Chapter II.—THE TEETH OF GEAR-WHEELS.—CAMS.

Wheel and Tangent Screw or Worm and Worm Gear.

In Fig. 74 are shown a worm and worm gear partly in section on the line of centres. The worm or tangent screw W is simply one long tooth wound around a cylinder, and its form may be determined by the rules laid down for a rack and pinion, the tangent screw or worm being considered as a rack and the wheel as an ordinary spur-wheel.

Worm gearing is employed for transmitting motion at a right

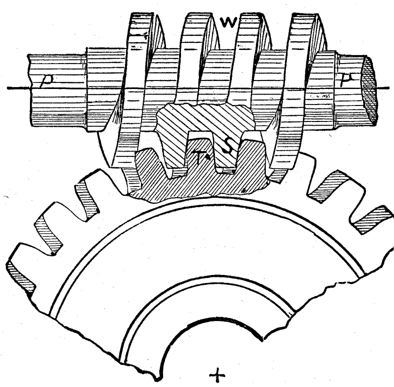

Fig. 74.

angle, while greatly reducing the motion. Thus one rotation of the screw will rotate the wheel to the amount of the pitch of its teeth only. Worm gearing possesses the qualification that, unless of very coarse pitch, the worm locks the wheel in any position in which the two may come to a state of rest, while at the same time the excess of movement of the worm over that of the wheel enables the movement of the latter, through a very minute

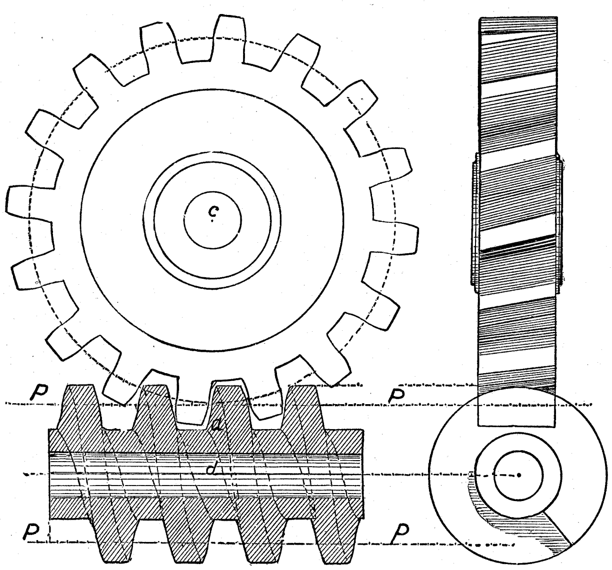

Fig. 75.

portion of a revolution. And it is evident that, when the plane of rotation of the worm is at a right angle to that of the wheel, the contact of the teeth is wholly a sliding one. The wear of the worm is greater than that of the wheel, because its teeth are in continuous contact, whereas the wheel teeth are in contact only when passing through the angle of action. It may be noted, however, that each tooth upon the worm is longer than the teeth on the wheel in proportion as the circumference of the worm is to the length of wheel tooth.

If the teeth of the wheel are straight and are set at an angle equal to the angle of the worm thread to its axis, as in Fig. 75, P P representing the pitch line of the worm, C D the line of centres, and *d* the worm axis, the contact of tooth upon tooth will be at the centre only of the sides of the wheel teeth. It is generally preferred, however, to have the wheel teeth curved to envelop a part of the circumference of the worm, and thus increase the line of contact of tooth upon tooth, and thereby provide more ample wearing surface.

In this case the form of the teeth upon the worm wheel varies at every point in its length as the line of centres is departed from. Thus in Fig. 76 is shown an end view of a worm and a worm gear in section, *c d* being the line of centres, and it will be readily perceived that the shape of the teeth if taken on the line *e f*, will differ from that on the line of centres *c d*; hence the form of the wheel teeth must, if contact is to occur along the full length of the tooth, be conformed to fit to the worm, which may be done by taking a series of section of the worm thread at varying distances from, and parallel to, the line of centres and joining the wheel teeth to the shape so obtained. But if the teeth of the wheel are to be cut to shape, then obviously a worm may be provided with

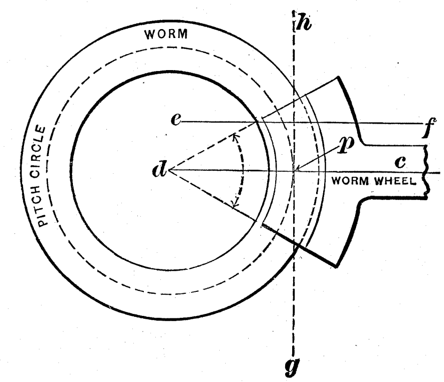

Fig. 76.

