at the circle being the true position of the point K. For circles struck upon flat surfaces, this plan may be advantageous; and in cases where there are not at hand compasses large enough, a pair of trammels may be used for the purpose; but our instructions are intended to apply also to marking off equidistant points on such circumferences as the faces of pulleys or on the outsides of small rings or cylinders, in which cases the use of compasses is impracticable. The experienced hand may, it is true, adjust the compasses as instructed, and mark off three or four of the marks B, C, &c,

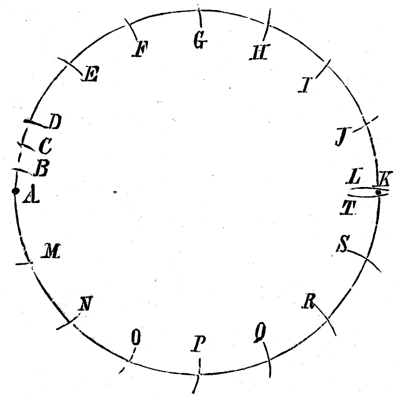

Fig. 154.

in Fig. 152, and then open out the compasses to the distance between the two extreme marks, and proceed as before to find the centre K, but as a rule, the time saved will scarcely repay the trouble; and all that can be done to save time in such cases is, if the holes come reasonably close together, to mark off, after the compasses are adjusted, three or four spaces, as shown in Fig. 154. Commencing at the point A, and marking off the points B, C, and D, we then set another pair of compasses to the distance between A and D, and then mark, from D on one side and from A on the other, the marks from F to L and from M to T, thus obtaining the point K. This method, however expeditious and correct for certain work, is not applicable to circumferential work of small diameter and in which the distance between two of the adjacent points is, at the most, $\frac{1}{20}$ of the circumference of the circle; because the angle of the surface of the metal to the compass point causes the latter to spring wider open in consequence of the pressure necessary to cause the compass point to mark the metal. This will be readily perceived on reference to Fig. 155 in which A represents the stationary, and B the scribing or marking point of the compasses.

The error in the set of the compasses as shown by the distance apart of the two marks E and I on the circle in Fig. 152 is too

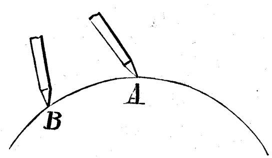

Fig. 155.

fine to render it practicable to remedy it by moving the compass legs, hence we effect the adjustment by oilstoning the points on the outside, throwing them closer together as the figure shows is necessary.

Having found the point K, we mark (on the outside of the circle, so as to keep the marks distinct from those first marked) the division B, C, D, Fig. 156, &c., up to G, the number of divisions between B and G being one quarter of those in the whole circle. Then, beginning at K, we mark off also one quarter of the number of divisions arriving at M in the figure and producing the point 3. By a similar operation on the other side of the circle, we get the true position of point No. 4. If, in obtaining points 3 and 4, the compasses are not found to be set dead true, the necessary adjustment must be made; and it will be seen that, so far, we have obtained four true positions, and the process of obtaining each of them has served as a justification of the distance of the compass points. From these four points we may proceed in like manner to mark off the holes or points between them; and the whole will be as true as it is practicable to mark them off upon that size of circle. In cases, however, where mathematical precision is required upon flat and not circumferential surfaces, the marking off may be performed upon a circle of larger diameter, as shown in Fig. 157. If it is required to mark off the circle A, Fig. 157, into any even number of equidistant points, and if, in consequence of the closeness together of the points, it becomes difficult to mark them (as described) with the compasses, we

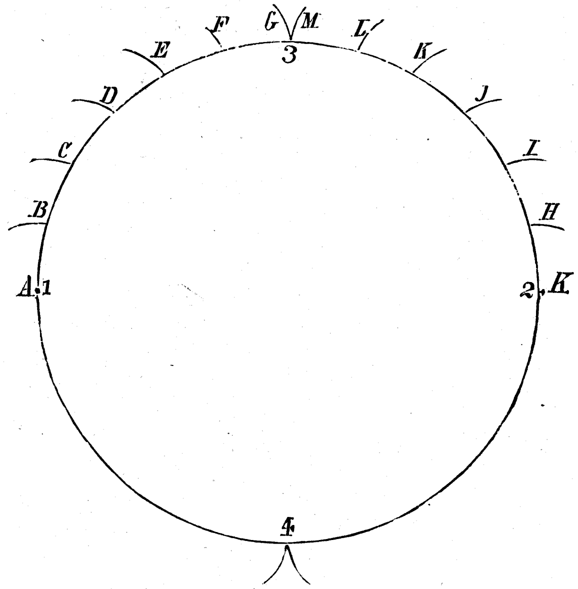

Fig. 156.

mark a circle B B of larger diameter, and perform our marking upon it, carrying the marks across the smaller circle with a straightedge placed to intersect the centres of the circles and the points marked on each side of the diameter. Thus, in Fig. 157, the lines 1 and 2 on the smaller circle would be obtained from a line struck through 1 and 4 on the outer circle; and supposing the larger circle to be three times the size of the smaller, the

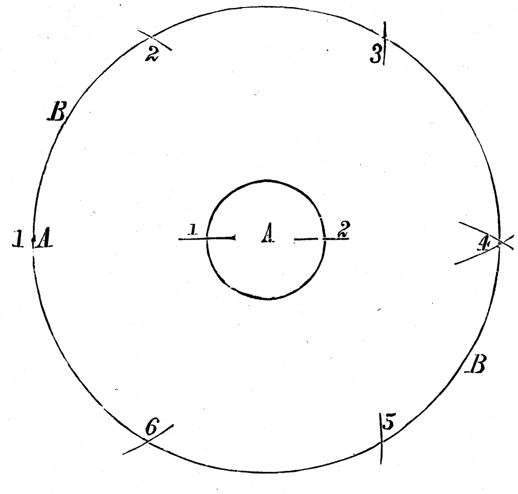

Fig. 157.

deviation from truth in the latter will be only $\frac{1}{3}$ of whatever it is in the former.

In this example we have supposed the number of divisions to be an even one, hence the point K, Fig. 152, falls diametrically opposite to A, whereas in an odd number of points of division this would not be the case, and we must proceed by either of the two following methods:—

In Fig. 158 is shown a circle requiring to be divided by 17 equidistant points. Starting from point 1 we mark on the outside of the circumference points 2, 3, 4, &c., up to point 9. Starting again from point 1 we mark points 10, 11, &c., up to 17. If, then, we try the compasses to 17 and 9 we shall find they come too close together, hence we take another pair of compasses (so as not to disturb the set of our first pair) and find the centre between 9 and 17 as shown by the point A. We then correct the set of our first pair of compasses, as near as the judgment dic-

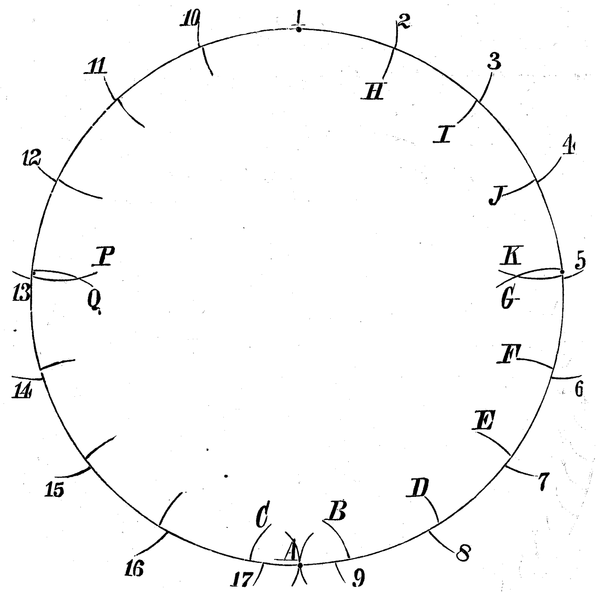

Fig. 158.

tates, and from point A, we mark with the second compasses (set to one half the new space of the first compasses) the points B, C. With the first pair of compasses, starting from B, we mark D, E, &c., to G; and from I, we mark divisions H, I, &c., to K, and if the compasses were set true, K and G would meet at the circle. We may, however, mark a point midway between K and G, as at 5. Starting again from points C and I, we mark the other side of the circle in a similar manner, producing the lines P and Q, midway between which (the compasses not being set quite correct as yet) is the true point for another division. After again correcting the compasses, we start from B and 5 respectively, and mark point 7, again correcting the compasses. Then from C and the point between P and Q, we may mark an intermediate point, and so on until all the points of division are made. This method is correct enough for most practical purposes, but the method shown in Fig. 159 is more correct for an odd number of points of division. Suppose that we have commenced at the point marked 1, we mark off half the required number of holes on one side and arrive at the point 2; and then, commencing at the point 1 again, we mark off the

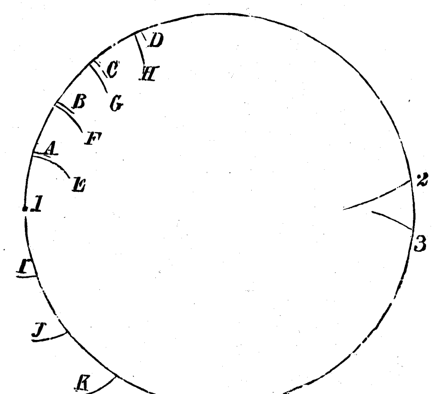

Fig. 159.

other half of the required number of holes, arriving at the point 3. We then apply our compasses to the distance between the points 2 and 3; and if that distance is not exactly the same to which the compasses are set, we make the necessary adjustment, and try again and again until correct adjustment is secured.

It is highly necessary, in this case, to make the lines drawn at each trial all on the same side of the circle and of equal length, but of a different length to those marked on previous trials. For example, left the lines A, B, C, D, in Fig. 159 represent those made on the first trial, and E, F, G, H, those made on the second trial; and when the adjustment is complete, let the last trial be made upon the outside or other side of the circle, as shown by the lines I, J, K, L. Having obtained the three true points, marked 1, 2, 3, we proceed to mark the intermediate divisions, as described for an even number of divisions, save that there will be a space, 2 and 3, opposite point 1, instead of a point, as in case of a circle having an even number of divisions.

The equal points of division thus obtained may be taken for the centres of the tooth at the pitch circle or for one side of the teeth, as the method to be pursued to mark the tooth curves may render most desirable. If, for example, a template be used to mark off the tooth curves, the marks may be used to best advan-

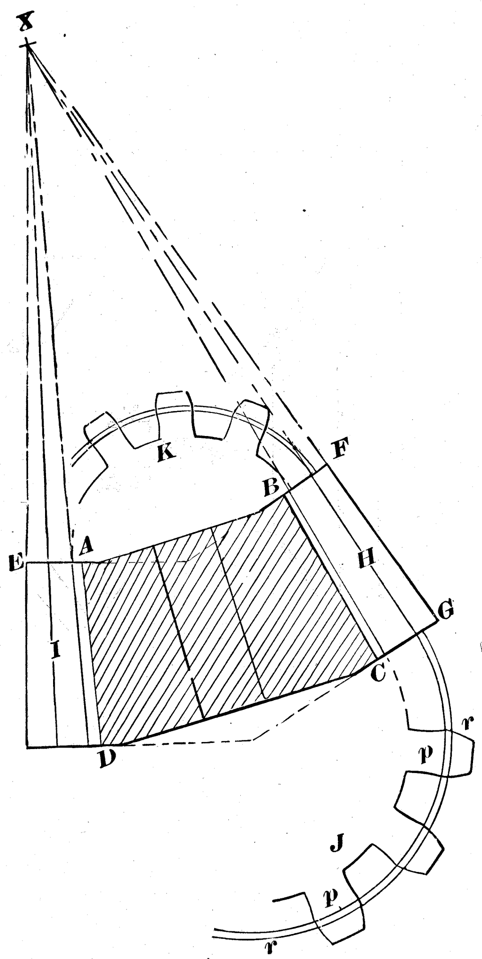

Fig. 160.

tage as representing the side of a tooth, and from them the thickness of the tooth may be marked or not as the kind of template used may require. Thus, if the template shown in Fig. 21 be used, no other marks will be used, because the sides of a tooth on each side of a space may be marked at one setting of the template to the lines or marks of division If, however, a template, such as shown in Fig. 81 be used, a second set of lines marked distant from the first to a radius equal to the thickness of a tooth becomes necessary so that the template may be set to each line marked. If the Willis odontograph or the Robinson template odontograph be used the second set of lines will also be necessary. In using the Walker scale a radial line, as G in Fig. 142, will require to be marked through the points of equal division, and the thickness of the tooth at the points on the pitch circle and at the root must be marked as was shown in Fig. 142.

But if the arcs for the tooth curves are to be marked by compasses, the location for the centres wherefrom to strike these arcs may be marked from the points of division as was shown in Fig. 130.

To construct a pattern wherefrom to cast a bevel gear-wheel.—When a pair of bevel-wheels are in gear and upon their respective shafts all the teeth on each wheel incline, as has been shown, to a single point, hence the pattern maker draws upon a piece of board a sketch representing the conditions under which the wheels are to operate. A sketch of this kind is shown in Fig. 160, in which A, B, C, D, represent in section the body of a bevel pinion. F G is the point of a tooth on one side, and E the point of a tooth on the other side of the pinion, while H I are pitch lines for the two teeth. Thus, the cone surface, the points, the pitch lines and the bottom of the spaces, projected as denoted by

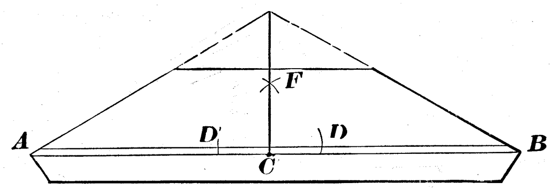

Fig. 161.

the dotted lines, would all meet at X, which represents the point where the axes of the shafts would meet.

In making wooden patterns wherefrom to cast the wheels, it is usual, therefore, to mark these lines on a drawing-board, so that they may be referred to by the workman in obtaining the degree of cone necessary for the body A B C D, to which the teeth are to be affixed. Suppose, then, that the diameter of the pinion is sufficiently small to permit the body A B C D to be formed of one piece instead of being put together in segments, the operation is as follows: The face D C is turned off on the lathe, and the piece is reversed on the lathe chuck, and the face A B is turned, leaving a slight recess at the centre to receive and hold the cone point true with the wheel. A bevel gauge is then set to the angle A B C, and the cone of the body is turned to coincide in angle with the gauge and to the required diameter, its surface being made true and straight so that the teeth may bed well. While turning the face D C in the lathe a fine line circle should be struck around the circumference of the cone and near D C, on which line the spacing for the teeth may be stepped off with the compasses. After this circle or line is divided off into as many equidistant points as there are to be teeth on the wheel, the points of division require to be drawn into lines, running across the cone surface of the wheel, and as the ordinary square is inapplicable for the purpose, a suitable square is improvised as follows: In Fig. 161 let the outline in full lines denote the body of a pinion ready to receive the teeth, and A B the circle referred to as necessary for the spacing or dividing with the compasses. On A B take any point, as C, as a centre, and with a pair of compasses mark equidistant on each side of it two lines, as D, D. From D, D as respective centres mark two lines, crossing each other as at F,

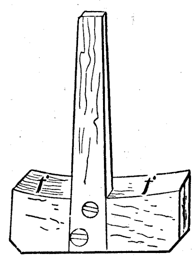

Fig. 162.

and draw a line, joining the intersection of the lines at F with C, and the last line, so produced, will be in the place in which the teeth are to lie; hence the wheel will require as many of these lines as it is to contain teeth, and the sides of the teeth, being set to these lines all around the pinion, will be in their proper positions, with the pitch lines pointing to X, in Fig. 160.

To avoid, however, the labor involved in producing these lines for each tooth, two other plans may be adopted. The first is to make a square, such as shown in Fig. 162, the face *f f* being fitted to the surface C, in Fig. 161, while the edges of its blade coincide with the line referred to; hence the edge of the blade may be placed coincident successively with each point of division, as D D, and the lines for the place of the length of each tooth be drawn. The second plan is to divide off the line A B before removing the body of the pinion from the lathe, and produce, as described, a line for one tooth. A piece of wood may then be placed so that when it lies on the surface of the hand-rest its upper surface will coincide with the line as shown in Fig. 163, in which W is the piece of wood, and A, B, C, &c., the lines referred to. If the teeth are to be glued and bradded to the body, they

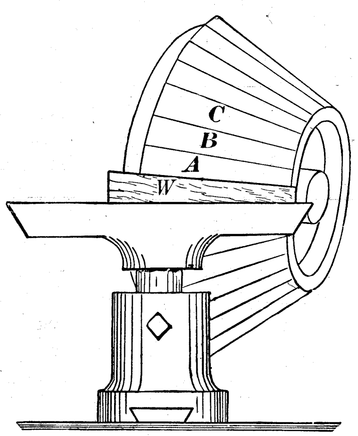

Fig. 163.

are first cut out in blocks, left a little larger every way than they are to be when finished, and the surfaces which are to bed on the cone are hollowed to fit it. Then blocks are glued to the body, one and the same relative side of each tooth being set fair to the lines. When the glue is dry, the pinion is again turned on the lathe, the gauge for the cone of the teeth being set in this case to the lines E, F G in Fig. 160. The pitch circles must then be struck at the ends of the teeth. The turned wheel is then ready to have the curves of the teeth marked. The wheel must now again be divided off on the pitch circle at the large end of the cone into as many equidistant points as there are to be teeth on

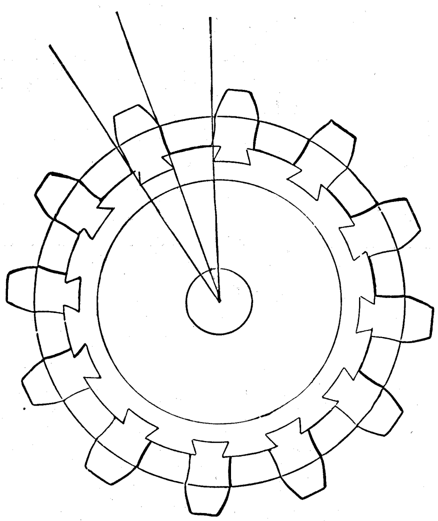

Fig. 164.

the wheel, and from these points, and on the same relative side of them, mark off a second series of points, distant from the points of division to an amount equal to the thickness the teeth are required to be. From these points draw in the outline of the teeth (upon the ends of the blocks to form the teeth) at the large end of the cone. Then, by use of the square, shown in Fig. 162, transfer the points of the teeth to the small end of the cone, and trace the outline of the teeth at the small end, taking centres and distances proportionate to the reduced diameter of the pitch circle at the small end, as shown in Fig. 160, where at J are three teeth so marked for the large end, and at K three for the small end, P P representing the pitch circle, and R R a circle for the compass points. The teeth for bevel pinions are sometimes put on by dovetails, as shown in Fig. 164, a plan which possesses points of advantage and disadvantage. Wood shrinks more across the grain than lengthwise with it, hence when the grain of the teeth crosses that of the body with every expansion or contraction of the wood (which always accompanies changes in the humidity of the atmosphere) there will be a movement between the two, because of the unequal expansion and contraction, causing the teeth to loosen or to move. In the employment of dovetails, however, a freedom of movement lengthways of the tooth is provided to accommodate the movement, while the teeth are detained in their proper positions. Again, if in making the founders' mould, one of the mould teeth should break or fall down when the pattern is withdrawn, a tooth may be removed from the pattern and used by the moulder to build up the damaged part of the mould again. And if the teeth of a bevel pinion are too much undercut on the flank curves to permit the whole pattern from being extracted from the mould without damaging it, dovetailed teeth may be drawn, leaving the body of the pattern to be extracted from the mould last. On the other hand, the dovetail is a costly construction if applied to large wheels. If the teeth are to be affixed by dovetails, the construction varies as follows: Cut out a wooden template of the dovetail, leaving it a little narrower than the thick-

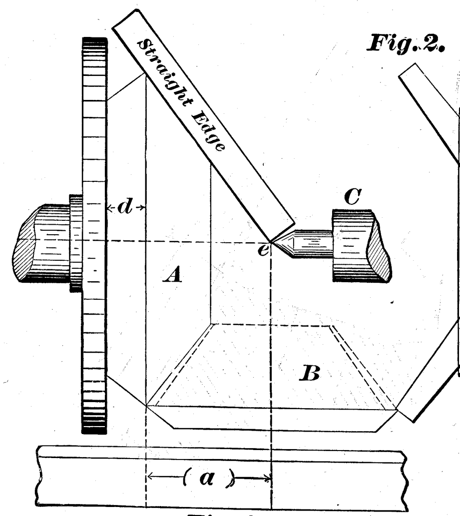

Fig. 1.

Fig. 165.

ness of the tooth at the root, and set the template on the cone at a distance from one of the lines A, B, C, Fig. 163, equal to the margin allowed between the edge of the dovetail and the side of the root of the tooth, and set it true by the employment of the square, shown in Fig. 162, and draw along the cone surface of the body lines representing the location of the dovetail grooves. The lines so drawn will give a taper toward X (Fig. 160), providing that, the template sides being parallel, each side is set to the square. While the body is in the lathe, a circle on each end may be struck for the depth of the dovetails, which should be cut out to gauge and to template, so that the teeth will interchange to any dovetail. The bottom of the dovetails need not be circular, but flat, which is easier to make. Dovetail pieces or strips are fitted to the grooves, being left to project slightly above the face of the cone or body. They are drawn in tight enough to enable them to keep their position while being turned in the lathe when the projecting points are turned down level with the cone of the body. The teeth may then be got out as described for glued teeth, and the dovetails added, each being marked to its place, and finally the teeth are cut to shape.