teeth (by serrating it along its length) and mounted in position upon the wheel so as to cut the teeth of the wheel to shape as the worm rotates. The pitch line of the wheel teeth, whether they be straight and are disposed at an angle as in Fig. 75, or curved as in Fig. 76, is at a right angle to the line of centres *c d*, or in other words in the plane of *g h*, in Fig. 76. This is evident because the pitch line must be parallel to the wheel axis, being at an equal radius from that axis, and therefore having an equal velocity of rotation at every point in the length of the pitch line of the wheel tooth.

If we multiply the number of teeth by their pitch to obtain the circumference of the pitch circle we shall obtain the circumference due to the radius of *g h*, from the wheel axis, and so long as *g h* is parallel to the wheel axis we shall by this means obtain the same diameter of pitch circle, so long as we measure it on a line parallel to the line of centres *c d*. The pitch of the worm is the same at whatever point in the tooth depth it may be measured, because the teeth curves are parallel one to the other, thus in Fig. 77 the pitch measures are equal at *m*, *n*, or *o*.

But the action of the worm and wheel will nevertheless not be correct unless the pitch line from which the curves were rolled coincides with the pitch line of the wheel on the line of centres, for although, if the pitch lines do not so coincide, the worm will at each revolution move the pitch line of the wheel through a distance equal to the pitch of the worm, yet the motion of the wheel will not be uniform because, supposing the two pitch lines not to meet, the faces of the pinion teeth will act against those of the wheel, as shown in Fig. 78, instead of against their flanks, and as the faces are not formed to work correctly together the motion will be irregular.

The diameter of the worm is usually made equal to four times

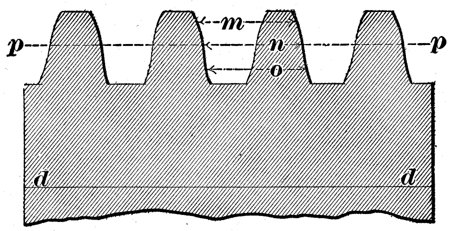

Fig. 77.

the pitch of the teeth, and if the teeth are curved as in figure 76 they are made to envelop not more than 30° of the worm.

The number of teeth in the wheel should not be less than thirty, a double worm being employed when a quicker ratio of wheel to worm motion is required.

When the teeth of the wheel are curved to partly envelop the worm circumference it has been found, from experiments made

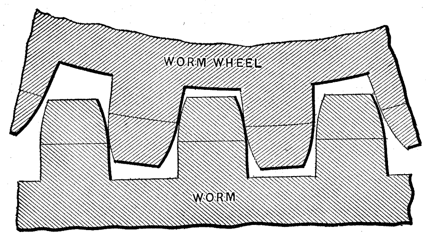

Fig. 78.

by Robert Briggs, that the worm and the wheel will be more durable, and will work with greatly diminished friction, if the pitch line of the worm be located to increase the length of face and diminish that of the flank, which will decrease the length of face and increase the length of flank on the wheel, as is shown

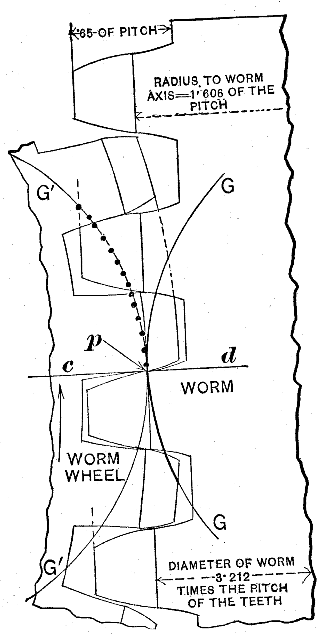

Fig. 79.

in Fig. 79; the location for the pitch line of the worm being determined as follows :—

The full radius of the worm is made equal to twice the pitch of its teeth, and the total depth of its teeth is made equal to ·65 of its pitch. The pitch line is then drawn at a radius of 1·606 of the pitch from the worm axis. The pitch line is thus determined in Fig. 76, with the result that the area of tooth face and of worm surface is equalized on the two sides of the pitch line in the figure. In addition to this, however, it may be observed that by thus locating the pitch line the arcs both of approach and of recess are altered. Thus in Fig. 80 is represented the same worm and wheel as in Fig. 79, but the pitch lines are here laid down as in ordinary gearing. In the two figures the arcs of approach are marked by the thickened part of the generating circle, while the arcs of recess are denoted by the dotted arc on the generating circle, and it is shown that increasing the worm face, as in Fig. 79, increases the arc of recess, while diminishing the worm flank diminishes the arc of approach, and the

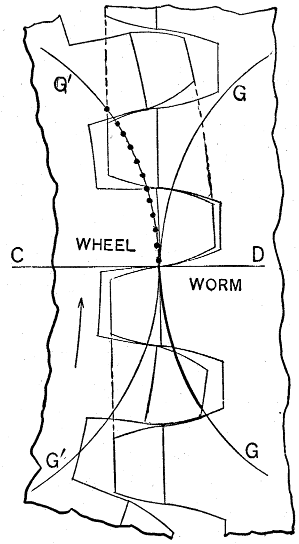

Fig. 80.

action of the worm is smoother because the worm exerts more pulling than pushing action, it being noted that the action of the worm on the wheel is a pushing one before reaching, and a pulling one after passing, the line of centres.

It may here be shown that a worm-wheel may be made to work correctly with a square thread. Suppose, for example, that the diameter of the generating circle be supposed to be infinite, and the sides of the thread may be accepted as rolled by the circle. On the wheel we roll a straight line, which gives a cycloidal curve suitable to work with the square thread. But the action will be confined to the points of the teeth, as is shown in Fig. 81, and also to the arc of approach. This is the same thing as taking the faces off the worm and filling in the flanks of the wheel.

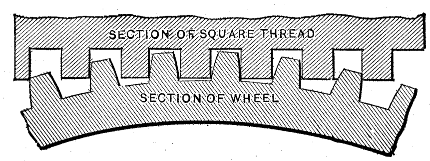

Fig. 81.

Obviously, then, we may reverse the process and give the worm faces only, and the wheel, flanks only, using such size of generating circle as will make the spaces of the wheel parallel in their depths and rolling the same generating circle upon the pitch line of the worm to obtain its face curve. This would enable the teeth on the wheel to be cut by a square-threaded tap, and would confine the contact of tooth upon tooth to the recess.

The diameter of generating circle used to roll the curves for a worm and worm-wheel should in all cases be larger than the radius of the worm-wheel, so that the flanks of the wheel teeth may be at least as thick at the root as they are at the pitch circle.

To find the diameter of a wheel, driven by a tangent-screw, which is required to make one revolution for a given number of turns of the screw, it is obvious, in the first place, that when the screw is single-threaded, the number of teeth in the wheel must be equal to the number of turns of the screw. Consequently, the pitch being also given, the radius of the wheel will be found by multiplying the pitch by the number of turns of the screw during one turn of the wheel, and dividing the product by 6·28.

When a wheel pattern is to be made, the first consideration is the determination of the diameter to suit the required speed; the next is the pitch which the teeth ought to have, so that the wheel

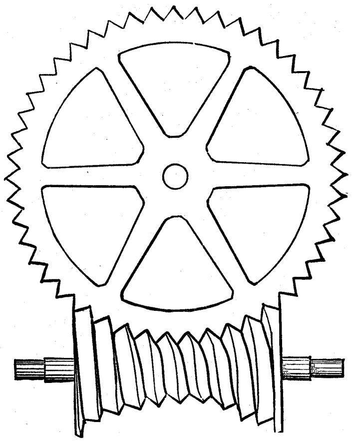

Fig. 82.

may be in accordance with the power which it is intended to transmit; the next, the number of the teeth in relation to the pitch and diameter; and, lastly, the proportions of the teeth, the clearance, length, and breadth.