In wheels too large to have their cones tested by a bevel gauge, a wooden gauge may be made by nailing two pieces of wood to stand at the required angle as shown in Fig. 165, which is extracted from *The American Machinist*, or the dead centre C and a straightedge may be used as follows. In the figure the other wheel of the pair is shown dotted in at B, and the dead centre is set at the point where the axes of A and B would meet; hence if the largest diameter of the cone of A is turned to correct size, the cone will be correct when a straightedge applied as shown lies flat on the cone and meets the point of the dead centre E. The pinion B, however, is merely introduced to explain the principle, and obviously could not be so applied practically, the distance to set *e*, however, is the radius *a*.

Skew Bevel.*—When the axles of the shaft are inclined to each other instead of being in a straight line, and it is proposed to connect and communicate motion to the shafts by means of a

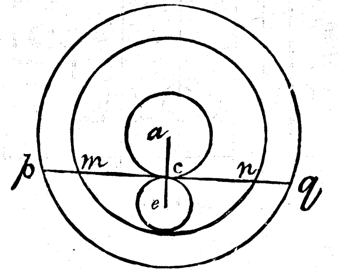

Fig. 166.

single pair of bevel-gears, the teeth must be inclined to the base of the frustra to allow them to come into contact.

To find the line of contact upon a given frustrum of the tangent-cone; let the Fig. 166 be the plane of the frustrum; *a* the centre. Set off *a e* equal to the shortest distance between the axes (called the *eccentricity*), and divide it in *c*, so that *a c* is to *e c* as the mean radius of the frustrum to the mean radius of that with which it is to work; draw *c p* perpendicular to *a e*, and meeting the circumference of the conical surface at *m*; perform a similar

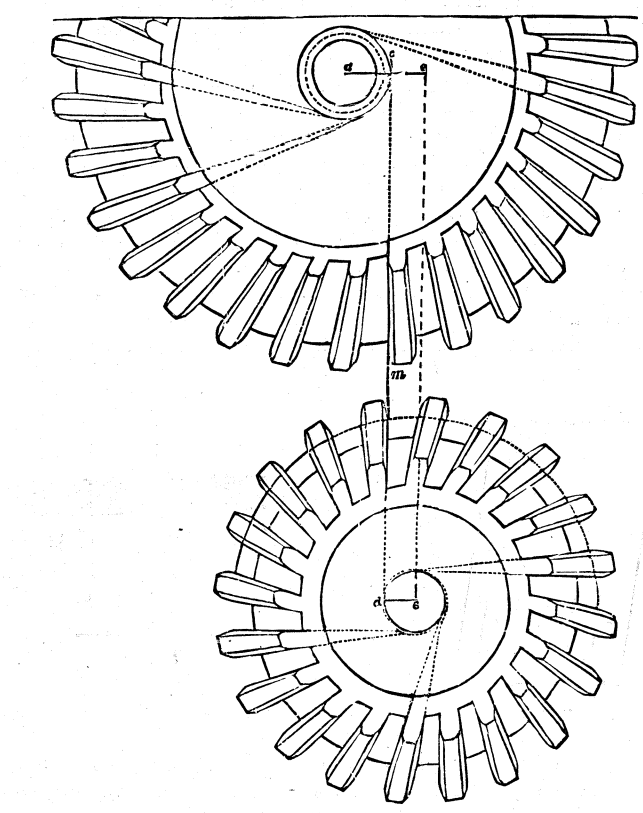

Fig. 167.

operation on the base of the frustrum by drawing a line parallel to *c m* and at the same distance *a c* from the centre, meeting the circumference in *p*.

The line *p c* is then plainly the line of direction of the teeth. We are also at liberty to employ the equally inclined line *c q* in the opposite direction, observing only that, in laying out the two wheels, the pair of directions be taken, of which the inclinations correspond.

Fig. 167 renders this mode of laying off the outlines of the wheels at once obvious. In this figure the line *a e* corresponds to the line marked by the same letters in Fig. 166; and the division of it at *c* is determined in the manner directed. The line *c m* being thus found in direction, it is drawn indefinitely to *d*. Parallel to this line and from the point *c* draw *e* to *e*, and in this line take the centre of the second wheel. The line *c m d* gives the direction of the teeth; and if from the centre *a* with radius at *c* a circle be described, the direction of any tooth of the wheel will be a tangent to it, as at *c*, and similarly if a centre *e* be taken in the line *e d*, and with radius *e d*, *c e* a circle be drawn, the direction of the teeth of the second wheel will be tangents to this last, as at *d*.

Having thus found the direction of the teeth, these outlines may be formed as in the case of ordinary bevel-wheels and with equal exactness and facility, all that is necessary being to find the curves for the teeth as described for bevel-wheels, and follow precisely the same construction, except that the square, Fig. 162, marking the lines across the cones, requires to be set to the angle for the tooth instead of at a right angle, and this angle may be found by the construction shown in Fig. 167, it being there represented by line *d c*. It is obvious, however, that the

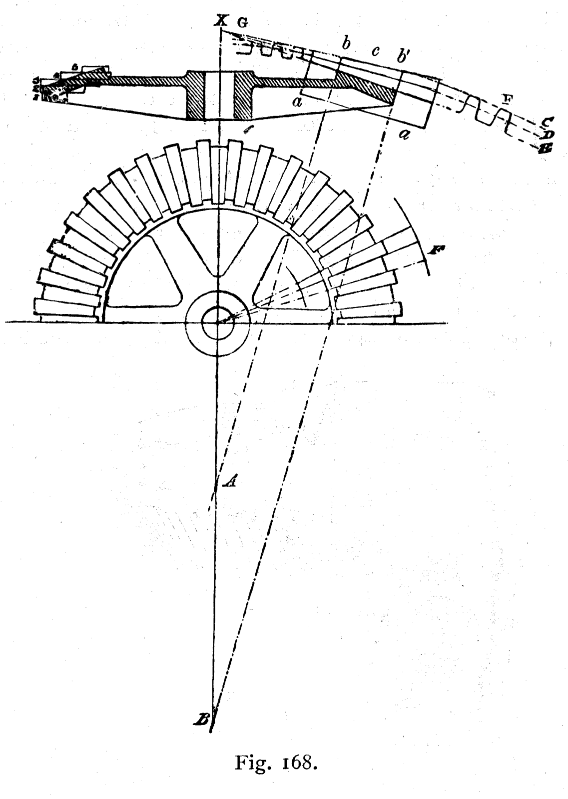

Fig. 168.

bottoms of the blocks to form the teeth must be curved to bed on the cone along the line *d c*, Fig. 167, and this may best be done by bedding two teeth, testing them by trial of the actual surfaces.

Then two teeth may be set in as No. 1 and No. 6 in the box shown in Fig. 148, the intermediate ones being dressed down to them.

Where a bevel-wheel pattern is too large to be constructed in one piece and requires to be built up in pieces, the construction is as in Fig. 168, in which on the left is shown the courses of segments 1, 2, 3, 4, 5, &c., of which the rim is built up (as described for spur wheels), and on the right is shown the finished rim with a tooth, *c*, in position.

The tooth proper is of the length of face of the wheel as denoted by *b b'*; now all the lines bounding the teeth must converge to the point X. Suppose, then, that the teeth are to be shaped for curve of face and flank in a box as described for spur-wheel teeth in Fig. 146, then in Fig. 168 let *a*, *a* represent the

* From the "Engineer and Machinists' Assistant."

bottom and *b b′* the top of the box, and *c* a tooth in the box, its ends filling the opening in the box at *b b′* then the curve on the sides of the box at *b′* must be of the form shown at F, and the curve on the sides of the box (at the point *b* of its length) must be as shown at G, the teeth shown in profile at G and F representing the forms of the teeth at their ends, on the outside of the wheel rim at *b′*, and on the inside at *b*; having thus made a box of the correct form on its sides, the teeth may be placed in it and planed down to it, thus giving all the teeth the same curve.

The spacing for the teeth and their fixing may be done as described for the bevel pinion.

To construct a pattern wherefrom to cast an endless screw, worm, or tangent screw, which is to have the worm or thread cut in a lathe.—Take two pieces, each to form one longitudinal half of the pattern; peg and screw them together at the ends, an

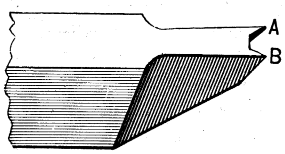

Fig. 169.

excess of stuff being allowed at each end for the accommodation of two screws to hold the two halves together while turning them in the lathe, or dogs, if the latter are more convenient, as they might be in a large pattern. Turn the piece down to the size over the top of the thread, after which the core prints are turned. The body thus formed will be ready to have the worm or thread cut, and for this purpose the tools shown in Figs. 169 and 140 are necessary.

That shown in Fig. 169 should be flat on the face similar to a parting tool for cast iron, but should have a great deal more bottom rake, as strength is not so much an object, and the tool is more easily sharpened. It has also in addition two little projections A B like the point of a penknife, formed by filing away the steel in the centre; these points are to cut the fibres of the wood, the severed portion being scraped away by the flat part of the tool.

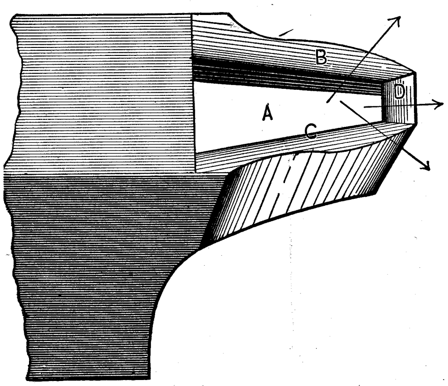

Fig. 170.

The degree of side rake given to the tool must be sufficient to let the tool sides well clear the thread or worm, and will therefore vary with the pitch of the worm.

The width of the tool must be a shade narrower than the narrowest part of the space in the worm. Having suitably adjusted the change wheels of the lathe to cut the pitch required the parting tool is fed in until the extreme points reach the bottom of the spaces, and a square nosed parting tool without any points or spurs will finish the worm to the required depth. This will have left a square thread, and this we have now to cut to the required curves on the thread or worm sides, and as the cutting will be performed on the end grain of the wood, the top face of the tool must be made keen by piercing through the tool a slot A, Fig. 170, and filing up the bevel faces B, C and D, and then carefully oilstoning them. This tool should be made slightly narrower than the width of the worm space, so that it may not cut on both sides at once, as it would have too great a length of cutting edge.

Furthermore, if the pattern is very large, it will be necessary to have two tools for finishing, one to cut from the pitch line inwards and the other to complete the form from the pitch line outwards. It is advisable to use hard wood for the pattern.

If it is decided to cut the thread by hand instead of with these lathe tools, then, the pattern being turned as before, separate the two halves by taking out the screws at the ends; select the half

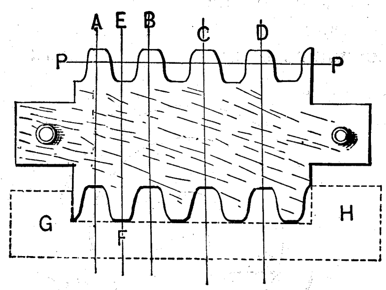

Fig. 171.

that has not the pegs, as being a little more convenient for tracing lines across. Set out the sections of the thread, A, B, C, and D, Fig. 171, similar to a rack; through the centres of A, B, C, and D, square lines across the piece; these lines, where they intersect the pitch line, will give the centres of teeth on that side: or if we draw lines, as E, F, through the centres of the spaces, they will pass through the centres of the teeth (so to speak) on the other side; in this position complete the outline on that side. It will be found, in drawing these outlines, that the centres of some of the arcs will lie outside the pattern. To obtain support for the compasses, we must fit over the pattern a piece of board such as shown by dotted lines at G H.

It now remains to draw in the top of the thread upon the curved surface of the half pattern; for this purpose take a piece of stiff card or other flexible material, wrap it around the pattern and fix it temporarily by tacks, we then trim off the edges true to the pattern, and mark upon the edges of the card the position of the tops of the thread upon each side; we remove the card and spread it out on a flat surface, join the points marked on the edges by lines as in Fig. 172, replace the card exactly as before upon the pattern, and with a fine scriber we prick through the lines. The cutting out is commenced by sawing, keeping, of course, well within the lines; and it is facilitated by attaching a stop to the saw so as to insure cutting at all parts nearly to the exact depth. This stop is a simple strip of wood and may be clamped to the saw, though it is much more convenient to have a couple of holes in the saw blade for the passage of screws. For

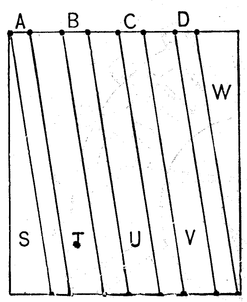

Fig. 172.

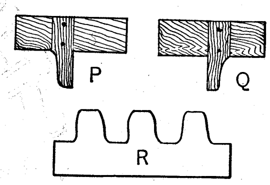

Fig. 173.

finishing, a pair of templates, P and Q, Fig. 173, right and left, will be found useful; and finally the work should be verified and slight imperfections corrected by the use of a form or template taking in three spaces, as shown at R in Fig. 173. In drawing the lines on the card, we must consider whether it is a right or left-handed worm that we desire. In the engraving the lines are those suitable for a right-handed thread. Having completed one half of the pattern, place the two halves together, and trace off the half that is uncut, using again the card template for drawing the lines on the curved surface. The cutting out will be the same as before.

As the teeth of cast wheels are, from their deviation from accuracy in the tooth curves and the concentricity of the teeth to the wheel centre, apt to create noise in running, it is not unusual to cast one or both wheels with mortises in the rim to receive wooden teeth. In this case the wheel is termed a mortise wheel, and the teeth are termed *cogs*. If only one of a pair of wheels is to be cogged, the largest of the pair is usually selected, because there are in that case more teeth to withstand the wear, it being obvious that the wear is greatest upon the wheel having the fewest teeth, and that the iron wheel or pinion can better withstand the wear than the mortise wheel. The woods most used for cogs are hickory, maple, hornbeam and locust. The blocks wherefrom the teeth are to be formed are usually cut out to nearly the required dimensions, and kept in stock, so as to be thoroughly well-seasoned when required for use, and, therefore

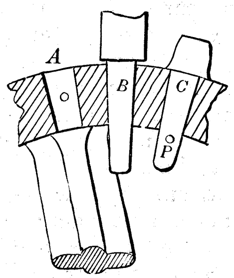

Fig. 174.

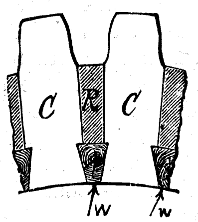

Fig. 175.

less liable to come loose from shrinkage after being fitted to the mortise in the wheel. The length of the shanks is made sufficient to project through the wheel rim and receive a pin, as shown in Fig. 174, in which B is a blank tooth, and C a finished tooth inserted in the wheel, the pin referred to being at P. But, if a mortise should fall in an arm of the wheel, this pin-hole must pass through the rim, as shown in the mortise A. The wheel, however, should be designed so that the mortises will not terminate in the arms of the wheel.

Another method of securing the teeth in the mortises is to dovetail them at the small end and drive wedges between them, as shown in Fig. 175, in which C C are two contiguous teeth, R the wheel rim and W W two of the wedges. On account of the dovetailing the wedges exert a pressure pressing the teeth into the mortises. This plan is preferable to that shown in the Fig. 174 inasmuch as from the small bearing area of the pins they become

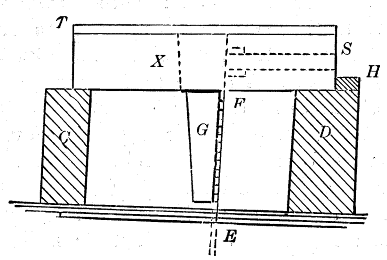

Fig. 176.

loose quicker, and furthermore there is more elasticity to take up the wear in the case of the wedges.

The mortises are first dressed out to a uniform size and taper, using two templates to test them with, one of which is for the breadth and the other for the width of the mortise. The height above the wheel requires to be considerably more than that due to the depth of the teeth, so that the surface bruised by driving the cogs or when fitting them into the mortises may be cut off. To avoid this damage as much as possible, a broad-face hammer should be employed—a copper, lead, lignum vitæ, or a raw hide hammer being preferable, and the last the best. The teeth are got out in a box and two guides, such as shown in Figs. 176, 177, and 178, similar letters of reference denoting the same parts in all three illustrations.

In Fig. 176, X is a frame or box containing and holding the operative part of the tooth, and resting on two guides C D. The height of D from the saw table is sufficiently greater than that of C to give the shank G the correct taper, E F representing the circular saw. T is a plain piece of the full size of the box or frame, and serving simply to close up on that side the mortise in the frame. The grain of T should run at a right angle to the other piece of the frame so as to strengthen it. S is a binding screw to hold the cog on the frame, and H is a guide for the edge of the frame to slide against. It is obvious, now, that if the piece D be adjusted at a proper distance from the circular saw E F, and the edge of the frame be moved in contact with the guide H,

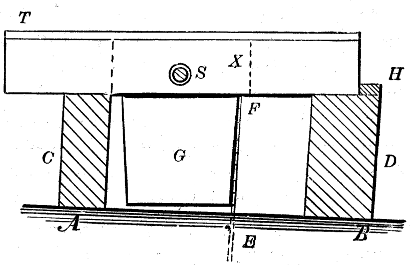

Fig. 177.

one side of the tooth shank will be sawn Then, by reversing the frame end for end, the other side of the shank may be sawn. Turning the frame to a right angle the edges of the cog shank can be sawn from the same box or frame, and pieces C, D, as shown in Fig. 177.

The frame is now stood on edge, as in Fig. 178, and the underneath surfaces sawed off to the depth the saw entered when the shank taper was sawn. This operation requires to be performed on all four sides of the tooth.

After this operation is performed on one cog, it should be tried in the wheel mortises, to test its correctness before cutting out the shanks on all the teeth.

The shanks, being correctly sawn, may then be fitted to the mortises, and let in within $\frac{1}{8}$ of butting down on the face of the wheel, this amount being left for the final driving. The cogs should be numbered to their places, and two of the mortises must be numbered to show the direction in which the numbers proceed. To mark the shoulders (which are now square) to the curvature of the rim, a fork scriber should be used, and the shanks of the cogs should have marked on them a line coincident with the inner edge of the wheel rim. This line serves as a guide in marking the pin-holes and for cutting the shanks to length; but it is to be remembered that the shanks will pass farther through to the amount of the distance marked by the fork scriber. The holes for the pins which pass through the shanks should be made

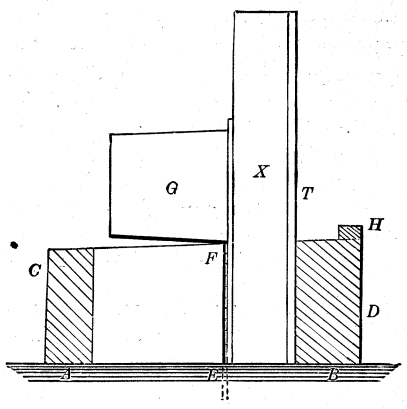

Fig 178.

slightly less in their distances (measured from the nearest edge of the pin-hole) from the shoulders of the cogs than is the thickness of the rim of the wheel, so that when the cogs are driven fully home the pin-holes will appear not quite full circles on the inside of the wheel rim; hence, the pins will bind tightly against the inside of the wheel rim, and act somewhat as keys, locking and drawing the shanks to their seats in the mortises.

In cases where quietness of running is of more consequence than the durability of the teeth, or where the wear is not great, both wheels may be cogged, but as a rule the larger wheel is cogged, the smaller being of metal. This is done because the teeth of the smaller wheel are the most subject to wear. The teeth of the cogged wheel are usually made the thickest, so as to somewhat equalise the strength of the teeth on the two wheels.

Since the power transmitted by a wheel in a given time is composed of the pressure or weight upon the wheel, and the space a point on the pitch circle moves through in the given time, it is obvious that in a train of wheels single geared, the velocities of all the wheels in the train being equal at the pitch circle, the teeth require to be of equal pitch and thickness throughout the train. But when the gearing is compounded the variation of velocity at the pitch circle, which is due to the compounding, has an important bearing upon the necessary strength of the teeth.

Suppose, for example, that a wheel receives a tooth pressure of 100 lbs. at the pitch circle, which travels at the velocity of 100 feet per minute, and is keyed to the same shaft with another wheel whose velocity is 50 feet per minute. Now, in the power transmitted by the two wheels the element of time is 50 for one wheel and 100 for the other, hence the latter (supposing both wheels to have an equal number of teeth in contact with their driver or follower as the case may be) will be twice as strong in proportion to the duty, and it appears that in compounded gearing the strength in proportion to the duty may be varied in proportion as the velocity is modified by compounding of the wheels. Thus, when the velocity at the pitch circle is increased its strength is increased, and per contra when its velocity is decreased its strength is decreased, when considered in proportion to the duty. When, however, the wheels are upon long shafts, or when they overhang the bearing of the shaft, the corner contact will from tension of the shaft, continue much longer than when the shaft is maintained rigid.

It is obvious that if a wheel transmits a certain amount of power, the pressure of tooth upon tooth will depend upon the number of teeth in contact, but since, in the case of very small wheels, that is to say, pinions of the smallest diameter of the given pitch that will transmit continuous motion, it occurs that only one tooth is in continuous contact, it is obvious that each single tooth must have sufficient strength to withstand the whole of the pressure when worn to the limits to which the teeth are supposed to wear. But when the pinion is so small that it has but one tooth in continuous contact, that contact takes place nearer the line of centres and to the root of the tooth, and therefore at a less leverage to the line of fracture, hence the ultimate strength of the tooth is proportionately increased. On the other hand, however, the whole stress of the wheel being concentrated on the arc of contact of one tooth only (instead of upon two or more teeth as in larger wheels), the wear is proportionately greater; hence, in a short time the teeth of the pinion are found to be thinner than those on the other wheel or wheels. The multiplicity of conditions under which small wheels may work with relation to the number of teeth in contact, the average leverage of the point of contact from the root of the tooth, the shape of the tooth, &c., renders it desirable in a general rule to suppose that the whole strain falls upon one tooth, so that the calculation shall give results to meet the requirements when a single tooth only is in continuous contact.