When the amount of power to be transmitted is sufficient to cause excessive wear, or when the velocity is so great as to cause rapid wear, the worm instead of being made parallel in diameter

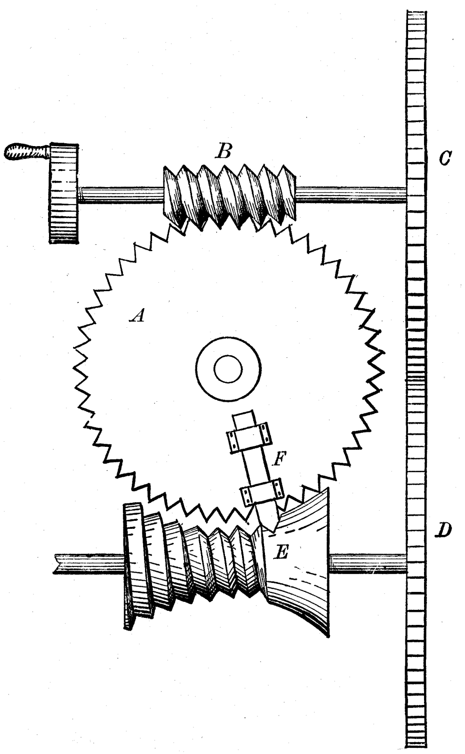

Fig. 83.

from end to end, is sometimes given a curvature equal to that of the worm-wheel, as is shown in Fig. 82.

The object of this design is to increase the bearing area, and thus, by causing the power transmitted to be spread over a larger area of contact, to diminish the wear. A mechanical means of cutting a worm to the required form for this arrangement is shown in Fig. 83, which is extracted from "Willis' Principles of Mechanism." "A is a wheel driven by an endless screw or worm-wheel, B, C is a toothed wheel fixed to the axis of the endless screw B and in gear with another and equal toothed gear D, upon whose axis is mounted the smooth surfaced solid E, which it is desired to cut into Hindleys'* endless screw. For this purpose a cutting tooth F is clamped to the face of the wheel A. When the handle attached to the axis of B C is turned round, the wheel A and solid wheel E will revolve with the same relative velocity as A and B, and the tool F will trace upon the surface of the solid E a thread which will correspond to the conditions. For from the very mode of its formation the section of every thread through the axis will

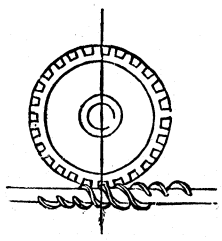

Fig. 84.

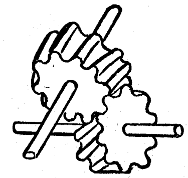

Fig. 85.

point to the centre of the wheel A. The axis of E lies considerably higher than that of B to enable the solid E to clear the wheel A.

"The edges of the section of the solid E along its horizontal centre line exactly fit the segment of the toothed wheel, but if a section be made by a plane parallel to this the teeth will no longer be equally divided as they are in the common screw, and therefore this kind of screw can only be in contact with each tooth along a line corresponding to its middle section. So that the advantage of this form over the common one is not so great as appears at first sight.

"If the inclination of the thread of a screw be very great, one or more intermediate threads may be added, as in Fig. 84, in which case the screw is said to be double or triple according to the number of separate spiral threads that are so placed upon its surface. As every one of these will pass its own wheel-tooth across the line of centres in each revolution of the screw, it follows that as many teeth of the wheel will pass that line during one revolution of the screw as there are threads to the screw. If we suppose the number of these threads to be considerable, for

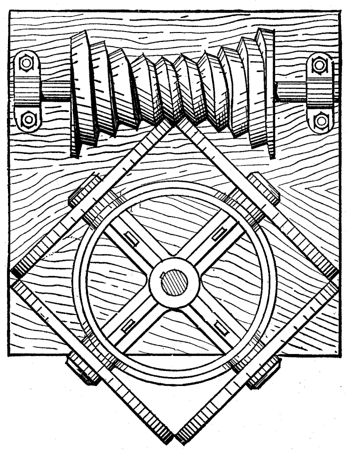

Fig. 86.

example, equal to those of the wheel teeth, then the screw and wheel may be made exactly alike, as in Fig. 85; which may serve as an example of the disguised forms which some common arrangements may assume."

In Fig. 86 is shown Hawkins's worm gearing. The object of this ingenious mechanical device is to transmit motion by means of screw or worm gearing, either by a screw in which the threads are of equal diameter throughout its length, or by a spiral worm, in which the threads are not of equal diameter throughout, but increase in diameter each way from the centre of its length, or

* The inventor of this form of endless screw.

about the centre of its length outwardly. Parallel screws are most applicable to this device when rectilinear motions are produced from circular motions of the driver, and spiral worms are applied when a circular motion is given by the driver, and imparted to the driven wheel. The threads of a spiral worm instead of gearing into teeth like those of an ordinary worm-wheel, actuate a series of rollers turning upon studs, which studs are attached to a wheel whose axis is not parallel to that of the worm, but placed at a suitable inclination thereto. When motion is given to the worm then rotation is produced in the roller wheel at a rate proportionable to the pitch of worm and diameter of wheel respectively.

In the arrangement for transmitting rectilinear motion from a screw, rollers may be employed whose axes are inclined to the axis of the driving screw, or else at right angles to or parallel to the same. When separate rollers are employed with inclined axes, or axes at right angles with that of the main driving screw, each thread in gear touches a roller at one part only; but when the rollers are employed with axes parallel to that of the driving screw a succession of grooves are turned in these rollers, into which the threads of the driving screw will be in gear throughout the entire length of the roller. These grooves may be separate and apart from each other, or else form a screw whose pitch is equal to that of the driving screw or some multiple thereof.

In Fig. 86 the spiral worm is made of such a length that the edge of one roller does not cease contact until the edge of the next comes into contact; a wheel carries four rollers which turn on studs, the latter being secured by cottars; the axis of the

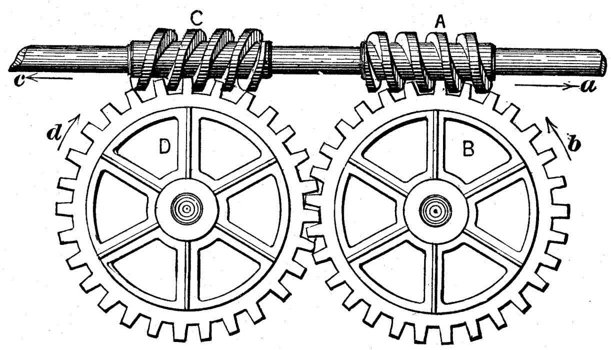

Fig. 87.

worm is at right angles with that of the wheel. The edges of the rollers come near together, leaving sufficient space for the thread of the worm to fit between any two contiguous rollers. The pitch line of the screw thread forms an arc of a circle, whose centre coincides with that of the wheel, therefore the thread will always bear fairly against the rollers and maintain rolling contact therewith during the whole of the time each roller is in gear, and by turning the screw in either direction the wheel will rotate.

To prevent end thrust on a worm shaft it may have a right-hand worm A, and a left-hand one C (Fig. 87), driving two wheels B and D which are in gear, and either of which may transmit the power. The thrust of the two worms A and C, being in opposite directions, one neutralizes the other, and it is obvious that as each revolution of the worm shaft moves both wheels to an amount equal to the pitch of the worms, the two wheels B D may, if desirable, be of different diameters.

Involute teeth.—These are teeth having their whole operative surfaces formed of one continuous involute curve. The diameter of the generating circle being supposed as infinite, then a portion of its circumference may be represented by a straight line, such as A in Fig. 88, and if this straight line be made to roll upon the circumference of a circle, as shown, then the curve traced will be involute P. In practice, a piece of flat spring steel, such as a piece of clock spring, is used for tracing involutes. It may be of any length, but at one end it should be filed so as to leave a scribing point that will come close to the base circle or line, and have a short handle, as shown in Fig. 89, in which S represents the piece of spring, having the point P′, and the handle H. The operation is, to make a template for the base circle, rest this template on drawing paper and mark a circle round its edge to represent on the paper the pitch circle, and to then bend the spring around the circle B, holding the point P′ in contact with the drawing paper, securing the other end of the piece of steel, so that it cannot slip upon B, and allowing the steel to unwind from the cylinder or circle B. The point P′ will mark the involute curve P. Another way to mark an involute is to use a piece of twine in place of the spring and a pencil instead of the tracing point; but this is not so accurate, unless, indeed, a piece

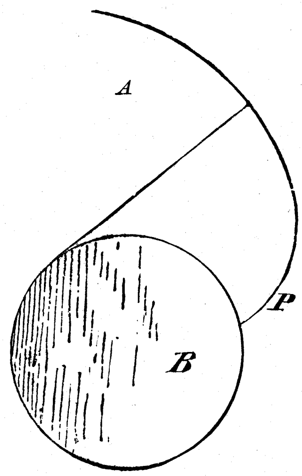

Fig. 88.