It follows, then, that the thickness of tooth arrived at by calculation should be that which will give to a tooth, when worn to the extreme thinness allowed, sufficient strength (with a proper margin of safety) to transmit the whole of the power transmitted by the wheel.

The margin (or factor) of safety, or in other words, the number of times the strength of the tooth should exceed the amount of power transmitted, varies (according to the conditions under which the wheels work) between 5 and 10.

The lesser factor may be used for slow speeds when the power is continuously and uniformly transmitted. The greater factor is necessary when the wheels are subjected to violent shocks and the direction of revolution requires to be reversed.

In pattern-cast teeth, contact between the teeth of one wheel and those of the other frequently occurs at one corner only, as shown in Fig. 179, and the line of fracture is in the direction denoted by the diagonal dotted lines. The causes of this corner contact have been already explained, but it may be added that as the wheels wear, the contact extends across the full breadths of the teeth, and the strength in proportion to the duty, therefore, steadily increases from the time the new wheels have action until the wear has caused contact fully across the breadth. Tredgold's rule for finding the proper thickness of tooth for a given stress upon cast-iron teeth loaded at the corner as in Fig. 179 and supposed to have a velocity of three feet per second of time, is as follows :—

Rule.—Divide the stress in pounds at the pitch circle by 1500, and the square root of the quotient is the required thickness of tooth in inches or parts of an inch.

In the results obtained by the employment of this rule, an allowance of one-third the thickness for wear, and the margin for safety is included, so that the thickness of tooth arrived at is that to be given to the actual tooth. Further, the rule supposes the breadth of the tooth to be not less than twice the height of the same, any extra breadth not affecting the result (as already explained), when the pressure falls on a corner of the tooth.

In practical application, however, the diameter of the wheel at the pitch circle is generally, or at least often a fixed quantity, as well as the amount of stress, and it will happen as a rule that taking the stress as a fixed element and arriving at the thickness of the tooth by calculation, the required diameter of wheel, or what is the same thing, its circumference, will not be such as to

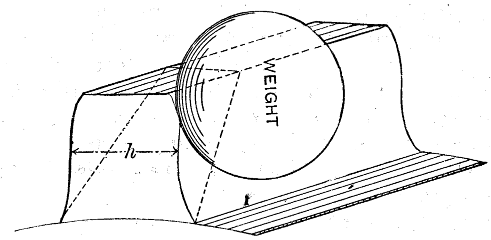

Fig. 179.

contain the exact number of teeth of the thickness found by the calculation, and still give the desired amount of side clearance. It is desirable, therefore, to deal with the stress upon the tooth at the pitch circle, and the diameter, radius, or circumference of the pitch circle, and its velocity, and deduce therefrom the required thickness for the teeth, and conform the pitch to the requirements as to clearance from the tooth thickness thus obtained.

To deduce the thickness of the teeth from these elements we have Robertson Buchanan's rule, which is as follows :—

Find the amount of horse-power employed to move the wheel, and divide such horse-power by the velocity in feet per second of the pitch line of the wheel. Extract the square root of the quotient, and three-fourths of this root will be the least thickness of the tooth. To the result thus obtained, there must be added the allowance for wear of the teeth and the width of the space including the clearance which will determine the number of teeth in the wheel.

In conforming strictly to this rule the difficulty is met with that it would give fractional pitches not usually employed and difficult to measure on an existing wheel. Cast wheels kept on hand or in stock by machinists have usually the following standard :—

Beginning with an inch pitch, the pitches increase by $\frac{1}{8}$ inch up to 3-inch pitch, from 3 to 4-inch pitches the increase is by $\frac{1}{4}$ inch, and from 4-inch pitch and upwards the increase is by $\frac{1}{2}$ inch. Now, under the rule the pitches would, with the clearance made to bear a certain proportion to the pitch, be in odd fractions of an inch.

It appears then, that, if in a calculation to obtain the necessary thickness of tooth, the diameter of the pitch circle is not an element, the rule cannot be strictly adhered to unless the diameter of the pitch circle be varied to suit the calculated thickness of tooth; or unless either the clearance, factor of safety, or amount of tooth thickness allowed for wear be varied to admit of the thickness of tooth arrived at by the calculation. But if the diameter of the pitch circle is one of the elements considered in arriving at the thickness of tooth requisite under given conditions, the pitch must, as a rule, either be in odd fractions, or else the allowance for wear, factor of safety, or amount of side clearance cannot bear a definite proportion to the pitch. But the allowance for clearance is in practice always a constant proportion of the pitch, and under these circumstances, all that can be done when the circumstances require a definite circumference of pitch circle, is to select such a pitch as will nearest meet the requirements of tooth thickness as found by calculation, while following the rule of making the clearance a constant proportion of the pitch. When following this plan gives a thinner tooth than the calculation calls for, the factor of safety and the allowance for wear are reduced. But this is of little consequence whenever more than one tooth on each wheel is in contact, because the rules provide for all the stress falling on one tooth. When, however, the number of teeth in the pinion is so small that one tooth only is in contact, it is better to select a pitch that will give a thicker rather than a thinner tooth than called for by the calculation, providing, of course, that the pitch be less than the arc of contact, so that the motion shall be continuous.

But when the pinions are shrouded, that is, have flanges at each end, the teeth are strengthened; and since the wear will continue greater than in wheels having more teeth in contact, the shrouding may be regarded as a provision against breakage in consequence of the reduction of tooth thickness resulting from wear.

In the following table is given the thickness of the tooth for a given stress at the pitch circle, calculated from Tredgold's rule for teeth supposed to have contact when new at one corner only.

Stress in lbs. at pitch circle.	Thickness of tooth in inches.	Actual pitches to which wheels may be made.
400	·52	$1\frac{1}{8}$ to $1\frac{1}{4}$
800	·75	$1\frac{1}{2}$,, $1\frac{5}{8}$
1,200	·90	$1\frac{7}{8}$,, 2
1,600	1·03	2 ,, $2\frac{1}{8}$
2,000	1·15	$2\frac{1}{4}$,, $2\frac{3}{8}$
2,400	1·26	$2\frac{1}{2}$,, $2\frac{5}{8}$
2,800	1·36	$2\frac{5}{8}$,, $2\frac{3}{4}$
3,200	1·43	$2\frac{7}{8}$,, 3
3,600	1·56	$3\frac{1}{8}$,, $3\frac{1}{4}$
4,000	1·63	$3\frac{1}{4}$,, $3\frac{3}{8}$
4,400	1·70	$3\frac{3}{8}$,, $3\frac{1}{2}$
4,800	1·78	$3\frac{1}{2}$,, $3\frac{5}{8}$
5,200	1·86	$3\frac{5}{8}$,, $3\frac{3}{4}$
5,600	1·93	$3\frac{3}{4}$,, 4
6,000	2·00	4 ,, $4\frac{1}{4}$

In wheels that have their teeth cut to form in a gear-cutting machine the thickness of tooth at any point in the depth is equal at any point across the breadth; hence, supposing the wheels to be properly keyed to their shafts so that the pitch line across the breadth of the wheel stands parallel to the axis of the shaft, the contact of tooth upon tooth occurs across the full breadth of the tooth.

As the practical result of these conditions we have three important advantages: first, that the stress being exerted along the full breadth of the tooth instead of on one corner only, the tooth is stronger (with a given breadth and thickness) in proportion to the duty; second, that with a given pitch, the thickness and therefore the margin for safety and allowance for wear are increased, because the tooth may be increased in thickness at the expense of the clearance, which need be merely sufficient to prevent contact on both sides of the spaces so as to prevent the teeth from locking in the spaces; and thirdly, because the teeth will not be subject to sudden impacts or shocks of tooth upon tooth by reason of back lash.

In determining the strength of cut gear-teeth we may suppose the weight to be disposed along the face at the extreme height of the tooth, in which case the theoretical shape of the tooth to possess equal strength at every point from the addendum circle to the root would be a parabola, as shown by the dotted lines in Fig. 180, which represents a tooth having radial flanks. In this case it is evident that the ultimate strength of the tooth is that due to the thickness at the root, because it is less than that at the pitch circle, and the strength, as a whole, is not greater than that at the weakest part. But since teeth with radial flanks are produced, as has been shown, with a generating circle equal in diameter to the radius of the pinion, and since with a generating circle bearing that ratio of diameter to diameter of pitch circle the acting part of the flank is limited, it is usual to fill in the

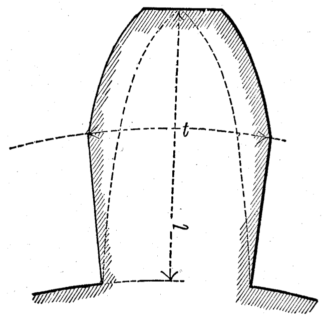

Fig. 180.

corners with fillets or rounded corners, as shown in Fig. 129; hence, the weakest part of the tooth will be where the radial line of the flank joins the fillet and, therefore, nearer the pitch circle than is the root. But as only the smallest wheel of the set has radial flanks and the flanks thicken as the diameter of the wheels increase, it is usual to take the thickness of the tooth at the pitch circle as representing the weakest part of the tooth, and, therefore, that from which the strength of the tooth is to be computed. This, however, is not actually the case even in teeth which have considerable spread at the roots, as is shown in Fig. 181, in which the shape of the tooth to possess equal strength throughout its depth is denoted by the parabolic dotted lines.

Considering a tooth as simply a beam supporting the strain as a weight we may calculate its strength as follows:—

Multiply the breadth of the tooth by the square of its thickness, and the product by the strength of the material, per square inch of section, of which the teeth are composed, and divide this last

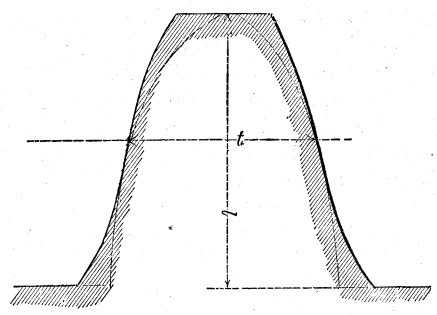

Fig. 181.

product by the distance of the pitch line from the root, and the quotient will give a tooth thickness having a strength equal to the weight of the load, but having no margin for safety, and no allowance for wear; hence, the result thus obtained must be multiplied by the factor of safety (which for this class of tooth may be taken as 6), and must have an additional thickness added to allow for wear, so that the factor of safety will be constant notwithstanding the wear.

Another, and in some respects more convenient method, for obtaining the strength of a tooth, is to take the strength of a tooth having 1-inch pitch, and 1 inch of breadth, and multiply this quantity of strength by the pitch and the face of the tooth it is required to find the strength of, both teeth being of the same material.

Example.—The safe working pressure for a cast-iron tooth of an inch pitch, and an inch broad will transmit, being taken as 400 lbs., what pressure will a tooth of $\frac{3}{4}$-inch pitch and 3 inches broad transmit with safety?

Here 400 lbs. × $\frac{3}{4}$ pitch × 3 breadth = 900 = safe working pressure of tooth $\frac{3}{4}$-inch pitch and 3 inches broad.

Again, the safe working pressure of a cast-iron tooth, 1 inch in breadth and of 1-inch pitch, being considered as 400 lbs., what is the safe working pressure of a tooth of 1-inch pitch and 4-inch breadth?

$$400 \times 1 \times 4 = 1600.$$

The philosophy of this is apparent when we consider that four wheels of 1-inch pitch and an inch face, placed together side by side, would constitute, if welded together, one wheel of an inch pitch and 4 inches face. (The term *face* is applied to the wheel, and the term breadth to the tooth, because such is the custom of the workshop, both terms, however, mean, in the case of spur-wheels, the dimension of the tooth in a direction parallel to the axis of the wheel shaft or wheel bore.)

The following table gives the safe working pressures for wheels having an inch pitch and an inch face when working at the given velocities, S.W.P. standing for "safe working pressure:"—

Velocity of pitch circle in feet per second.	S.W.P. for cast-iron spur gears.	S.W.P. for spur mortise gears.	S.W.P. for cast-iron bevel gears.	S.W.P. for bevel mortise gear.
2	368	178	258	178
3	322	178	225	157
6	255	178	178	125
12	203	142	142	99
18	177	124	124	87
24	161	113	113	79
30	150	105	105	74
36	140	98	98	69
42	133	93	93	65
48	127	88	88	62

For velocities less than 2 feet per second, use the same value as for 2 feet per second.

The proportions, in terms of the pitch, upon which this table is based, are as follows:—

Thickness of iron teeth	.	395 of the pitch.
" wooden "	.	·595 "
Height of addendum	. .	·28 "
Depth below pitch line	. .	·32 "

The table is based upon 400 lbs. per inch of face for an inch pitch, as the safe working pressure of mortise wheel teeth or cogs; it may be noted that there is considerable difference of opinion. They are claimed by some to be in many cases practically stronger than teeth of cast iron. This may be, and probably is, the case when the conditions are such that the teeth being rigid and rigidly held (as in the case of cast-iron teeth), there is but one tooth on each wheel in contact. But when there is so nearly contact between two teeth on each wheel that but little elasticity in the teeth would cause a second pair of teeth to have contact, then the elasticity of the wood would cause this second contact. Added to this, however, we have the fact that under conditions where violent shock occurs the cog would have sufficient elasticity to give, or spring, and thus break the shock which cast iron would resist to the point of rupture. It is under these conditions, which mainly occur in high velocities with one of the wheels having cast teeth, that mortise wheels, or cogging, is employed, possessing the advantage that a broken or worn-out tooth, or teeth, may be readily replaced. It is usual, however, to assign to wooden teeth a value of strength more nearly equal to that of its strength in proportion to that of cast iron; hence, Thomas Box allows a wood tooth a value of about $\frac{3}{10}$ths the strength of cast iron; a value as high as $\frac{7}{10}$ths is, however, assigned by other authorities. But the strength of the tooth cannot exceed that at the top of the shank, where it fits into the mortise of the wheel, and on account of the leverage of the pressure the width of the mortise should exceed the thickness of the tooth.

In some practice, the mortise teeth, or cogs, are made thicker in proportion to the pitch than the teeth on the iron wheel; thus Professor Unwin, in his "Elements of Machine Design," gives the following as "good proportions":—

Thickness of iron teeth	. . . .	0·395 of the pitch.
" wood cogs	. . . .	0·595 " "

which makes the cogs $\frac{2}{10}$ths inch thicker than the teeth.

The mortises in the wheel rim are made taper in both the breadth and the width, which enables the tooth shank to be more accurately fitted, and also of being driven more tightly home, than if parallel. The amount of this taper is a matter of judgment, but it may be observed that the greater the taper the more labor there is involved in fitting, and the more strain there is thrown upon the pins when locking the teeth with a given amount of strain. While the less the taper, the more care required to obtain an accurate fit. Taking these two elements into consideration, $\frac{1}{8}$th inch of taper in a length of 4 inches may be given as a desirable proportion.

As an evidence of the durability of wooden teeth, there appeared in *Engineering* of January 7th, 1879, the illustration shown in Fig. 182, which represents a cog from a wheel of 14 ft. $\frac{1}{2}$ in. diameter, and having a 10-inch face, its pinion being 4 ft. in diameter. This cog had been running for $26\frac{1}{2}$ years, day and night; not a cog in the wheel having been touched during that time. Its average revolutions were 38 per minute, the power

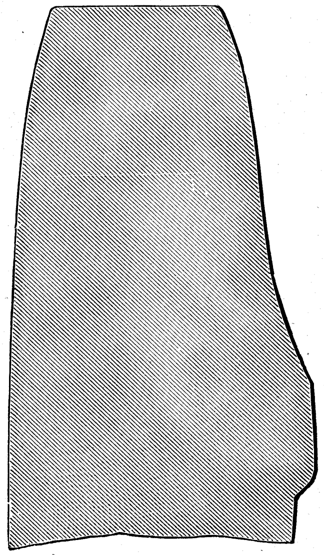

Fig. 182.

developed by the engine being from 90 to 100 indicated horse-power. The teeth were composed of beech, and had been greased twice a week, with tallow and plumbago ore.

Since the width of the face of a wheel influences its wear (by providing a larger area of contact over which the pressure may be distributed, as well as increasing the strength), two methods of proportioning the breadth may be adopted. First, it may be made a certain proportion of the pitch; and secondly, it may be proportioned to the pressure transmitted and the number of revolutions. The desirability of the second is manifest when we consider that each tooth will pass through the arcs of contact (and thus be subjected to wear) once during each revolution; hence, by making the number of revolutions an element in the calculation to find the breadth, the latter is more in proportion to the wear than it would be if proportioned to the pitch.

It is obvious that the breadth should be sufficient to afford the required degree of strength with a suitable factor of safety, and allowance for wear of the smallest wheel in the pair or set, as the case may be.

According to Reuleaux, the face of a wheel should never be less than that obtained by multiplying the gross pressure, transmitted in lbs., by the revolutions per minute, and dividing the product by 28,000.

In the case of bevel-wheels the pitch increases, as the perimeter of the wheel is approached, and the maximum pitch is usually taken as the designated pitch of the wheel. But the mean pitch is that which should be taken for the purposes of calculating the strength, it being in the middle of the tooth breadth. The mean pitch is also the diameter of the pitch circle, used for ascertaining the velocity of the wheel as an element in calculating the safe pressure, or the amount of power the wheel is capable of transmitting, and it is upon this basis that the values for bevel-wheels in the above table are computed.

In many cases it is required to find the amount of horse-power a wheel will transmit, or the proportions requisite for a wheel to transmit a given horse-power; and as an aid to the necessary calculations, the following table is given of the amount of horse-power that may be transmitted with safety, by the various wheels at the given velocities, with a wheel of an inch pitch and an inch face, from which that for other pitches and faces may be obtained by proportion.

TABLE SHOWING THE HORSE-POWER WHICH DIFFERENT KINDS OF GEAR-WHEELS OF ONE INCH PITCH AND ONE INCH FACE WILL SAFELY TRANSMIT AT VARIOUS VELOCITIES OF PITCH CIRCLE.

Velocity of Pitch Circle in Feet per Second.	Spur-Wheels. H.P.	Spur Mortise Wheels. H.P.	Bevel-Wheels. H.P.	Bevel Mortise Wheels. H.P.
2	1·338	·647	·938	·647
3	1·756	·971	1·227	·856
6	2·782	1·76	1·76	1·363
12	4·43	3·1	3·1	2·16
18	5·793	4·058	4·058	2·847
24	7·025	4·931	4·931	3·447
30	8·182	5·727	5·727	4·036
36	9·163	6·414	6·414	4·516
42	10·156	7·102	7·102	4·963
48	11·083	7·680	7·680	5·411

In this table, as in the preceding one, the safe working pressure for 1-inch pitch and 1-inch breadth of face is supposed to be 400 lbs.

In cast gearing, the mould for which is made by a gear moulding machine, the element of draft to permit the extraction of the pattern is reduced: hence, the pressure of tooth upon tooth may be supposed to be along the full breadth of the tooth instead of at one corner only, as in the case of pattern-moulded teeth. But from the inaccuracies which may occur from unequal contraction in the cooling of the casting, and from possible warping of the casting while cooling, which is sure to occur to some extent, however small the amount may be, it is not to be presumed that the contact of the teeth of one wheel will be in all the teeth as perfect across the full breadth as in the case of machine-cut teeth. Furthermore, the clearance allowed for machine-moulded teeth, while considerably less than that allowed for pattern-moulded teeth, is greater than that allowed for machine-cut teeth; hence, the strength of machine-moulded teeth in proportion to the pitch lies somewhere between that of pattern-moulded and machine-cut teeth—but exactly where, it would be difficult to determine in the absence of experiments made for the purpose of ascertaining.

It is not improbable, however, that the contact of tooth upon tooth extends in cast gears across at least two-thirds of the breadth of the tooth, in which case the rules for ascertaining the strength of cut teeth of equal thickness may be employed, substituting ⅔rds of the actual tooth breadth as the breadth for the purposes of the calculation.

If instead of supposing all the strain to fall upon one tooth and calculating the necessary strength of the teeth upon that basis (as is necessary in interchangeable gearing, because these conditions may exist in the case of the smallest pinion that can be used in pitch), the actual working condition of each separate application of gears be considered, it will appear that with a given diameter of pitch circle, all other things being equal, the arc of contact will remain constant whatever the pitch of the teeth, or in other words is independent of the pitch, and it follows that when the thickness of iron necessary to withstand (with the allowances for wear and factor of safety) the given stress under the given velocity has been determined, it may be disposed in a coarse pitch that will give one tooth always in contact, or a finer pitch that will give two or more teeth always in contact, the strength in proportion to the duty remaining the same in both cases.

In this case the expense of producing the wheel patterns or in trimming the teeth is to be considered, because if there are a train of wheels the finer pitch would obviously involve the construction and dressing to shape of a much greater number of teeth on each wheel in the train, thus increasing the labor. When, however, it is required to reduce the pinion to a minimum diameter, it is obvious that this may be accomplished by selecting the finer pitch, because the finer the pitch, the less the diameter of the wheel may be. Thus with a given diameter of pitch circle it is possible to select a pitch so fine that motion from one wheel may be communicated to another, whatever the diameter of the pitch circle may be, the limit being bounded by the practicability of casting or producing teeth of the necessary fineness of pitch. The durability of a wheel having a fine pitch is greater for two reasons: first, because the metal nearest the cast surface of cast iron is stronger than the internal metal, and the finer pitch would have more of this surface to withstand the wear; and second, because in a wheel of a given width there would be two points, or twice the area of metal, to withstand the abrasion, it being remembered that the point of contact is a line which partly rolls and partly slides along the depth of the tooth as the wheel rotates, and that with two teeth in contact on each wheel there are two of such lines. There is also less sliding or rubbing action of the teeth, but this is offset by the fact that there are more teeth in contact, and that there are therefore a greater number of teeth simultaneously rubbing or sliding one upon the other.