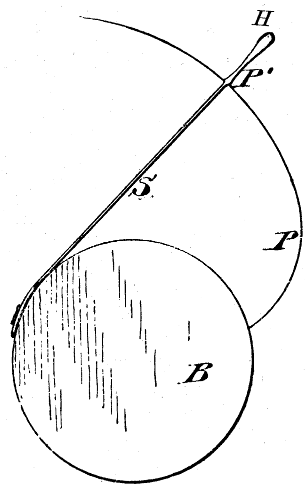

Fig. 89.

of wood be laid on the drawing-board and the pencil held firmly against it, so as to steady the pencil point and prevent the variation in the curve that would arise from variation in the vertical position of the pencil.

The flanks being composed of the same curve as the faces of the teeth, it is obvious that the circle from which the tracing point starts, or around which the straight line rolls, must be of less diameter than the pitch circle, or the teeth would have no flanks.

A circle of less diameter than the pitch circle of the wheel is, therefore, introduced, wherefrom to produce the involute curves forming the full side of the tooth.

The depth below pitch line or the length of flank is, therefore, the distance between the pitch circle and the base circle. Now

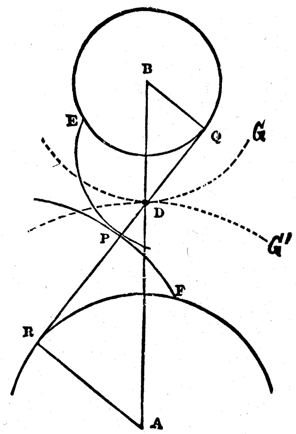

Fig. 90.

even supposing a straight line to be a portion of the circumference of a circle of infinite diameter or radius, the conditions would here appear to be imperfect, because the generating circle is not rolled upon the pitch circle but upon a circle of lesser diameter. But it can be shown that the requirements of a proper velocity ratio will be met, notwithstanding the employment of the base instead of the pitch circle. Thus, in Fig. 90, let A and B represent the respective centres of the two pitch circles, marked in dotted lines. Draw the base circle for B as E Q, which may be of any radius less than that of the pitch circle of B. Draw the straight line Q D R touching this base circle at its perimeter and passing through the point of contact on the pitch circles as at D. Draw the circle whose radius is A R forming the base circle for wheel A. Thus the line R P Q will meet the perimeters of the two circles while passing through the point of contact D at the line of centres (a condition which the relative diameters of the base circles must always be so proportioned as to attain).

If now we take any point on R Q, as P in the figure, as a tracing point, and suppose the radius or distance P Q to represent the steel spring shown in Fig. 89, and move the tracing point back to the base circle of B, it will trace the involute E P. Again we may take the tracing point P (supposing the line P R to represent the steel spring), and trace the involute P F, and these two involutes represent each one side of the teeth on the respective wheels.

The line R P Q is at a right angle to the curves P E and P F, at their point of contact, and, therefore, fills the conditions referred to in Fig. 41. Now the line R P Q denotes the path of contact of tooth upon tooth as the wheels revolve; or, in other words, the point of contact between the side of a tooth on one wheel, and the side of a tooth on the other wheel, will always move along the line Q R, or upon a similar line passing through D, but meeting the base circles upon the opposite sides of the line of centres, and since line Q R always cuts the line of centres at the point of contact of the pitch circles, the conditions necessary to obtain a correct angular velocity are completely fulfilled. The velocity ratio is, therefore, as the length of B Q is to that of A R, or, what is the same thing, as the radius of the base circle of one wheel is to that of the other. It is to be observed that the line Q R will vary in its angle to the line of centres A B, according to the diameter of the base circle from which it is struck, and it becomes a consideration as to what is its most desirable angle to produce the least possible amount of thrust tending to separate the wheels, because this thrust (described in Fig. 39) tends to wear the journals and bearings carrying the wheel shafts, and thus to permit the pitch circles to separate. To avoid, as far as possible, this thrust the propor-

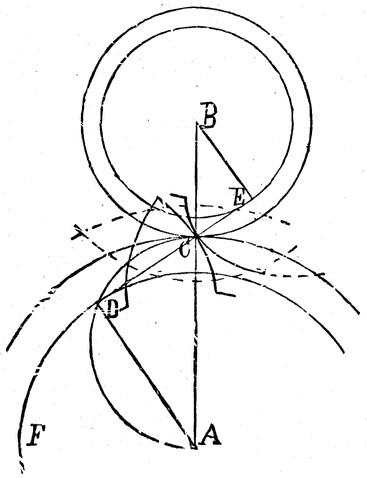

Fig. 91.