But when we deal with the number of teeth the circumstances are altered; thus with teeth of epicycloidal form it is manifestly impossible to communicate constant motion with a driving wheel having but one tooth, or to receive motion on a follower having but one tooth. The number of teeth must always be such that there is at all times a tooth of each wheel within the arc of action, or in contact, so that one pair of teeth may come into contact before the contact of the preceding teeth has ceased.

In the construction of wheels designed to transmit power as well as simple motion, as is the case with the wheels employed in machine work, however, it is not considered desirable to employ wheels containing a less number of teeth than 12. The diameter of the wheel bearing such a relation to the pitch that both wheels containing the same number of teeth (12), the motion will be communicated from one to the other continuously.

It is obvious that as the number of teeth in one of the wheels (of a pair in gear) is increased the number of teeth in the other may be (within certain limits) diminished, and still be capable of transmitting continuous motion. Thus a pinion containing, say 8 teeth, may be capable of receiving continuous motion from a rack in continuous motion, while it would not be capable of receiving continuous motion from a pinion having 4 teeth; and as the requirements of machine construction often call for the transmission of motion from one pinion to another of equal diameters, and as small as possible, 12 teeth are the smallest number it is considered desirable for a pinion to contain, except it be in the case of an internal wheel, in which the arc of contact is greater in proportion to the diameters than in spur-wheels, and continuous motion can therefore be transmitted either with coarser pitches or smaller diameters of pinion.

For convenience in calculating the pitch diameter at pitch circle, or pitch diameter as it is termed, and the number of teeth of wheels, the following rules and table extracted from the *Cincinnati Artisan* and arranged from a table by D. A. Clarke, are given. The first column gives the pitch, the following nine columns give the pitch diameters of wheels for each pitch from 1 tooth to 9. By multiplying these numbers by 10 we have the pitch diameters from 10 to 90 teeth, increasing by *tens;* by multiplying by 100 we likewise have the pitch diameters from 100 to 900, increasing by *hundreds.*

TABLE FOR DETERMINING THE RELATION BETWEEN PITCH DIAMETER, PITCH, AND NUMBER OF TEETH IN GEAR-WHEELS.

Pitch.	Number of Teeth.								
	1.	2.	3.	4.	5.	6.	7.	8.	9.
1	·3183	·6366	·9549	1·2732	1·5915	1·9099	2·2282	2·5465	2·8648
$1\frac{1}{8}$	·3581	·7162	1·0743	1·4324	1·7905	2·1486	2·5067	2·8648	3·2229
$1\frac{1}{4}$	·3979	·7958	1·1937	1·5915	1·9894	2·3873	2·7852	3·1831	3·5810
$1\frac{3}{8}$	·4377	·8753	1·3130	1·7507	2·1884	2·6260	3·0637	3·5014	3·9391
$1\frac{1}{2}$	·4775	·9549	1·4324	1·9099	2·3873	2·8648	3·3422	3·8197	4·2971
$1\frac{5}{8}$	·5173	1·0345	1·5517	2·0690	2·5862	3·1035	3·6207	4·1380	4·6552
$1\frac{3}{4}$	·5570	1·1141	1·6711	2·2282	2·7852	3·3422	3·8993	4·4563	5·0134
$1\frac{7}{8}$	·5968	1·1937	1·7905	2·3873	2·9841	3·5810	4·1778	4·7746	5·3714
2	·6366	1·2732	1·9099	2·5465	3·1831	3·8197	4·4563	5·0929	5·7296
$2\frac{1}{8}$	·6764	1·3528	2·0292	2·7056	3·3820	4·0584	4·7348	5·4112	6·0877
$2\frac{1}{4}$	·7162	1·4324	2·1486	2·8648	3·5810	4·2972	5·0134	5·7296	6·4457
$2\frac{3}{8}$	·7560	1·5120	2·2679	3·0239	3·7799	4·5359	5·2919	6·0479	6·8038
$2\frac{1}{2}$	·7958	1·5915	2·3873	3·1831	3·9789	4·7746	5·5704	6·3662	7·1619
$2\frac{5}{8}$	·8355	1·6711	2·5067	3·3422	4·1778	5·0133	5·8499	6·6845	7·5200
$2\frac{3}{4}$	·8753	1·7507	2·6260	3·5014	4·3767	5·2521	6·1274	7·0028	7·8781
$2\frac{7}{8}$	·9151	1·8303	2·7454	3·6605	4·5757	5·4908	6·4059	7·3211	8.2362
3	·9549	1·9099	2·8648	3·8197	4·7746	5·7296	6·6845	7·6394	8·5943
$3\frac{1}{4}$	1·0345	2·0690	3·1035	4·1380	5·1725	6·2070	7·2415	8·2760	9·3105
$3\frac{1}{2}$	1·1141	2·2282	3·3422	4·4563	5·5704	6·6845	7·7986	8·9126	10·0268
$3\frac{3}{4}$	1·1937	2·3873	3·5810	4·7746	5·9683	7·1619	8·3556	9·5493	10·7429
4	1·2732	2·5465	3·8197	5·0929	6·3662	7·6394	8·9127	10·1839	11·4591
$4\frac{1}{2}$	1·4324	2·8648	4·2972	5·7296	7·1619	8·5943	10·0267	11·4591	12·8915
5	1·5915	3·1831	4·7746	6·3662	7·9577	9·5493	11·1408	12·7324	14·3240
$5\frac{1}{2}$	1·7507	3·5014	5·2521	7·0028	8·7535	10·5042	12·2549	14·0056	15·7563
6	1·9099	3·8196	5·7295	7·6394	9·5493	11·4591	13·3690	15·2788	17·1887

The following rules and examples show how the table is used:

Rule 1.—Given —— number of teeth and pitch; to find —— pitch diameter.

Select from table in columns opposite the given pitch—

First, the value corresponding to the number of units in the number of teeth.

Second, the value corresponding to the number of tens, and multiply this by 10.

Third, the value corresponding to the number of hundreds, and multiply this by 100. Add these together, and their sum is the pitch diameter required.

Example.—What is the pitch diameter of a wheel with 128 teeth, $1\frac{1}{2}$ inches pitch?

We find in line corresponding to $1\frac{1}{2}$ inch pitch—

Pitch diameter for	8 teeth	3·8197
" "	20 "	9·549
" "	100 "	47·75
" "	128 "	61·1187

Or about $61\frac{1}{8}''$. Answer.

Rule 2.—Given —— pitch diameter and number of teeth; to find —— pitch.

First, ascertain by Rule 1 the pitch diameter for a wheel of 1-*inch pitch*, and the given *number of teeth*.

Second, divide *given pitch diameter* by the *pitch diameter* for 1-*inch pitch*.

The quotient is the pitch desired.

Example.—What is the pitch of a wheel with 148 teeth, the pitch diameter being 72″?

First, pitch diameter for 148 teeth, 1-inch pitch, is—

8 teeth	2·5465
40 "	12·732
100 "	31·83
148	47·1085

Second, $\frac{72}{47 \cdot 1} = 1 \cdot 53$ inch equal to the pitch.

This is nearly $1\frac{1}{2}$-inch pitch, and if possible the diameter would be reduced or the number of teeth increased so as to make the wheel exactly $1\frac{1}{2}$-inch pitch.

Rule 3.—Given —— pitch and pitch diamer; to find —— number of teeth.

First, ascertain from table the *pitch diameter* for 1 *tooth* of the given *pitch*.

Second, divide the *given pitch diameter* by the *value* found in table.

The quotient is the number required.

Example.—What is the number of teeth in a wheel whose pitch diameter is 42 inches, and pitch is $2\frac{1}{2}$ inches?

First, the pitch diameter, 1 tooth, $2\frac{1}{2}$-inch pitch, is 0·7958 inches.

Second. $\frac{42}{0 \cdot 7958} = 52 \cdot 8$. Answer

This gives a fractional number of teeth, which is impossible; so the pitch diameter will have to be increased to correspond to 53 teeth, or the pitch changed so as to have the number of teeth come an even number.

Whenever two parallel shafts are connected together by gearing, the distance between centres being a fixed quantity, and the speeds of the shafts being of a fixed ratio, then the pitch is generally the best proportion to be changed, and necessarily may not be of standard size. Suppose there are two shafts situated in this manner, so that the distance between their centres is 84 inches, and the speed of one is $2\frac{1}{2}$ times that of the other, what size wheels shall be used? In this case the pitch diameter and number of teeth of the wheel on the slow-running shaft have to be $2\frac{1}{2}$ times those of the wheel on the fast-running shaft; so that 84 inches must be divided into two parts, one of which is $2\frac{1}{2}$ times the other, and these quantities will be the pitch radii of the wheels; that is, 84 inches are to be divided into $3\frac{1}{2}$ equal parts, 1 of which is the radius of one wheel, and $2\frac{1}{2}$ of which the radius of the other, thus $\frac{84''}{3\frac{1}{2}} = 24$ inches. So that 24 inches is the pitch radius of pinion, pitch diameter = 48 inches; and $2\frac{1}{2} \times 24$ inches = 60 inches is the pitch radius of the wheel, pitch diameter = 120 inches. The pitch used depends upon the power to be transmitted; suppose that $2\frac{5}{8}$ inches had been decided as about the pitch to be used, it is found by Rule 3 that the number of teeth are respectively 143·6, and 57·4 for wheel and pinion. As this is impossible, some whole number of teeth, nearest these in value, have to be taken, one of which is $2\frac{1}{2}$ times the other; thus 145 and 58 are the nearest, and the pitch for these values is found by Rule 2 to be 2·6 inches, being the best that can be done under the circumstances.

The forms of spur-gearing having their teeth at an angle to the axis, or formed in advancing steps shown in Figs. 183 and 184, were designed by Dr. Hooke, and "were intended," says the inventor, "first to make a piece of wheel work so that both the

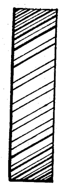

Fig. 183

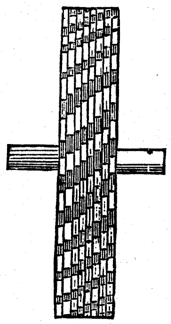

Fig. 184.

wheel and pinion, though of never so small a size, shall have as great a number of teeth as shall be desired, and yet neither weaken the wheels nor make the teeth so small as not to be practicable by any ordinary workman. Next that the motion shall be so equally communicated from the wheel to the pinion that the work being well made there can be no inequality of force or motion communicated.

"Thirdly, that the point of touching and bearing shall be always in the line that joins the two centres together.

"Fourthly, that it shall have *no manner* of *rubbing*, nor be more difficult to make than common wheel work."

The objections to this form of wheel lies in the difficulty of making the pattern and of moulding it in the foundry, and as a result it is rarely employed at the present day. For racks, however, two or more separate racks are cast and bolted together to form the full width of rack as shown in Fig. 185. This arrangement permits of the adjustment of the width of step so as to take up the lost motion due to the wear of the tooth curves.

Another objection to the sloping of the teeth, as in Fig. 183, is that it induces an end pressure tending to force the wheels apart *laterally*, and this causes *end* wear on the journals and bearings.

To obviate this difficulty the form of gear shown in Fig. 186 is employed, the angles of the teeth from each side of the wheel to

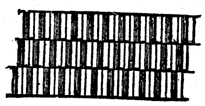

Fig. 185.

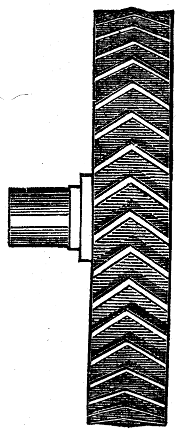

Fig. 186.

its centre being made equal so as to equalize the lateral pressure. It is obvious that the stepped gear, Fig. 184, is simply equivalent to a number of thin wheels bolted together to form a thick one, but possessing the advantage that with a sufficient number of steps, as in the figure, there is always contact on the line of centres, and that the condition of constant contact at the line of centres will be approached in proportion to the number of steps in the wheel, providing that the steps progress in one continuous direction across the wheel as in Fig. 184. The action of the wheels will, in this event, be smoother, because there will be less pressure tending to force the wheels apart.

But in the form of gearing shown in Fig. 183, the contact of the teeth will bear every instant at a single point, which, as the wheels revolve, will pass from one end to the other of the tooth, a fresh contact always beginning on the first side immediately before the preceding contact has ceased on the opposite side. The contact, moreover, being always in the plane of the centres of the pair, the action is reduced to that of rolling, and as there is no sliding motion there is consequently no rubbing friction between the teeth.

A further modification of Dr. Hooke's gearing has been somewhat extensively adopted, especially in cotton-spinning machines. This consists, when the direction of the motion is simply to be changed to an angle of 90°, in forming the teeth upon the periphery of the pair at an angle of 45° to the respective axes of the wheels, as in Figs. 187 and 188; it will then be perceived that if the sloped teeth be presented to each other in such a way as to have exactly the same horizontal angle, the wheels will gear together, and motion being communicated to one axis the same will be transmitted to the other at a right angle to it, as in a common bevel pair. Thus if the wheel A upon a horizontal

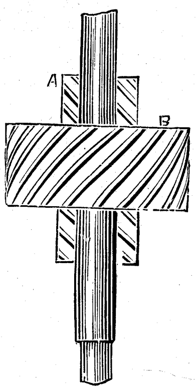

Fig. 187.

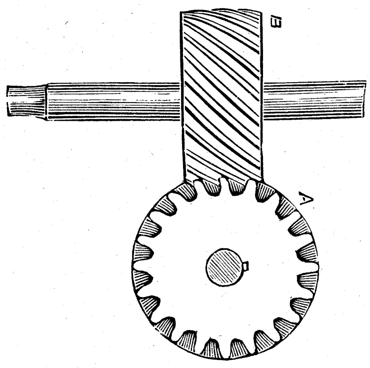

Fig. 188.

shaft have the teeth formed upon its circumference at an angle of 45° to the plane of its axis it can gear with a similar wheel B upon a vertical axis. Let it be upon the driving shaft and the motion will be changed in direction as if A and B were a pair of bevel-wheels of the ordinary kind, and, as with bevels generally, the direction of motion will be changed through an equal angle to the sum of the angles which the teeth of the wheels of the pair form with their respective axes. The objection in respect of lateral or end pressure, however, applies to this form equally with that shown in Fig. 183, but in the case of a vertical shaft the end pressure may be (by sloping the teeth in the necessary direction) made to tend to lift the shaft and not force it down into the step bearing. This would act to keep the wheels in close contact by reason of the weight of the vertical shaft and at the same time reduce the friction between the end of that shaft and its step bearing. This renders this form of gearing preferable to skew bevels when employed upon vertical shafts.

It is obvious that gears, such as shown in Figs. 187 and 188 may be turned up in the lathe, because the teeth are simply portions of spirals wound about the circumference of the wheel. For a pair of wheels of equal diameter a cylindrical piece equal in length to the required breadth of the two wheels is turned up in the lathe, and the teeth may be cut in the same manner as cutting a thread in the lathe, that is to say, by traversing the tool the requisite distance per lathe revolution. In pitches above about $\frac{1}{4}$ inch, it will be necessary to shape one side of the tooth at a time on account of the broadness of the cutting edges. After the spiral (for the teeth are really spirals) is finished the piece may be cut in two in the lathe and each half will form a wheel.

To find the full diameter to which to turn a cylinder for a pair of these wheels we proceed as in the following example: Required to cut a spiral wheel 5 inches in diameter and to have 30 teeth. First find the diametral pitch, thus 30 (number of teeth) ÷ 5 (diameter of wheel at pitch circle) = 6; thus there are 6 teeth or 6 parts to every inch of the wheel's diameter at the pitch circle; adding 2 of these parts to the diameter of the wheel, at the pitch circle we have 5 and $\frac{2}{6}$ of another inch, or $5\frac{2}{6}$ inches,

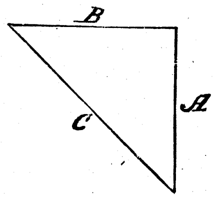

Fig. 189.

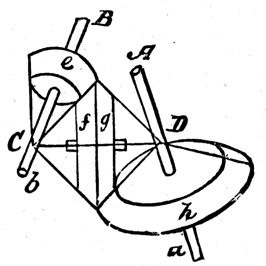

Fig. 190.

which is the full diameter of the wheel, or the diameter of the addendum, as it is termed.

It is now necessary to find what change wheels to put on the lathe to cut the teeth out the proper angle. Suppose then the axes of the shafts are at a right angle one to the other, and that the teeth therefore require to be at an angle of 45° to the axes of the respective wheels, then we have the following considerations. In Fig. 189 let the line A represent the circumference of the wheel, and B a line of equal length but at a right angle to it, then the line C, joining A,B, is at an angle of 45°. It is obvious then that if the traverse of the lathe tool be equal at each lathe revolution to the circumference of the wheel at the pitch circle, the angle of the teeth will be 45° to the axis of the wheel.

Hence, the change wheels on the lathe must be such as will traverse the tool a distance equal to the circumference at pitch circle of the wheel, and the wheels may be found as for ordinary screw cutting.

If, however, the axes of the shafts are at any other angle we may find the distance the lathe tool must travel per lathe revolution to give teeth of the required angle (or in other words the pitch of the spiral) by direct proportion, thus: Let it be required to find the angle or pitch for wheels to connect shafts at an angle of 25°, the wheels to have 20 teeth, and to be of 10 diametral pitch.

Here, 20 ÷ 10 = 2 = diameter of wheel at the pitch circle. The circumference of 2 inches being 6·28 inches we have, as the degrees of angle of the axes of the shafts are to 45°, so is 6·28 inches (the circumference of the wheels, to the pitch sought).

Here, 6·28 inches × 45° ÷ 25° = 11·3 inches, which is the required pitch for the spiral.

When the axes of the shafts are neither parallel nor meeting, motion from one shaft to another may be transmitted by means of a double gear. Thus (taking rolling cones of the diameters of the respective pitch circles as representing the wheels) in Fig. 190, let A be the shaft of gear *h*, and B *b* that of wheel *e*. Then a double gear-wheel having teeth on *f*, *g* may be placed as shown, and the face *f* will gear with *e*, while face *g* will gear with *h*, the cone surfaces meeting in a point as at C and D respectively, hence the velocity will be equal.

When the axial line of the shafts for two gear-wheels are nearly in line one with the other, motion may be transmitted by gearing the wheels as in Fig 191. This is a very strong method of gearing, because there are a large number of teeth in contact, hence the strain is distributed by a larger number of teeth and the wear is diminished.

Fig. 192 (from Willis's "Principles of Mechanism") is another method of constructing the same combination, which admits of

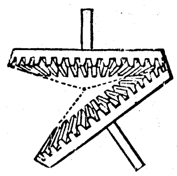

Fig. 191.

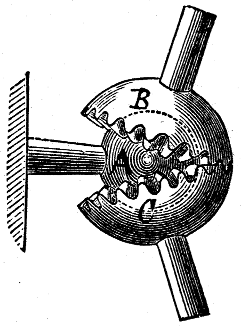

Fig. 192.

a steady support for the shafts at their point of intersection, A being a spherical bearing, and B,C being cupped to fit to A.

Rotary motion variable at different parts of a rotation may be obtained by means of gear-wheels varied in form from the true circle.

The commonest form of gearing for this purpose is elliptical gearing, the principles governing the construction of which are thus given by Professer McCord. "It is as well to begin at the foundation by defining the ellipse as a closed plane-curve, generated by the motion of a point subject to the condition that the sum of its distances from two fixed points within shall be constant: Thus, in Fig. 193, A and B are the two fixed points, called the *foci;* L, E, F, G, P are points in the curve; and A F + F B = A E + E B. Also, A L + L B = A P + P B = A G + G B. From this it follows that A G = L O, O being the centre of the curve, and G the extremity of the minor axis, whence the foci may be found if the axes be assumed, or, if the foci and one axis be given, the other axis may be determined. It is also apparent that if about either focus, as B, we describe an arc with a radius greater than B P and less than B L, for instance B E, and about A another arc with radius A E = L P—B E, the intersection, E, of these arcs will be on the ellipse; and in this manner any desired number of points may be found, and the curve drawn by the aid of sweeps.

"Having completed this ellipse, prolong its major axis, and draw

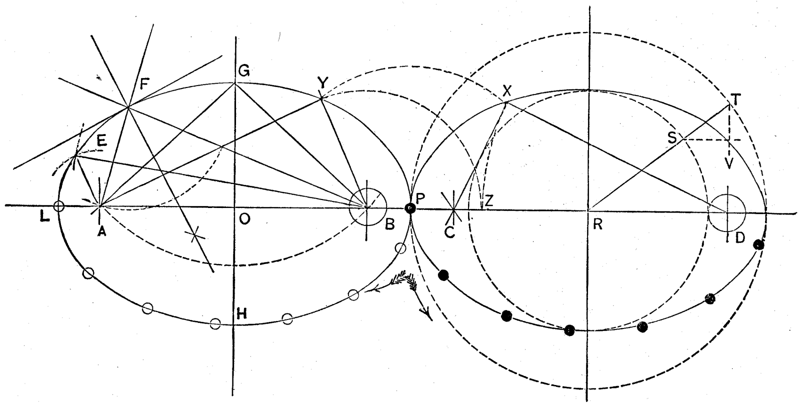

Fig. 193.

a similar and equal one, with its foci, C, D, upon that prolongation, and tangent to the first one at P; then B D = L P. About B describe an arc with any radius, cutting the first ellipse at Y and the line L at Z; about D describe an arc with radius D Z, cutting the second ellipse in X; draw A Y, B Y, C X, and D X. Then A Y = D X, and B Y = C X, and because the ellipses are alike, the arcs P Y and P X are equal. If then B and D are taken as fixed centres, and the ellipses turn about them as shown by the arrows, X and Y will come together at Z on the line of centres; and the same is true of any points equally distant from P on the two curves. But this is the condition of rolling contact. We see, then, that in order that two ellipses may roll together, and serve as the pitch-lines of wheels, they must be equal and similar, the fixed centres must be at corresponding foci, and the distance between these centres must be equal to the major axis. Were they to be toothless wheels, it would evidently be essential that the outlines should be truly elliptical; but the changes of curvature in the ellipse are gradual, and circular arcs may be drawn

"In Fig. 194, A A and B B are centre lines passing through the major and minor axes of the ellipse, of which *a* is the axis or centre, *b c* is the major and *a e* half of the minor axis. Draw the rectangle *b f g c*, and then the diagonal line *b e;* at a right angle to *b e* draw line *f h* cutting B B at *i*. With radius *a e* and from *a* as a centre draw the dotted arc *e j*, giving the point *j* on the line B B. From centre *k*, which is on line B B, and central between *b* and *j*, draw the semicircle *b m j*, cutting A A at *l*. Draw the radius of the semicircle *b m j* cutting *f g* at *n*. With radius *m n* mark on A A, at and from *a* as a centre, the point *o*. With radius *h o* and from centre *h* draw the arc *p o q*. With radius *a l* and from *b* and *c* as centres draw arcs cutting *p o q* at the points *p q*. Draw the lines *h p r* and *h q s*, and also the lines *p i t* and *q v w*. From *h* as centre draw that part of the ellipse lying between *r* and *s*. With radius *p r* and from *p* as a centre draw that part of the ellipse lying between *r* and *t*. With radius *q s* and from *q* draw the ellipse from *s* to *w*. With radius *i t* and from *i* as a centre draw the ellipse from *t* to *b*. With radius *v w*

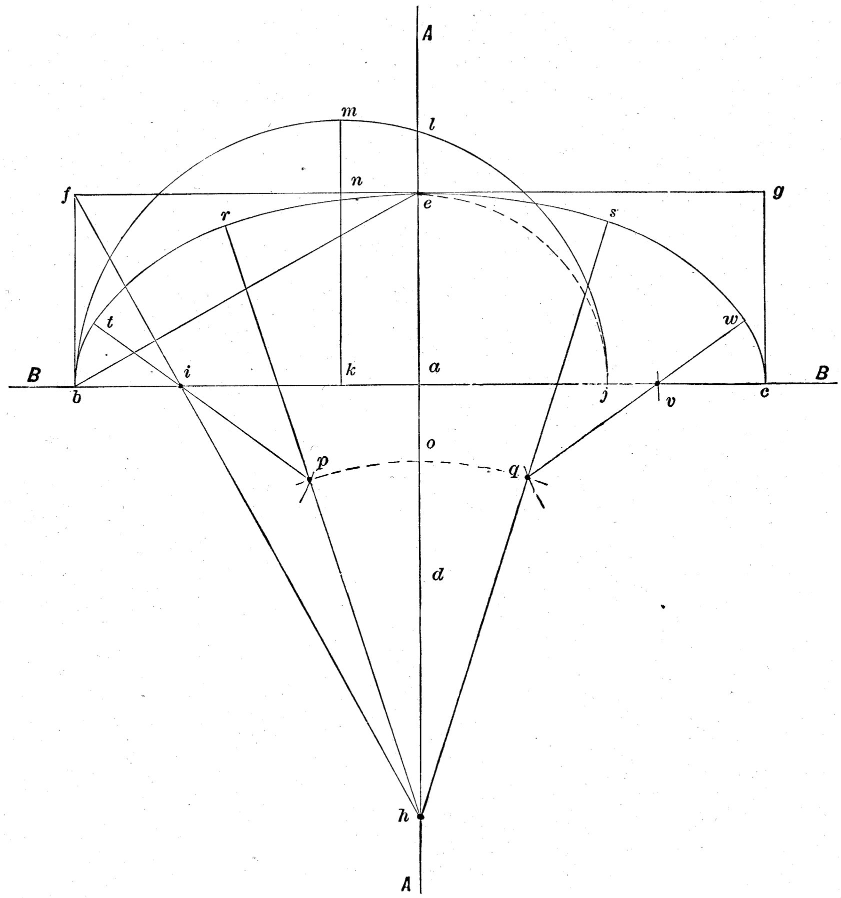

Fig. 194.

so nearly coinciding with it, that when teeth are employed, the errors resulting from the substitution are quite inappreciable. Nevertheless, the rapidity of these changes varies so much in ellipses of different proportions, that we believe it to be practically better to draw the curve accurately first, and to find the radii of the approximating arcs by trial and error, than to trust to any definite rule for determining them; and for this reason we give a second and more convenient method of finding points, in connection with the ellipse whose centre is R, Fig. 193. About the centre describe two circles, as shown, whose diameters are the major and minor axes; draw any radius, as R T, cutting the first circle in T, and the second in S; through T draw a parallel to one axis, through S a parallel to the other, and the intersection, V, will lie on the curve. In the left hand ellipse, the line bisecting the angle A F B is normal to the curve at F, and the perpendicular to it is tangent at the same point, and bisects the angles adjacent to A F B, formed by prolonging A F, B F.

"To mark the pitch line we proceed as follows:—

and from *v* as a centre draw the ellipse from *w* to *c*, and one half the ellipse will be drawn. It will be seen that the whole construction has been performed to find the centres *h p q i* and *v*, and that while *v* and *i* may be used to carry the curve around the other side or half of the ellipse, new centres must be provided for *h p* and *q;* these new centres correspond in position to *h p q*.

"If it were possible to subdivide the ellipse into equal parts it would be unnecessary to resort to these processes of approximately representing the two curves by arcs of circles; but unless this be done, the spacing of the teeth can only be effected by the laborious process of stepping off the perimeter into such small subdivisions that the chords may be regarded as equal to the arcs, which after all is but an approximation; unless, indeed, we adopt the mechanical expedient of cutting out the ellipse in metal or other substance, measuring and subdividing it with a strip of paper or a steel tape, and wrapping back the divided measure in order to find the points of division on the curve.

"But these circular arcs may be rectified and subdivided with great facility and accuracy by a very simple process, which we take from Prof. Rankine's "Machinery and Mill Work," and is illustrated in Fig. 195. Let O B be tangent at O to the arc O D, of which C is the centre. Draw the chord D O, bisect it in E, and produce it to A, making O A = O E; with centre A and radius A D describe an arc cutting the tangent in B; then O B will be very nearly equal in length to the arc O D, which, however, should not exceed about 60°; if it be 60°, the error is theoretically about

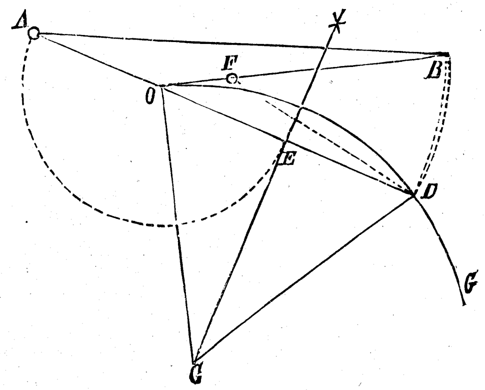

Fig. 195.

$\frac{1}{900}$ of the length of the arc, O B being so much too short; but this error varies with the fourth power of the angle subtended by the arc, so that for 30° it is reduced to $\frac{1}{16}$ of that amount, that is, to $\frac{1}{14400}$. Conversely, let O B be a tangent of given length; make O F = $\frac{1}{4}$ O B; then with centre F and radius F B describe an arc cutting the circle O D G (tangent to O B at O) in the point D; then O D will be approximately equal to O B, the error being the same as in the other construction and following the same law.

"The extreme simplicity of these two constructions and the facility with which they may be made with ordinary drawing instruments make them exceedingly convenient, and they should be more widely known than they are. Their application to the present problem is shown in Fig. 196, which represents a quadrant of an ellipse, the approximate arcs C D, D E, E F, F A having been determined by trial and error. In order to space this off, for the positions of the teeth, a tangent is drawn at D, upon which is constructed the rectification of D C, which is D G, and also that of D E in the opposite direction, that is, D H, by the process just explained. Then, drawing the tangent at F, we set off in the same manner F I = F E, and F K = F A, and then measuring H L = I K, we have finally G L, equal to the whole quadrant of the ellipse.

"Let it now be required to lay out 24 teeth upon this ellipse; that is, 6 in each quadrant; and for symmetry's sake we will suppose that the centre of one tooth is to be at A, and that of another at C, Fig. 196. We therefore divide L G into six equal parts at the points 1, 2, 3, &c., which will be the centres of the teeth upon the rectified ellipse. It is practically necessary to make the spaces a little greater than the teeth; but if the greatest attainable exactness in the operation of the wheel is aimed at, it is important to observe that backlash, in elliptical gearing, has an effect quite different from that resulting in the case of circular wheels. When the pitch-curves are circles, they are always in contact; and we may, if we choose, make the tooth only half the breadth of the space, so long as its outline is correct. When the motion of the driver is reversed, the follower will stand still until the backlash is taken up, when the motion will go on with a perfectly constant velocity ratio as before. But in the case of two elliptical wheels, if the follower stand still while the driver moves, which must happen when the motion is reversed if backlash exists, the pitch-curves are thrown out of contact, and, although the continuity of the motion will not be interrupted, the velocity ratio will be affected. If the motion is never to be reversed, the perfect law of the velocity ratio due to the elliptical pitch-curve may be preserved by reducing the thickness of the tooth, not equally on each side, as is done in circular wheels, but wholly on the side not in action. But if the machine must be capable of acting indifferently in both directions, the reduction must be made on both sides of the tooth: evidently the action will be slightly impaired, for which reason the backlash should be reduced to a minimum. Precisely what *is* the minimum is not so easy to say, as it evidently depends much upon the excellence of the tools and the skill of the workmen. In many treatises on constructive mechanism it is variously stated that the backlash should be from one-fifteenth to one-eleventh of the pitch, which would seem to be an ample allowance in reasonably good castings not intended to be finished, and quite excessive if the teeth are to be cut; nor is it very obvious that its amount should depend upon the pitch any more than upon the precession of the equinoxes. On paper, at any rate, we may reduce it to zero, and make the teeth and spaces equal in breadth, as shown in the figure, the teeth being indicated by the double lines. Those upon the portion L H are then laid off upon K I, after which these divisions are transferred to curves. And since under that condition the motion of this third line, relatively to each of the others, is the same as though it rolled along each of them separately while they remained fixed, the process of constructing the generated curves becomes comparatively simple. For the describing line, we naturally select a circle, which, in order to fulfil the condition, must be small enough to roll within the pitch ellipse; its diameter is determined by the consideration, that if it be equal to A P, the radius of the arc A F, the flanks of the teeth in that region will be radial. We have, therefore, chosen a circle whose diameter, A B, is three-fourths of A P, as shown, so that the teeth, even at the ends of the wheels, will be broader at the base than on the pitch line. This circle ought strictly to roll upon the true elliptical curve, and assuming as usual the

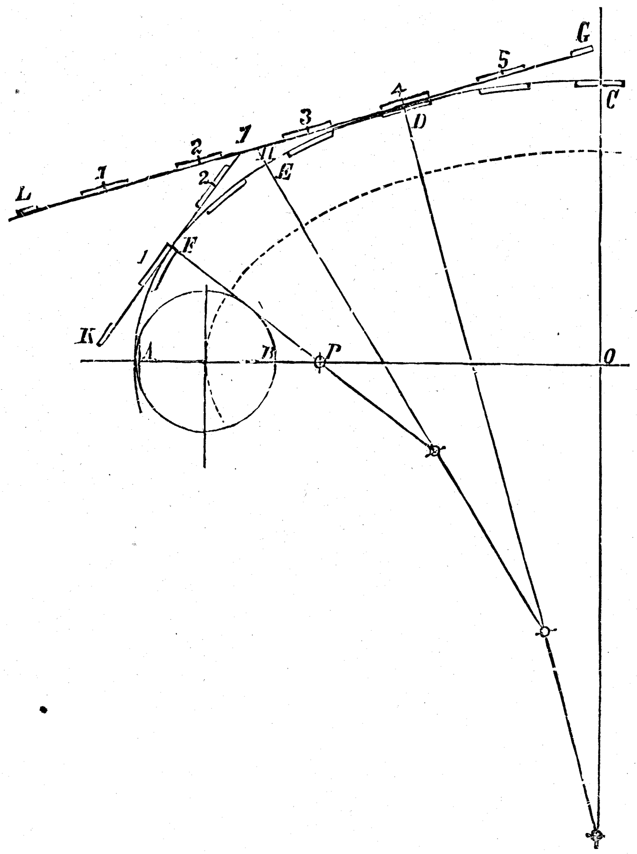

Fig. 196.

tracing-point upon the circumference, the generated curves would vary slightly from true epicycloids, and no two of those used in the same quadrant of the ellipse would be exactly alike. Were it possible to divide the ellipse accurately, there would be no difficulty in laying out these curves; but having substituted the circular arcs, we must now roll the generating circle upon these as bases, thus forming true epicycloidal teeth, of which those lying upon the same approximating arc will be exactly alike.

Should the junction of two of these arcs fall within the breadth of a tooth, as at D, evidently both the face and the flank on one side of that tooth will be different from those on the other side; should the junction coincide with the edge of a tooth, which is very nearly the case at F, then the face on that side will be the epicycloid belonging to one of the arcs, its flank a hypocycloid belonging to the other; and it is possible that either the face or the flank on one side should be generated by the rolling of the describing circle partly on one arc, partly on the one adjacent,

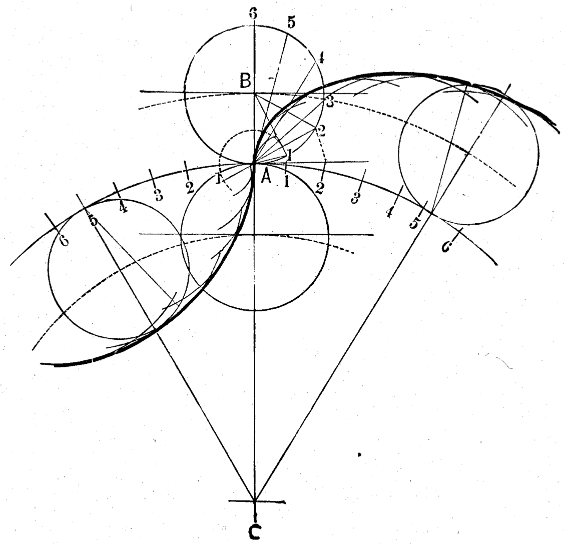

Fig. 197.

which, upon a large scale and where the best results are aimed at, may make a sensible change in the form of the curve

"The convenience of the constructions given in Fig. 194 is nowhere more apparent than in the drawing of the epicycloids, when, as in the case in hand, the base and generating circles may be of incommensurable diameters; for which reason we have, in Fig. 197, shown its application in connection with the most rapid and accurate mode yet known of describing those curves. Let C be the centre of the base circle; B that of the rolling one; A the point of contact. Divide the semi-circumference of B into six equal parts at 1, 2, 3, &c.; draw the common tangent at A, upon which rectify the arc A2 by process No. 1, then by process No. 2 set out an equal arc A2 on the base circle, and stepping it off three times to the right and left, bisect these spaces, thus making subdivisions on the base circle equal in length to those on the rolling one. Take in succession as radii the chords A1, A2, A3, &c., of the describing circle, and with centres 1, 2, 3, &c., on the base circle, strike arcs either externally or internally, as shown respectively on the right and left; the curve tangent to the external arcs is the epicycloid, that tangent to the internal ones the hypocycloid, forming the face and flank of a tooth for the base circle.

"In the diagram, Fig. 196, we have shown a part of an ellipse whose length is 10 inches and breadth 6, the figure being half size. In order to give an idea of the actual appearance of the combination when complete, we show in Fig. 198 the pair in gear, on a scale of 3 inches to the foot. The excessive eccentricity was selected merely for the purpose of illustration. Fig. 198 will serve also to call attention to another serious circumstance, which is that although the ellipses are alike, the wheels are not; nor can they be made so if there be an even number of teeth, for the obvious reason that a tooth upon one wheel must fit into a space on the other; and since in the first wheel, Fig. 196, we chose to place a tooth at the extremity of each axis, we must in the second one place there a space instead; because at one time the major axes must coincide, at another the minor axis, as in Fig. 191. If then we use even numbers, the distribution and even the forms of the teeth are not the same in the two wheels of the pair. But this complication may be avoided by using an odd number of teeth, since, placing a tooth at one extremity of the major axis, a space will come at the other.

It is not, however, always necessary to cut teeth all round these wheels, as will be seen by an examination of Fig. 199, C and D being the fixed centres of the two ellipses in contact at P. Now P must be on the line C D, whence, considering the free foci, we see PB is equal to PC, and PA to PD; and the common tangent at P makes equal angles with C P and P A, as is also with P B and P D; therefore, C D being a straight line, A B is also a straight line and equal to C D. If then the wheels be overhung, that is, fixed on the ends of the shafts outside the bearings, leaving the outer faces free, the moving foci may be connected by a rigid link A B, as shown.

"This link will then communicate the same motion that would result from the use of the complete elliptical wheels, and we may therefore dispense with most of the teeth, retaining only those near the extremities of the major axes which are necessary in order to assist and control the motion of the link at and near the dead-points. The arc of the pitch-curves through which the teeth must extend will vary with their eccentricity: but in many cases it would not be greater than that which in the approximation may be struck about one centre, so that, in fact, it would not be necessary to go through the process of rectifying and subdividing the quarter of the ellipse at all, as in this case it can make no possible difference whether the spacing adopted for the teeth to be cut would "come out even" or not if carried around the curve. By this expedient, then, we may save not only the trouble of drawing, but a great deal of labor in making, the teeth round the whole ellipse. We might even omit the intermediate portions of the pitch ellipses themselves; but as they move in rolling contact their retention can do no harm, and in one part of the movement will be beneficial, as they will do part of the work; for if, when turning, as shown by the arrows, we consider the wheel whose axis is D as the driver, it will be noted that its radius of contact, C P, is on the increase; and so long as this is the case the other

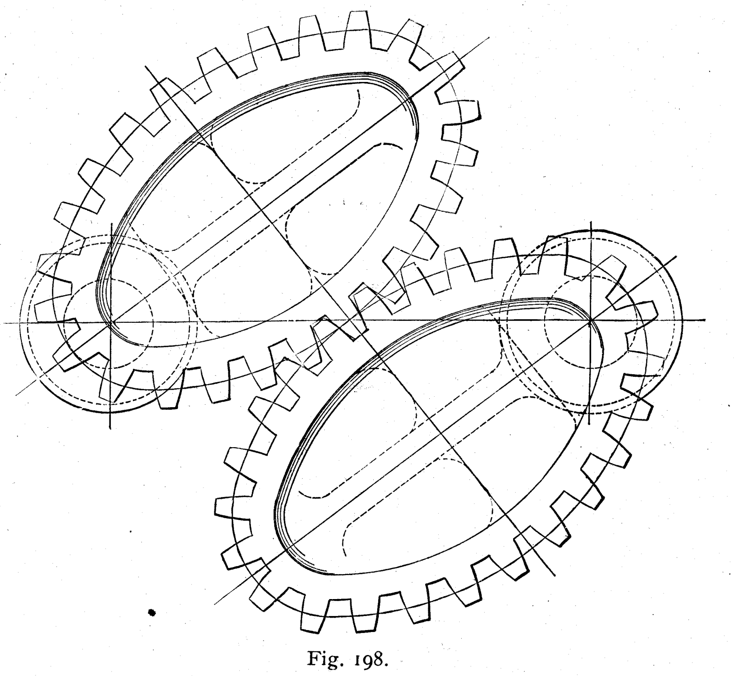

Fig. 198.

wheel will be compelled to move by contact of the pitch lines, although the link be omitted. And even if teeth be cut all round the wheels, this link is a comparatively inexpensive and a useful addition to the combination, especially if the eccentricity be considerable. Of course the wheels shown in Fig. 198 might also have been made alike, by placing a tooth at one end of the major axis and a space at the other, as above suggested. In regard to the variation in the velocity ratio, it will be seen, by reference to Fig. 199, that if D be the axis of the driver, the follower will in the position there shown move faster, the ratio of the angular velocities being $\frac{PD}{PB}$; if the driver turn uniformly the velocity of the follower will diminish, until at the end of half a revolution, the velocity ratio will be $\frac{PB}{PD}$; in the other half of the revolution these changes will occur in a reverse order. But P D = L B; if then the centres B D are given in position, we know L P, the major axis; and in order to produce any assumed maximum or minimum velocity ratio, we have only to divide L P into segments whose ratio is equal to that assumed value, which will give the foci of the ellipse, whence the minor axis may be found and the curve described. For instance, in Fig. 198 the velocity ratio being nine to one at the maximum, the major axis is divided into two parts, of which one is nine times as long as the other; in Fig. 199 the ratio is as one to three, so that, the major axis being divided into four parts, the distance A C between the foci is equal to two of them, and the distance of either focus from the nearer extremity of the major axis equal to one, and from the more remote extremity equal to three of these parts."

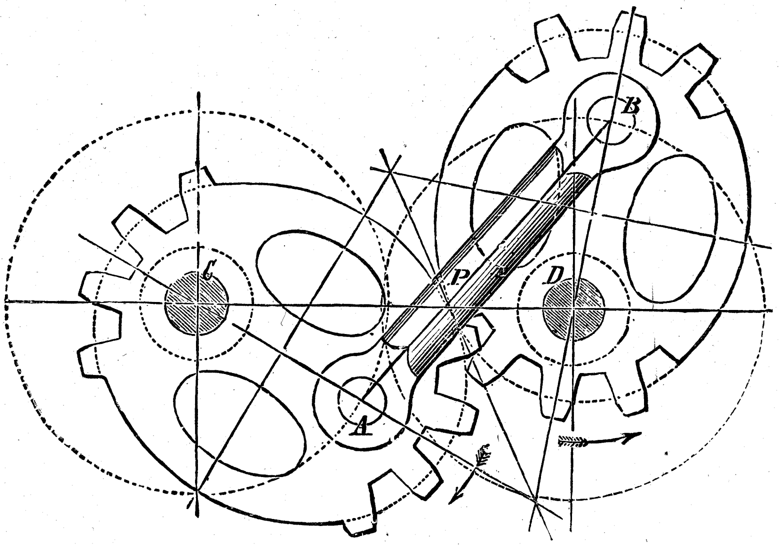

Fig. 199.

Another example of obtaining a variable motion is given in Fig. 200, The only condition necessary to the construction of wheels of this class is that the sum of the radii of the pitch circles on the line of centres shall equal the distance between the axes of the two wheels. The pitch curves are to be considered the same as pitch circles, "so that," says Willis, "if any given circle or curve be assumed as a describing (or generating) curve, and if it be made to roll on the inside of one of these pitch curves and on the outside of the corresponding portion of the other pitch curve, then the motion communicated by the pressure and sliding contact of one of the curved teeth so traced upon the other will be exactly the same as that effected by the rolling contact (by friction) of the original pitch curves."

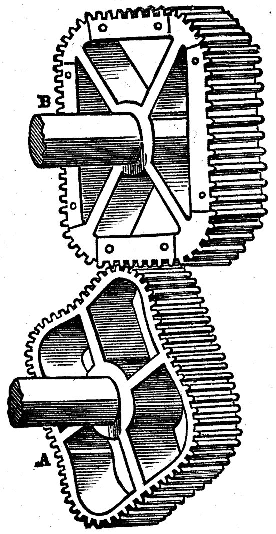

Fig. 200.

It is obvious that on B the corner sections are formed of simple segments of a circle of which the centre is the axis of the shaft, and that the sections between them are simply racks. The corners of A are segments of a circle of which the axis of A is the centre, and the sections between the corners curves meeting the pitch circles of the rack at every point as it passes the line of centres.

Intermittent motion may also be obtained by means of a worm-wheel constructed as in Fig. 201, the worm having its teeth at a right angle to its axis for a distance around the circumference proportioned to the required duration of the period of rest; or the motion may be made variable by giving the worm teeth different degrees of inclination (to the axis), on different portions of the circumference.

In addition to the simple operation of two or more wheels transmitting motion by rotating about their fixed centres and in fixed positions, the following examples of wheel motion may be given.

In Fig. 202 are two gear-wheels, A, which is fast upon its stationary shaft, and B, which is free to rotate upon its shaft, the link C affording journal bearing to the two shafts. Suppose that A has 40 teeth, while B has 20 teeth, and that the link C is rotated once around the axis of A, how many revolutions will B make? By reason of there being twice as many teeth in A as in B the latter will make two rotations, and in addition to this it will, by reason of its connection to the arm C, also make a revolution, these being two distinct motions, one a rotation of B about the axis of A, and the other two rotations of B upon its own axis.

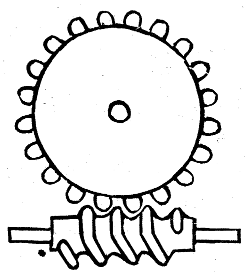
Fig. 201.

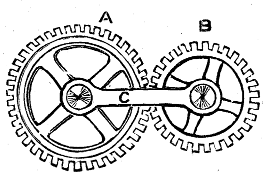

Fig. 202.

A simple arrangement of gearing for reversing the direction of rotation of a shaft is shown in Fig. 203. I and F are fast and loose pulleys for the shaft D, A and C are gears free to rotate upon D, N is a clutch driven by D; hence if N be moved so as to engage with C the latter will act as a driver to rotate the shaft B, the wheel upon B rotating A in an opposite direction to the rotation of D. But if N be moved to engage with A the latter becomes the driving wheel, and B will be caused to rotate in the opposite direction. Since, however, the engagement of the clutch N with the clutch on the nut of the gear-wheels is accompanied with a violent shock and with noise, a preferable arrangement is shown in Fig. 204, in which the gears are all fast to their shafts, and the driving shaft for C passes through the core or bore of that for

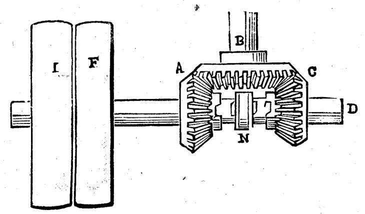

Fig. 203.

A, which is a sleeve, so that when the driving belt acts upon pulley F the shaft B rotates in one direction, while when the belt acts upon E, B rotates in the opposite direction, I being a loose pulley.

If the speed of rotation of B require to be greater in one direction than in the other, then the bevel-wheel on B is made a double one, that is to say, it has two annular toothed surfaces on its radial face, one of larger diameter than the other; A gearing

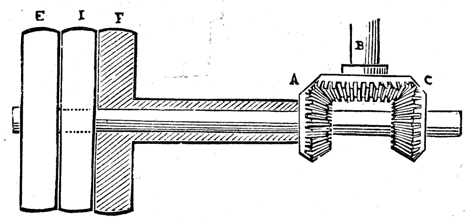

Fig. 204.

with one of these toothed surfaces, and C with the other. It is obvious that the pinions A C, being of equal diameters, that gearing with the surface or gear of largest diameter will give to B the slowest speed of rotation.

Fig. 205 represents Watt's sun-and-planet motion for converting reciprocating into rotary motion; B D is the working beam of the engine, whose centre of motion is at D. The gear A is so connected to the connecting rod that it cannot rotate, and is kept in gear with the wheel C on the fly-wheel shaft by means of

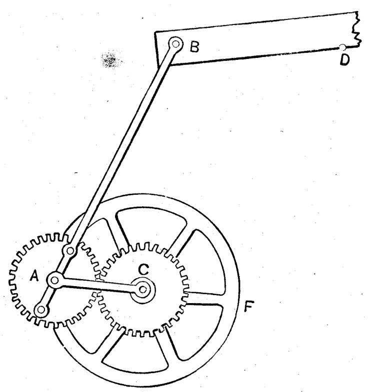

Fig. 205.

the link shown. The wheel A being prevented from rotation on its axis causes rotary motion to the wheel C, which makes two revolutions for one orbit of A.

An arrangement for the rapid increase of motion by means of gears is shown in Fig. 206, in which A is a stationary gear, B is free to rotate upon its shaft, and being pivoted upon the shaft of A, at D, is capable of rotation around A while remaining in gear with C. Suppose now that the wheel A were absent, then if B were rotated around C with D as a centre of motion, C and its shaft E would make a revolution even though B would have no rotation upon its axis. But A will cause B to rotate upon its axis and thus communicate a second degree of motion to C, with the result that one revolution of B causes two rotations of C.

The relation of motion between B and C is in this case constant (2 to 1), but this relation may be made variable by a construction such as shown in Fig. 207, in which the wheel B is carried in a gear-wheel H, which rides upon the shaft D. Suppose now that H remains stationary while A revolves, then motion will be transmitted through B to C, and this motion will be constant and in proportion to the relative diameters of A and C. But suppose by means of an independent pinion the wheel H be rotated upon its axis, then increased motion will be imparted to

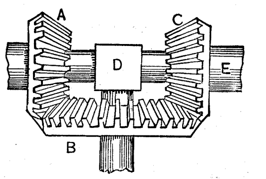

Fig. 206.

C, and the amount of the increase will be determined by the speed of rotation of H, which may be made variable by means of cone pulleys or other suitable mechanical devices.

Fig. 208 represents an arrangement of gearing used upon steam fire-engines and traction engines to enable them to turn easily in a short radius, as in turning corners in narrow streets. The object is to enable the driving wheel on either side of the engine to increase or diminish its rotation to suit the conditions caused by the leading or front pair of steering wheels.

In the figures A is a plate wheel having the lugs L, by means of which it may be rotated by a chain. A is a working fit on the shaft S, and carries three pinions E pivoted upon their axes P. F is a bevel-gear, a working fit on S, while C is a similar gear fast

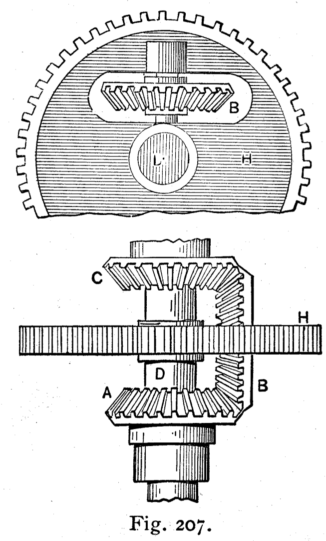

Fig. 207.

to S. The pinions B, D are to drive gears on the wheels of the engine, the wheels being a working fit on the axle. Let it now be noted that if S be rotated, C and F will rotate in opposite directions and A will remain stationary. But if A be rotated, then all the gears will rotate with it, but E will not rotate upon P unless there be an unequal resistance to the motion of pinions D and B. So soon, however, as there exists an inequality of resistance between D and B then pinions E operate. For example, let B have more resistance than D, and B will rotate more slowly, causing pinion E to rotate and move C faster than is due to the motion of the chain wheel A, thus causing the wheel on one side of the engine to retard and the other to increase its motion, and thus enable the engine to turn easily. From its action this arrangement is termed the equalizing gear.

In Figs. 209 to 214 are shown what are known as mangle-wheels from their having been first used in clothes mangling machines.

The mangle-wheel* in its simplest form is a revolving disc of metal with a centre of motion C (Fig. 209). Upon the face of the disc is fixed a projecting annulus *a m*, the outer and inner edges of which are cut into teeth. This annulus is interrupted at *f*, and the teeth are continued round the edges of the interrupted portion and the motion again be reversed. The *velocity ratio* in either direction will remain constant, but the ratio when the pinion is inside will differ slightly from the ratio when it is outside, because the pitch radius of the annular or internal teeth is necessarily somewhat less than that of the spur teeth. However, the change of direction is not instantaneous, for the form of the groove S *f t*, which connects the inner and outer grooves, is a semicircle, and when the axis of the pinion reaches S the velocity of the mangle-

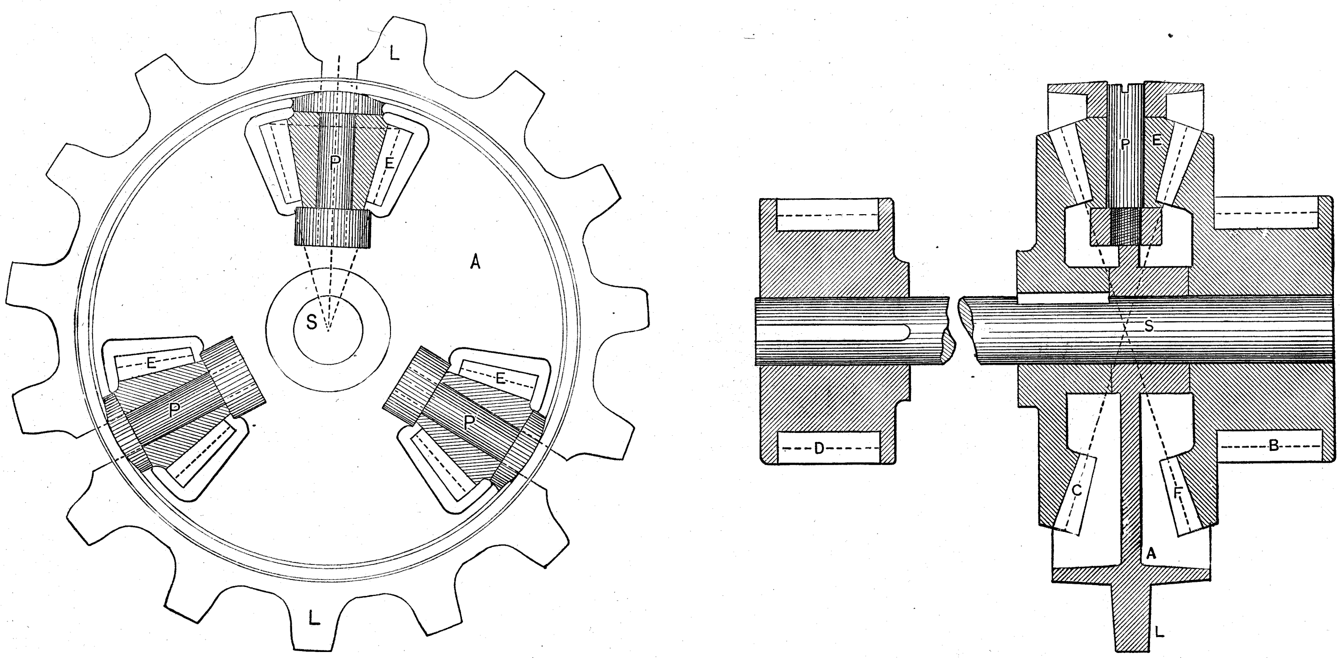

Fig. 208.

so as to form a continued series passing from the outer to the inner edge and back again.

A pinion B, whose teeth are of the same pitch as those of the wheel, is fixed to the end of an axis, and this axis is mounted so as to allow of a short travelling motion in the direction B C. This may be effected by supporting this end of it either in a swing-frame moving upon a centre as at D, or in a sliding piece, according to the nature of the train with which it is connected. A short pivot projects from the centre of the pinion, and this rests in and is guided by a groove B S *f t b h* K, which is cut in the

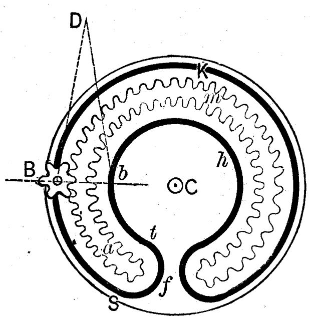

Fig. 209.

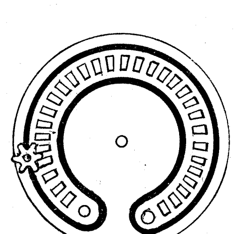

Fig. 210.

surface of the disc, and made concentric to the pitch circles of the inner and outer rays of teeth, and at a normal distance from them equal to the pitch radius of the pinion.

Now when the pinion revolves it will, if it be on the outside, as in Fig. 209, act upon the spur teeth and turn the wheel in the opposite direction to its own, but when the interrupted portion *f* of the teeth is thus brought to the pinion the groove will guide the pinion while it passes from the outside to the inside, and thus bring its teeth into action with the annular or internal teeth. The wheel will then receive motion in the same direction as that of the pinion, and this will continue until the gap *f* is again brought to the pinion, when the latter will be carried outwards

* From Willis's "Principles of Mechanism."

wheel begins to diminish gradually until it is brought to rest at *f*, and is again gradually set in motion from *f* to *t*, when the constant ratio begins; and this retardation will be increased by increasing the difference between the radius of the inner and outer pitch circles.

The teeth of a mangle-wheel are, however, most commonly formed by pins projecting from the face of the disc as in Fig. 210. In this manner the pitch circles for the inner and outer wheels coincide, and therefore the velocity ratio is the same within and without, also the space through which the pinion moves in shifting is reduced.

This space may be still further reduced by arranging the teeth as in Fig. 211, that is, by placing the spur-wheel within the

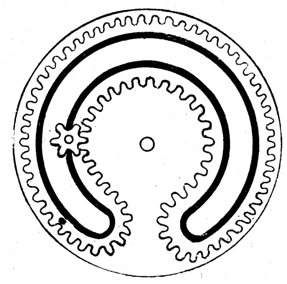

Fig. 211.

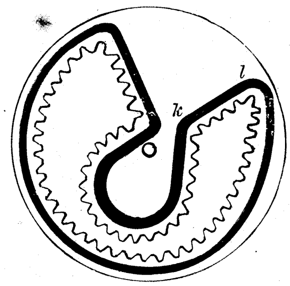

Fig. 212.

annular or internal one; but at the same time the difference of the two velocity ratios is increased.

If it be required that the velocity ratio vary, then the pitch lines of the mangle-wheel must no longer be concentric.

Thus in Fig. 212 the groove *k l* is directed to the centre of the mangle-wheel, and therefore the pinion will proceed during this portion of its path without giving any motion to the wheel, and in the other lines of teeth the pitch radius varies, hence the angular velocity ratio will vary.

In Figs. 209, 210, and 211 the curves of the teeth are readily obtained by employing the same describing circle for the whole of them. But when the form Fig. 212 is adopted, the shape of the teeth requires some consideration.

Every tooth of such a mangle-wheel may be considered as formed of two ordinary teeth set back to back, the pitch line passing through the middle. The outer half, therefore, appropriated to the action of the pinion on the outside of the wheel, resembles that portion of an ordinary spur-wheel tooth that lies beyond its pitch line, and the inner half which receives the inside action of the pinion resembles the half of an annular wheel that lies within the pitch circle. But the consequence of this arrangement is, that in both positions the action of the driving teeth must be confined to the approach of its teeth to the line of centres, and consequently these teeth must be wholly within their pitch line.

To obtain the forms of the teeth, therefore, take any convenient describing circle, and employ it to describe the teeth of the pinion by rolling within its pitch circle, and to describe the teeth of the wheel by rolling within and without its pitch circle, and the pinion will then work truly with the teeth of the wheel in both positions. The tooth at each extremity of the series must be a circular

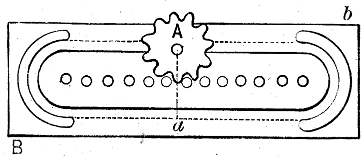

Fig. 213.

one, whose centre lies on the pitch line and whose diameter is equal to half the pitch.

If the reciprocating piece move in a straight line, as it very often does, then the mangle-*wheel* is transformed into a *mangle-rack* (Fig. 213) and its teeth may be simply made cylindrical pins, which those of the mangle-wheel do not admit of on correct principle. B *b* is the sliding piece, and A the driving pinion, whose axis must have the power of shifting from A to *a* through a space equal to its own diameter, to allow of the change from one side of the rack to the other at each extremity of the motion. The teeth of the mangle-rack may receive any of the forms which are given to common rack-teeth, if the arrangement be derived from either Fig. 210 or Fig. 211.

But the mangle-rack admits of an arrangement by which the shifting motion of the driving pinion, which is often inconvenient, may be dispensed with.

B *b*, Fig. 214, is the piece which receives the reciprocating motion, and which may be either guided between rollers, as

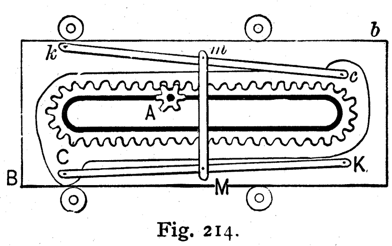

Fig. 214.

shown, or in any other usual way; A the driving pinion, whose axis of motion is fixed; the mangle rack C *c* is formed upon a separate plate, and in this example has the teeth upon the inside of the projecting ridge which borders it, and the guide-groove formed within the ring of teeth, similar to Fig. 211.

This rack is connected with the piece B *b* in such a manner as to allow of a short transverse motion with respect to that piece, by which the pinion, when it arrives at either end of the course, is enabled by shifting the rack to follow the course of the guide-groove, and thus to reverse the motion by acting upon the opposite row of teeth.

The best mode of connecting the rack and its sliding piece is that represented in the figure, and is the same which is adopted in the well-known cylinder printing-engines of Mr. Cowper. Two guide-rods K C, *k c* are jointed at one end K *k* to the reciprocating piece B *b*, and at the other end C *c* to the shifting-rack; these rods are moreover connected by a rod M *m* which is jointed to each midway between their extremities, so that the angular motion of these guide-rods round their centres K *k* will be the same; and as the angular motion is small and the rods nearly parallel to the path of the slide, their extremities C *c* may be supposed to move at a right angle to that path, and consequently the rack which is jointed to those extremities will also move upon B *b* in a direction at a right angle to its path, which is the thing required, and admits of no other motion with respect to B *b*.

To multiply plane motion the construction shown in Fig. 215 is frequently employed. A and B are two racks, and C is a wheel

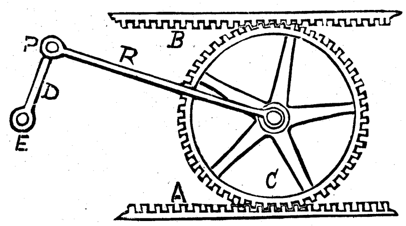

Fig. 215.

between them pivoted upon the rod R. A crank shaft or lever D is pivoted at E and also (at P) to R. If D be operated C traverses along A and also rotates upon its axis, thus giving to B a velocity equal to twice that of the lateral motion of C.

The diameter of the wheel is immaterial, for the motion of B will always be twice that of C.

Friction gearing-wheels which communicate motion one to the other by simple contact of their surfaces are termed friction-wheels, or friction-gearing. Thus in Fig. 216 let A and B be two wheels that touch each other at C, each being suspended upon a central shaft; then if either be made to revolve, it will cause the other to revolve also, by the friction of the surfaces meeting at C. The degree of force which will be thus conveyed from one to the other will depend upon the character of the surface and the length of the line of contact at C.

These surfaces should be made as concentric to the axis of the wheel and as flat and smooth as possible in order to obtain a maximum power of transmission. Mr. E. S. Wicklin states that under these conditions and proper forms of construction as much as 300 horse-power may be (and is in some of the Western States) transmitted.

In practice, small wheels of this class are often covered with some softer material, as leather; sometimes one wheel only is so covered, and it is preferred that the covered wheel drive the iron one, because, if a slip takes place and the iron wheel was the driver, it would be apt to wear a concave spot in the wood

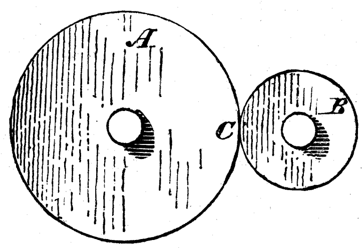

Fig. 216.

covered one, and the friction between the two would be so greatly diminished that there would be difficulty in starting them when the damaged spot was on the line of centre.

If, however, the iron wheel ceased motion, the wooden one continuing to revolve, the damage would be spread over that part of the circumference of the wooden one which continued while the iron one was at rest, and if this occurred throughout a whole revolution of the wooden wheel its roundness would not be apt to be impaired, except in so far as differences in the hardness of the wood and similar causes might effect.

"To select the best material for driving pulleys in friction-gearing has required considerable experience; nor is it certain that this object has yet been attained. Few, if any, well-arranged and careful experiments have been made with a view of determining the comparative value of different materials as a frictional medium for driving iron pulleys. The various theories and notions of builders have, however, caused the application to this use of several varieties of wood, and also of leather, india-rubber, and paper; and thus an opportunity has been given to judge of their different degrees of efficiency. The materials most easily obtained, and most used, are the different varieties of wood, and of these several have given good results.

"For driving light machinery, running at high speed, as in sash, door, and blind factories, basswood, the linden of the Southern and Middle States (*Tilia Americana*) has been found to possess good qualities, having considerable durability and being unsurpassed in the smoothness and softness of its movement. Cotton wood (*Populus monilifera*) has been tried for small machinery with results somewhat similar to those of basswood, but is found to be more affected by atmospheric changes. And even white pine makes a driving surface which is, considering the softness of the wood, of astonishing efficiency and durability. But for all heavy work, where from twenty to sixty horse-power is transmitted by a single contact, soft maple (*Acer rubrum*) has, at present, no rival. Driving pulleys of this wood, if correctly proportioned and well built, will run for years with no perceptible wear.

"For very small pulleys, leather is an excellent driver and is very durable; and rubber also possesses great adhesion as a driver; but a surface of soft rubber undoubtedly requires more power than one of a less elastic substance.

"Recently paper has been introduced as a driver for small machinery, and has been applied in some situations where the test was most severe; and the remarkable manner in which it has thus far withstood the severity of these tests appears to point to it as the most efficient material yet tried.

"The proportioning, however, of friction-pulleys to the work required and their substantial and accurate construction are matters of perhaps more importance than the selection of material.

"Friction-wheels must be most accurately and substantially made and kept in perfect line so that the contact between the surfaces may not be diminished. The bodies are usually of iron lagged or covered with wooden segments.

"All large drivers, say from four to ten feet diameter and from twelve to thirty inch face, should have rims of soft maple six or seven inches deep. These should be made up of plank, one and a half or two inches thick, cut into 'cants,' one-sixth, eighth, or tenth of the circle, so as to place the grain of the wood as nearly as practicable in the direction of the circumference. The cants should be closely fitted, and put together with white lead or glue, strongly nailed and bolted. The wooden rim, thus made up to within about three inches of the width required for the finished pulley, is mounted upon one or two heavy iron 'spiders,' with six or eight radial arms. If the pulley is above six feet in diameter, there should be eight arms, and two spiders when the width of face is more than eighteen inches.

"Upon the ends of the arms are flat 'pads,' which should be of just sufficient width to extend across the inner face of the wooden rim, as described; that is, three inches less than the width of the finished pulley. These pads are gained into the inner side of the rim; the gains being cut large enough to admit keys under and beside the pads. When the keys are well driven, strong 'lag' screws are put through the ends of the arm into the rim. This done, an additional 'round' is put upon each side of the rim to cover bolt heads and secure the keys from ever working out. The pulley is now put to its place on the shaft and keyed, the edges trued up, and the face turned off with the utmost exactness.

"For small drivers, the best construction is to make an iron pulley of about eight inches less diameter and three inches less face than the pulley required. Have four lugs, about an inch square, cast across the face of this pulley. Make a wooden rim, four inches deep, with face equal to that of the iron pulley, and the inside diameter equal to the outer diameter of the iron. Drive this rim snugly on over the rim of the iron pulley having cut gains to receive the lugs, together with a hard wood key beside each. Now add a round of cants upon each side, with their inner diameter less than the first, so as to cover the iron rim. If the pulley is designed for heavy work, the wood should be maple, and should be well fastened by lag screws put through the iron rim; but for light work, it may be of basswood or pine, and the lag screws omitted. But in all cases, the wood should be thoroughly seasoned.

"In the early use of friction-gearing, when it was used only as backing gear in saw-mills, and for hoisting in grist-mills, the pulleys were made so as to present the head of the wood to the surface; and we occasionally yet meet with an instance where they are so made. But such pulleys never run so smoothly nor drive

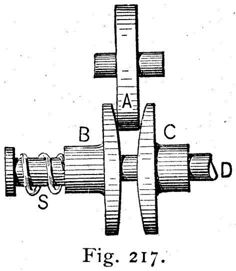

Fig. 217.

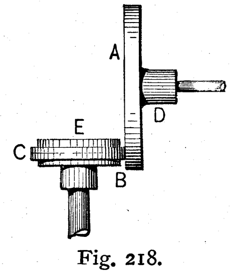

Fig. 218.

so well as those made with the fibre more nearly in a line with the work."*

The driving friction may be obtained from contact of the radial surfaces in two ways: thus, Fig. 217 represents three discs, A, B, and C; the edge of A being gripped by and between B and C, which must be held together by a spiral spring S or other equivalent device. These wheels may be made to give a variable speed of rotation by curving the surfaces of the pair B C as in the figure. By means of suitable lever-motion A may be made to advance towards or recede from the centre of B and C, giving to their shaft an increased or diminished speed of revolution.

A similar result may be obtained by the construction shown in Fig. 218, in which D and E are two discs fast upon their respective shafts, and C are discs of leather clamped in E. It is obvious that if D be the driver the speed of revolution of E will be dimin-

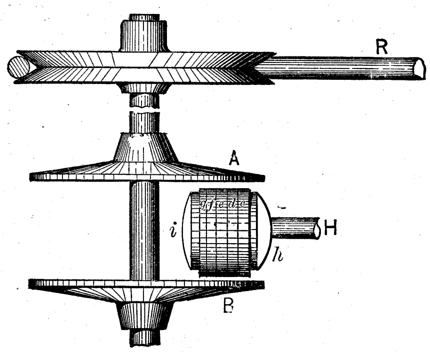

Fig. 219.

ished in proportion as it is moved nearer to the centre of D, and also that the direction of revolution of D remaining constant, that of E will be in one direction if on the side B of the centre of D, and in the other direction if it is on the side A of the centre of D, thus affording means of reversing the motion as well as of varying its speed. A similar arrangement is sometimes employed to enable the direction of rotation of the driver shaft to be reversed, or its motion to cease. Thus, in Fig. 219, R is a driving rope driving the discs A, B, and *c, d, e, f, g* are discs of yellow pine clamped between the flanges *h i;* when these five discs are forced (by lifting shaft H), against the face of A motion occurs in one direction, while if forced against B the direction of motion of H is reversed.

For many purposes, such as hoisting, for example, where considerable power requires to be transmitted, the form of friction wheels shown in Fig. 220 is employed, the object being to increase the line of contact between wheels of a given width of

* By E. S. Wicklin.

face. In this case the strain due to the length of the line of contact partly counteracts itself, thus relieving to that extent the journals from friction. Thus in Fig. 221 is shown a single

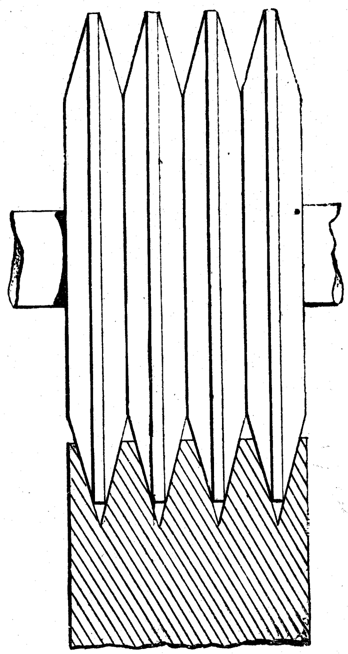

Fig. 220.

wedge and groove of a pair of wheels. The surface pressure on each side will be at a right angle to the face, or in the direction described by the arrows A and B. The surface contact acts to thrust the bearings of the two shafts apart. The effective length of surface acting to thrust the bearings apart being denoted by the dotted line C. The relative efficiency of this class of wheel, however, is not to be measured by the length of the line C, as compared to that of the two contacting sides of the groove, because it is increased from the wedge shape of the groove, and furthermore, no matter how solid the wheels may be, there will be some elasticity which will operate to increase the driving power due to the contact. It is to preserve the wedge principle that the wedges are made flat at the top, so that they shall not bottom in the grooves even after considerable wear has taken place. The object of employing this class of gear is to avoid noise and jar and to insure a uniform motion. The motion at the line of contact of such wheels is not a rolling,

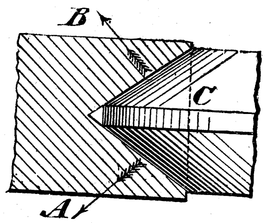

Fig. 221.

but, in part, a sliding one, which may readily be perceived from a consideration of the following. The circumference of the top of each wedge is greater than that of the bottom, and, in the case of the groove, the circumference of the top is greater than that of the bottom; and since the top or largest circumference of one contacts with the smallest circumference of the other, it follows that the difference between the two represents the amount of sliding motion that occurs in each revolution. Suppose, for example, we take two of such wheels 10 inches in diameter, having wedges and grooves $\frac{1}{4}$ inch high and deep respectively; then the top of the groove will travel 31·416 inches in a revolution, and it will contact with the bottom of the wedge which travels (on account of its lesser diameter) 29·845 inches per revolution.

Fig. 222 shows the construction for a pair of bevel wheels on the same principle.

A form of friction-gearing in which the journals are relieved of the strain due to the pressure of contact, and in which slip is

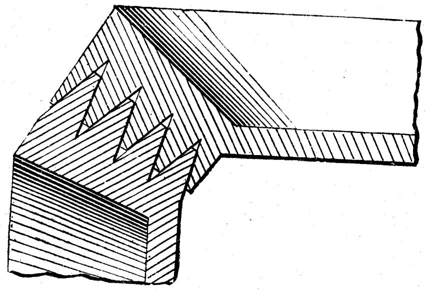

Fig. 222.

impossible, is shown in Fig. 223. It consists of projections on one wheel and corresponding depressions or cavities on the other. These projections and cavities are at opposite angles on each

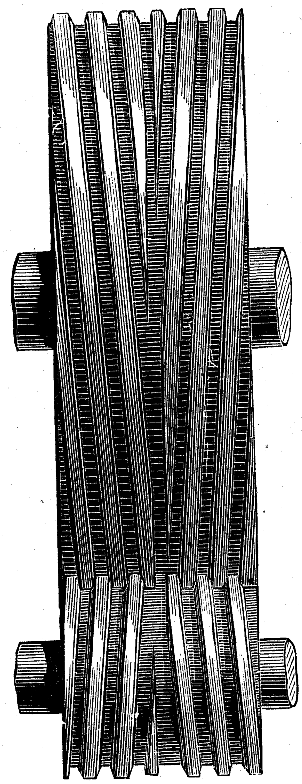

Fig. 223.

half of each wheel, so as to avoid the end pressure on the journals which would otherwise ensue. Their shapes may be formed at will, providing that the tops of the projections are narrower than

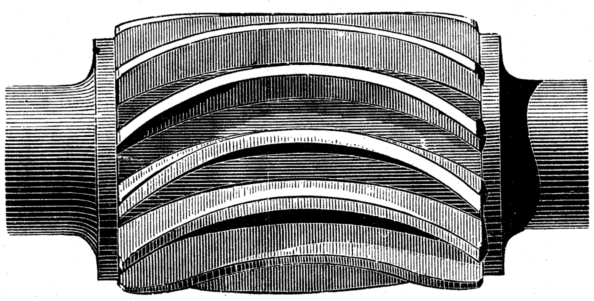

Fig. 224.

their bases, which is necessary to enable the projections to enter and leave the cavities. In this class of positive gear great truth or exactness is possible, because both the projections and cavities may be turned in a lathe. Fig. 224 represents a similar kind of gear with the projections running lengthways of the cylinder approaching more nearly in its action to toothed gearing, and in this case the curves for the teeth and groves should be formed by the rules already laid down for toothed gearing. The action of this latter class may be made very smooth, because a continuous contact on the line of centres may be maintained by reason of the longitudinal curve of the teeth.

Cams may be employed to impart either a uniform, an irregular,

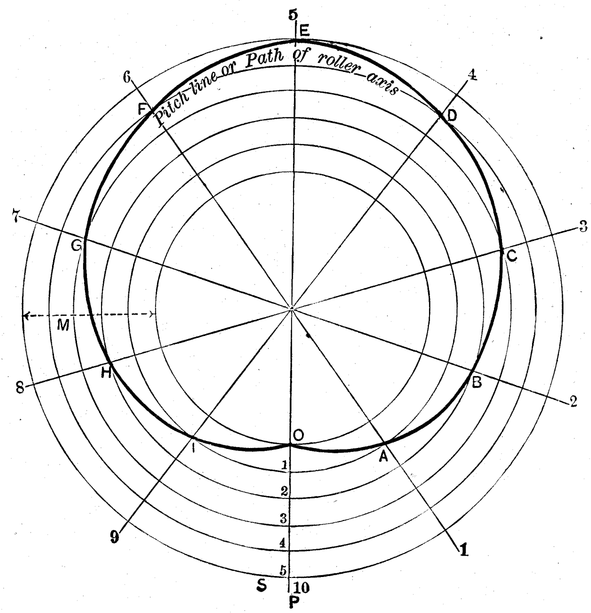

Fig. 225.

or an intermittent motion, the principles involved in their construction being as follows:—Let it be required to construct a cam that being revolved at a uniform velocity shall impart a uniform reciprocating motion. First draw an inner circle O, Fig. 225, whose radius must equal the radius of the shaft that is to drive it, plus the depth of the cam at its shallowest part, plus the radius of the roller the cam is to actuate. Then from the same

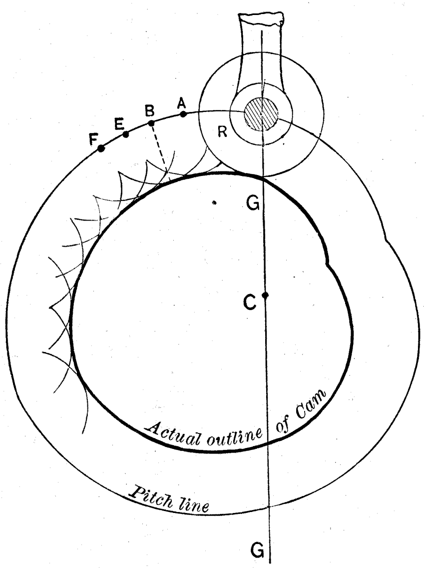

Fig. 226.

centre draw an outer circle S, the radius between these two circles being equal to the amount the cam is to move the roller. Draw a line O P, and divide it into any convenient numbers of divisions (five being shown in the figure), and through these points draw circles. Divide the outer circle S into twice as many equal divisions as the line O P is divided into (as from 1 to 10 in the figure), and where these lines pass through the circles will be points through which the pitch line of the cam may be drawn.

Thus where circle 1 meets line 1, or at point A, is one point in the pitch line of the cam; where circle 2 meets line 2, or at B, is another point in the pitch line of the cam, and so on until we reach the point E, where circle 5 meets line 5. From this point we simply repeat the process, the point E where line 6 cuts circle 4, being a point on the pitch line, and so on throughout the whole 10 divisions, and through the points so obtained we draw the pitch line.

If we were to cut out a cam to the outline thus obtained, and revolve it at a uniform velocity, it would move a point held against its perimeter at a uniform velocity throughout the whole of the cam revolution. But such a point would rapidly become

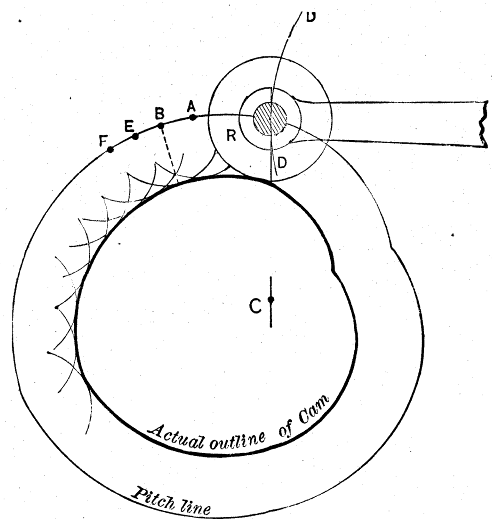

Fig. 227.

worn away and dulled, which would, as the point broadened, vary the motion imparted to it, as will be seen presently. To avoid this wear a roller is used in place of a point, and the diameter of the roller affects the action of the cam, causing it to accelerate the cam action at one and retard it at another part of the cam revolution, hence the pitch line obtained by the process in Fig. 225 represents the path of the centre of the roller, and from this pitch line we may mark out the actual cam by the construction shown in Fig. 226. A pair of compasses are set to the radius of the roller R, and from points (such as at A, B, E, F), as the pitch line, arcs of circles are struck, and a line drawn to just meet the crowns of these arcs will give the outline of the actual cam.

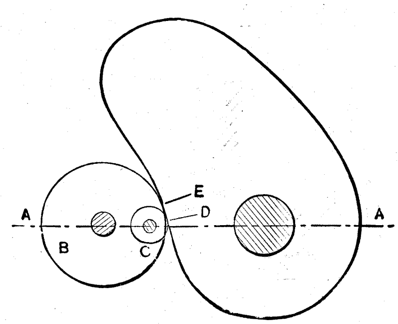

Fig. 228.

The motion of the roller, however, in approaching and receding from the cam centre C, must be in a straight line G G that passes through the centre C of the cam. Suppose, for example, that instead of the roller lifting and falling in the line G G its arm is horizontal, as in Fig. 227, and that this arm being pivoted the roller moves in an arc of a circle as D D, and the motion imparted to the arm will no longer be uniform. Furthermore, different diameters of roller require different forms of cam to accomplish the same motion, or, in other words, with a given cam the action will vary with different diameters of roller. Suppose, for example, that in Fig. 228 we have a cam that is to operate a roller along the line A A, and that B represents a large and C a small roller, and with the cam in the position shown in the figure, C will have contact with the cam edge at point D, while B will have contact at the point E, and it follows that on account of the enlarged diameter of roller B over roller C, its action is at this point quicker under a given amount of cam

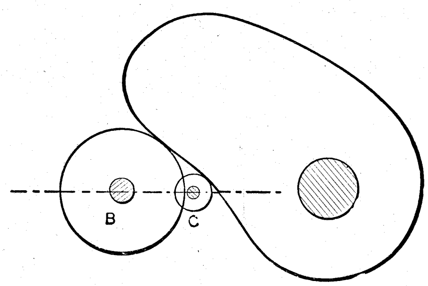

Fig. 229.

motion, which has occurred because the point of contact has advanced upon the roller surface—rolling along it, as it were. In Fig. 229 we find that as the cam moves forward this action continues on both the large and the small roller, its effect being greater upon the large than upon the small one, and as this

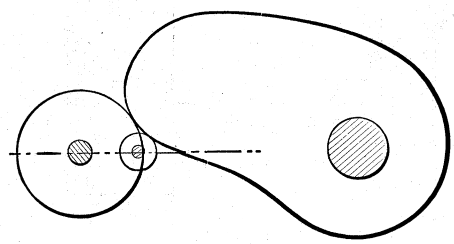

Fig. 230.

rolling motion of the point of contact evidently occurs easily, a quick roller motion is obtained without shock or vibration. Continuing the cam motion, we find in Fig. 230 that the point of contact is receding toward the line of motion on the large roller and advancing upon the small one, while in Fig. 231 the two

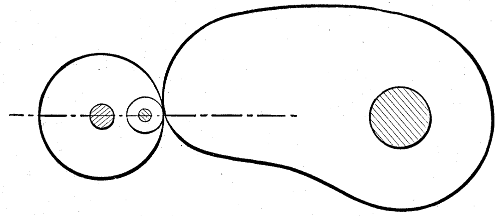

Fig. 231.

have contact at about the same point, the forward motion being about completed.

To compare the motions of the respective rollers along the line of motion A A we proceed as in Fig. 232, in which the two dots M and N are the same distance apart as are the centres of the two

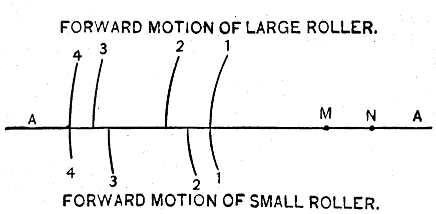

Fig. 232.

rollers B and C when in the positions they occupy in Fig. 228; hence a pair of compasses set to the radius from the axis of the cam to that of roller B will, if rested at N, strike the arc marked 1 above the line of motion A A, while a pair of compasses set to the radius from the axis of the cam to that of roller C in Fig. 228 will, if rested at M in Fig. 232, mark the arc 1 below the line of motion A A. Continuing this process, we set the compasses to the radius from the axis of the cam to that of roller B in Fig. 229, and mark this radius at arc 2 above the line A A in Fig. 232; hence the distance apart of these two arcs is the amount the roller travelled along the line A A while the cam moved from its position in Fig. 228 to its position in Fig. 229. Next we set the compasses from the axis of the cam to that of the large roller in Fig. 230, and then mark arc 3 above the line in Fig. 232, and

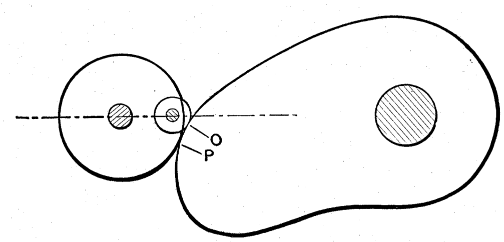

Fig. 233.

repeat the process for Fig. 233, thus using the centre N for all the positions of the large roller and marking its motion above the line A A. To get the motion of the small roller C, we set the compasses to the radius from the axis of the cam to the small roller in Fig. 228, and then resting one point of these compasses on centre M in Fig. 232, we mark arc 1 below the line A A. Turning to Fig. 229 we set the compasses from the cam axis to the centre of roller C, and from centre N in Fig. 232 mark arc 2 below line A. From Figs. 230 and 231 proceed in the same way

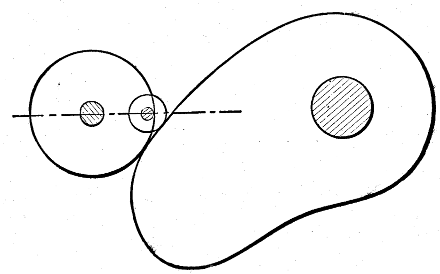

Fig. 234.

to get lines 3 and 4 below line A in Fig. 232, and we may at once compare the two motions. Thus we find that while the cam moved from the position in Fig. 228 to that in Fig. 229, the large roller moved twice as far as the small one, while at 230 the motions were rapidly equalizing again, the equalization being completed at 231.

We may now consider the return motion, and in Fig. 233 we find that the order of things is reversed, for the small roller has contact at O, while the large one has contact at P; hence the

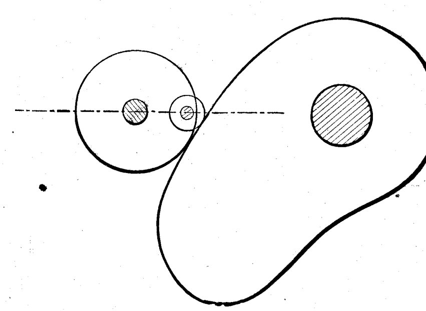

Fig. 235.

small one leads and gives the most rapid motion, which it continues to do, as is shown in Figs. 234, 235, and 236, and we may plot out the two motions as in Fig. 237—that for the large roller being above and that for the small one below the line A A. First we set a pair of compasses to the radius from the axis of the large and small roller when in the position shown in Fig. 231 (which corresponds to the same radius in Fig. 228), and mark two centres, M and N, as we did in Fig. 232. Of these N is the centre for plotting the motion of the large roller and M the centre for plotting the motion of the small one. We set a pair of compasses to the radius from the axis of the cam and that of the large roller in Fig. 231, and then resting the compasses at N we mark arc 5 above the line A A, Fig. 237. The compasses are then set from the cam to the roller axis in Fig. 233, and arc 6 is

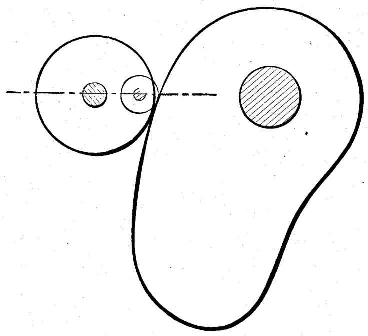

Fig. 236.

marked above line A A. From Figs. 234, 235, and 236 we get the radii to mark arcs 7, 8, 9 above A A, and the motion of the large roller is plotted. We proceed in the same way for the small one, but use the centre M, Fig. 237, to mark the arcs 5, 6, 7, 8, and 9 below the line A A, and find that the small roller has moved

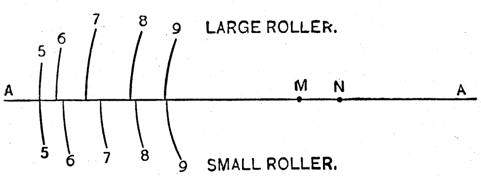

Fig. 237.

quickest throughout. It appears, then, that the larger the roller the quicker the forward motion and the slower the return one, which is advantageous, because the object is to move the roller out quickly and close it slowly, so that under a quick speed the cam shall not run away from the roller as it is apt to do in the absence of a return or backing cam, which consists of a separate cam for moving the roller on its return stroke, thus dispensing with the use of springs or weights to keep the roller upon the cam and making the motion positive.

The return or backing cam obviously depends for its shape upon the forward cam, and the latter having been determined, the requisite form for the return cam may be found as follows. In Fig. 238 let A represent the forward cam fastened in any suitable or convenient way to a disc of paper, or, what is better, sheet zinc, B. The cam is pivoted by a pin passing through it and the zinc, and driven into the drawing-board. A frame F is made to carry two rollers R and R′, whose width apart exactly equals the extreme length of the forward cam. The faces D D of the frame F are in a line with a line passing through the centres of the rolls R R′, and the cam is also pivoted on this line, so that when the four pins P are driven into the drawing-board, the frame F will be guided by them to move in a line that crosses the centre of the cam A. Suppose then that, the pieces occupying the position shown in the engraving, we slide F so that roller R touches the edge of cam A, and we may then take a needle and mark an arc or line around the edge of R′. We then revolve cam A a trifle, and, being fast to B, the two will move together, and with R against A we mark a second arc, coincident with the edge of roller

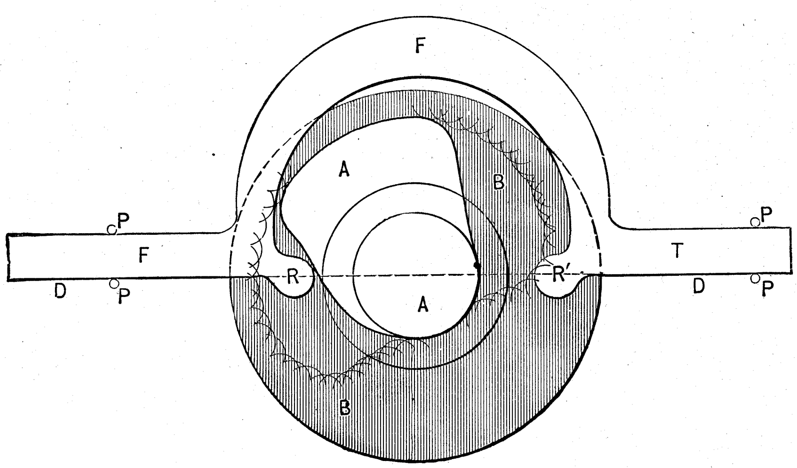

Fig. 238.

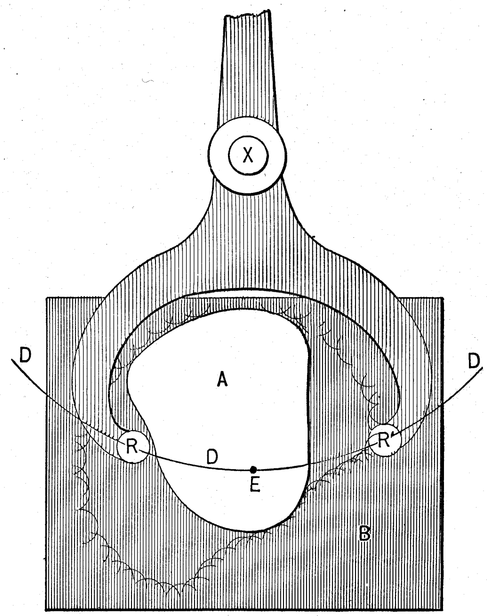

Fig. 239.

R′. By continuing this process we mark the numerous short arcs shown upon B, and the crowns of these arcs give us the outline of the return cam. It is obvious that, while the edge of the cam A will not let roller R (and therefore frame F) move to the right, roller R′ being against the edge of the backing or return cam as marked upon B, prevents the frame F from moving to the left; hence neither roll can leave its cam.

We have in this example supposed that the frame carrying the rollers is guided to move in a straight line, and it remains to give an example in which the rollers are carried on a pivoted shaft or rocking arm. In Fig. 239 we have the same cam A with a sheet of paper B fastened to it, the rollers R R′ being carried in a rock shaft pivoted at X. It is essential in this case that the rollers R and R′ and the centre upon which the cam revolves shall all three be in the arc of a circle whose centre is the axis of X, as is denoted by the arc D. The cam A is fastened to the piece of stiff paper or of sheet zinc B, and the two are pivoted by a pin passing through the axis E of the cam and into the drawing-board, while the lever is pivoted at X by a pin passing into the drawing-board. The backing or return cam is obviously marked out the same way as was described with reference to Fig. 238.

In Fig. 240 we have as an example the construction of a cam to operate the slide valve of an engine which is to have the steam

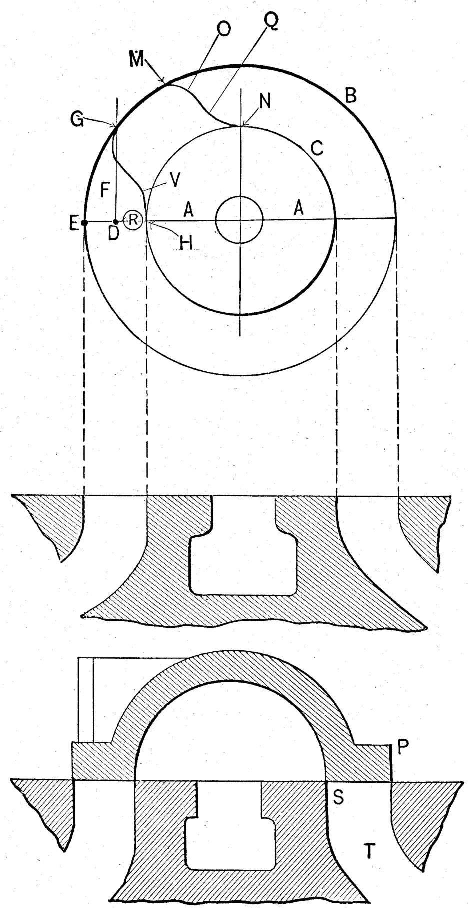

Fig. 240.

supply to the cylinder cut off at one-half the piston stroke, and that will admit the live steam as quickly as a valve having steam lap equal to, say, three-fourths the width of the port. In Fig. 240 let the line A represent a piston stroke of 24 inches, the outer circle B the path of the outer edge of the cam, and the inner circle C the inner edge of the cam, the radius between these circles representing the full width of the steam port. Now, in a valve

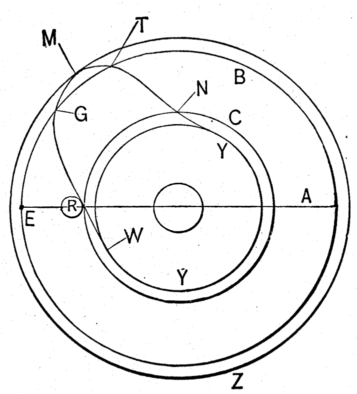

Fig. 241.

having lap equal to three-fourths the width of the steam port, and travel enough to open both ports fully, the piston of a 24-inch-stroke engine will have moved about 2 inches before the steam port is fully opened, and to construct a cam that will effect the same movement we mark a dot D, distant from the end E of piston stroke $\frac{2}{26}$ of the length of the line A, and by erecting the line F we get at point G, the point at which the cam must attain its greatest throw. It is obvious, therefore, that as the roller is at R the valve will be in mid-position, as shown at the bottom of the figure, and that when point G of the cam arrives at E the edge P of the valve will be moved fair with edge S of the steam port T, which will therefore be full open. To cut off at half stroke the valve must again be closed by the time point N of the cam meets the roller R; hence we may mark point N. We may then mark in the cam curve from N to M, making it as short as it will work properly without causing the roller to fail to follow the curve or strike a blow when reaching the circle C. To accomplish this end in a single cam, it is essential to make the curve as gradual as possible from point M to O, so as to start the roller motion easily. But once having fairly started, its motion may be rapidly accelerated, the descent from O to Q being rapid. To prevent the roller from meeting circle C with a blow, the curve from Q to N is again made gradual, so as to ease and retard the roller motion. The same remarks apply to the curve from R to G, the object being to cause the roller to begin and end its passage along the cam curve as slowly as the length of cam edge occupied by the curve will permit. There is one objection to starting the curve slowly at G, which is that the port S will be opened correspondingly slowly for the live steam. This, however, may be overcome by giving the valve an increased travel, as shown in Fig. 241, which will simply cause the valve edge to travel to a corresponding amount over the inside edge of the port. The increased travel is shown by the circles Y and Z, and it is seen that the cam curve from W to R is more gradual than in Fig. 240, while the

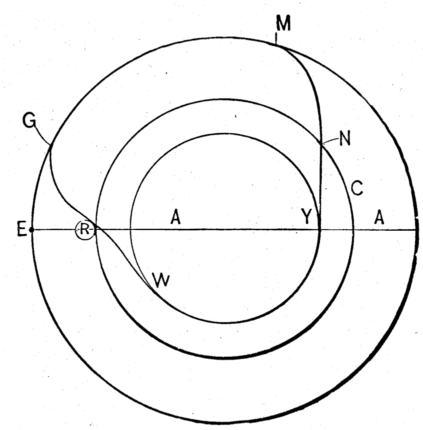

Fig. 242.

roller R will be moved much more quickly in the position shown in Fig. 241 than it will in that shown in Fig. 240, both positions being that when the piston is at the end of the stroke and the port about to open. While that part of the cam curve from G to M in Fig. 241 is moving past the roller R, the valve will be moving over the bridge, the steam port remaining wide open, and therefore not affecting the steam distribution. After point M, Fig. 241, has passed the roller, we have from M to T to start the roller gradually, so that when it has arrived at T and the port begins to close for the cut-off it may move rapidly, and continue to do so until the point N reaches the roller and the cut-off has occurred, after which it does not matter how slowly the valve moves; hence we may make the curve from N to the circle Y as gradual as we like.

Fig. 242 represents a cam for a valve having the amount of lap represented by the distance between circles C and Y, the cam occupying the position it would do with the piston at one end of the stroke, as at E. Obviously, a full port is obtained when point G reaches the roller, and as point N is distant from E three-quarters of the diameter of the outer circle, the cut-off occurs at three-quarter stroke, and we have from N to Y to make the curve as gradual as we like, and from W to R in moving the valve to open the port. We cannot, however, give more gradual curves at G and at M without retarding the roller motion, and therefore opening and closing the port slower, and it would simply be a matter of increase of speed to cause the roller to fail to follow the cam surface at these two points unless a return cam be employed.

We have in these engine cams considered the steam supply and point of cut-off only, and it is obvious that a second and separate cam would be required to operate the exhaust valves.

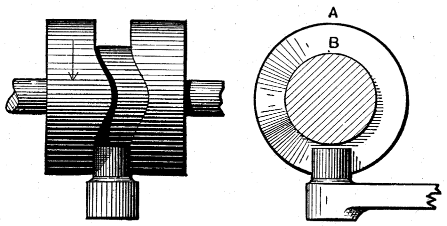

Fig. 243.

Fig. 243 represents a groove-cam, and it is to be observed that the roller cannot be maintained a close fit in the groove, because the friction on its two sides endeavours to drive it in opposite directions at the same time, causing an abrasion that soon widens

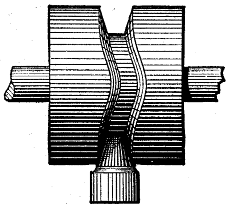

Fig. 244.

the groove and reduces the roller diameter; furthermore, when the grooves are made of equal width all the way down (and these cams are often made in this way) the roller cannot have a rolling action only, but must have some sliding motion. Thus, referring to Fig. 243, the amount of sliding motion will be equal to the differences in the circumferences of the outer circle A and the inner one B. To obviate this the groove and roller must be made of such a taper that the axis of the cam and of the roller will meet on the line of the cam axes and in the middle of the width, as is shown in Fig. 244; but even in this case the cam will grind away the roller to some extent, on account of rubbing its sides in opposite directions. To obviate this, Mr. James Brady, of Brooklyn, N. Y., has patented the use of two rollers, as in Fig. 245, one acting against one side and the other against the

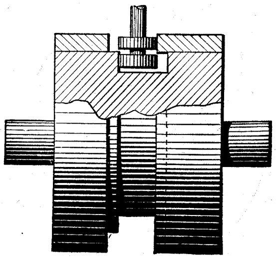

Fig. 245.

other side of the groove, by which means lost motion and rapid wear are successfully avoided.

In making a cam of this form, the body of the cam is covered by a sleeve. The groove is cut through the sleeve and into the body, and is made wider than the diameter of the roller. When the rollers are in place on the spindle or journal, the sleeve is pushed forward, or rather endways, and fastened by a set-screw. This gives the desired bearing on both sides of the groove, while each roller touches one side only of the groove. The edges of the sleeve are then faced off even with the cam body, the whole appearing as in the figure.

THE V-THREAD.

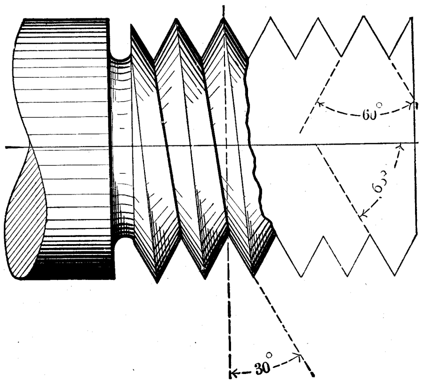

Fig. 246.

THE UNITED STATES STANDARD THREAD.

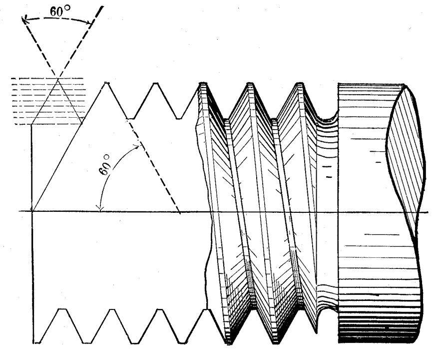

Fig. 247.

THE WHITWORTH, OR ENGLISH STANDARD THREAD.

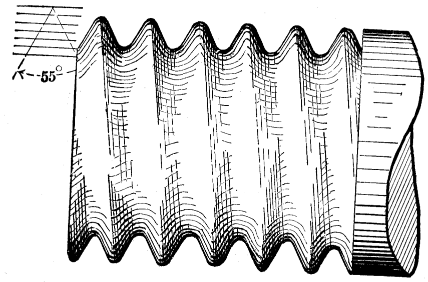

Fig. 248.

THE SQUARE THREAD.

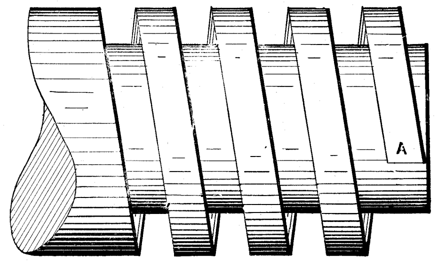

Fig. 249.

THE PITCH OF A THREAD.

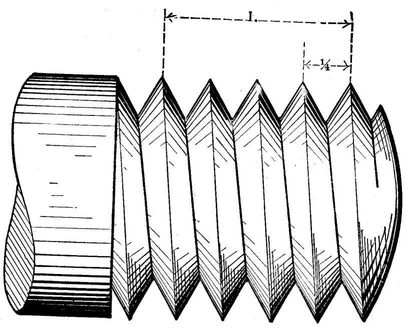

Fig. 250.

A DOUBLE THREAD.

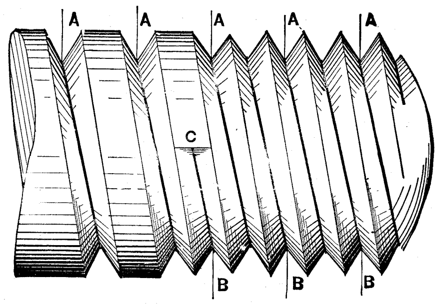

Fig. 251.

THE RATCHET THREAD.

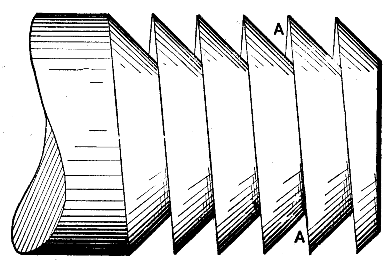

Fig. 252.

A "DRUNKEN" THREAD.

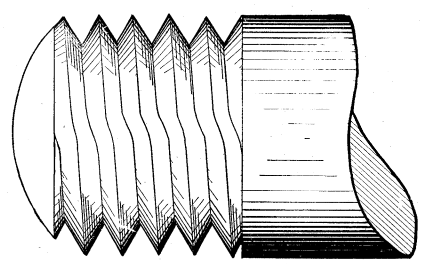

Fig. 253.

RIGHT AND LEFT HAND THREAD.

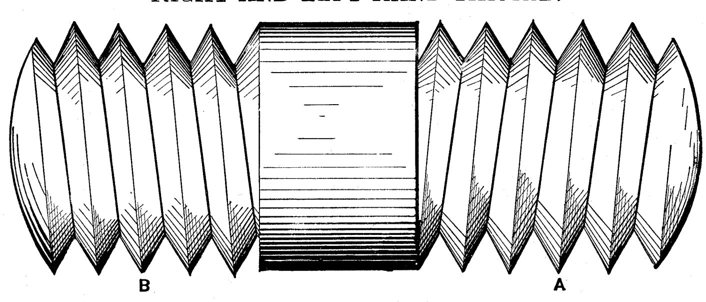

Fig. 254.