tions between the diameters of the base circles D and E, Fig. 91, must be such that the line D E passes through the point of contact on the line of centres, as at C, while the angles of the straight line D E should be as nearly 90° to a radial line, meeting it from the centres of the wheels (as shown in the figure, by the lines B E and D E), as is consistent with the length of D E, which in order to impart continuous motion must at least equal the pitch of the teeth. It is obvious, also, that, to give continuous motion, the length of D E must be more than the pitch in proportion, as the points of the teeth come short of passing through the base circles at D and E, as denoted by the dotted arcs, which should therefore represent the addendum circles. The least possible obliquity, or angle of D E, will be when the construction under any given conditions be made such by trial, that the base circles D and E coincide with the addendum circles on the line of centres, and thus, with a given depth of both beyond, the pitch circle, or addenda as it is termed, will cause the tooth contacts to extend over the greatest attainable length of line between the limits of the addendum circles, thus giving a maximum number of teeth in contact at any instant of time. These conditions are fulfilled in Fig. 92,* the addendum on the small wheel being longer than the depth below pitch line, while the faces of the teeth are the narrowest.

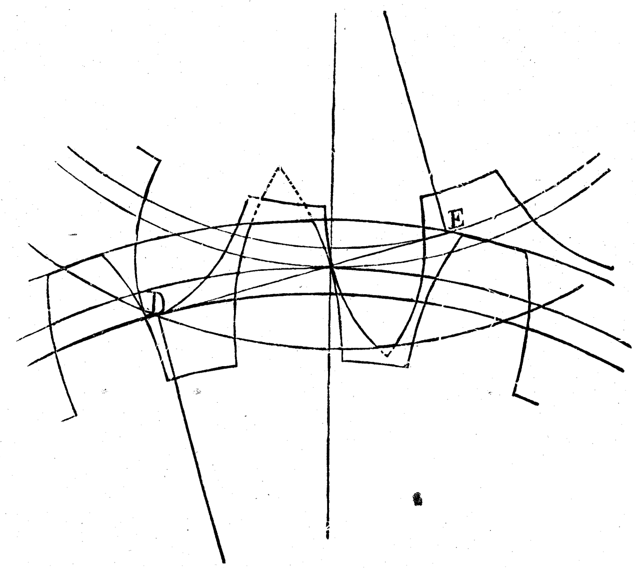

Fig. 92.

In seeking the minimum obliquity or angle of D E in the figure, it is to be observed that the less it is, the nearer the base circle approaches the pitch circle; hence, the shorter the operative length of tooth flank and the greater its wear.

In comparing the merits of involute with those of epicycloidal teeth, the direction of the line of pressure at each point of contact must always be the common perpendicular to the surfaces at the point of contact, and these perpendiculars or normals must pass through the pitch circles on the line of centres, as was shown in Fig. 41, and it follows that a line drawn from C (Fig. 91) to any point of contact, is in the direction of the pressure on the surfaces at that point of contact. In involute teeth, the contact will always be on the line D E (Fig. 92), but in epicycloidal, on the line of the generating circle, when that circle is tangent at the line of centres; hence, the direction of pressure will be a chord of the circle drawn from the pitch circle at the line of centres to the position of contact considered. Comparing involute with radial flanked epicycloidal teeth, let C D A (Fig. 91) represent the rolling circle for the latter, and D C will be the direction of pressure for the contact at D; but for point of contact nearer C, the direction will be much nearer 90°, reaching that angle as the point of contact approaches C. Now, D is the most remote legitimate contact for involute teeth (and considering it so far as epicycloidal struck with a generating circle of infinite diameter), we find that the aggregate directions of the pressures of the teeth upon each other is much nearer perpendicular in epicycloidal, than in involute gearing; hence, the latter exert a greater pressure, tending to force the wheels apart. Hence, the former are, in this respect, preferable.

It is to be observed, however, that in some experiments made by Mr. Hawkins, he states that he found "no tendency to press the wheels apart, which tendency would exist if the angle of the line D E (Fig. 92) deviated more than 20° from the line of centres A B of the two wheels."

A method commonly employed in practice to strike the curves of involute teeth, is as follows:—

In Fig. 93 let C represent the centre of a wheel, D D the full diameter, P P the pitch circle, and E the circle of the roots of the

* From an article by Prof. Robinson.

teeth, while R is a radial line. Divide on R, the distance between the pitch circle and the wheel centre, into four equal parts, by 1, 2, 3, &c. From point or division 2, as a centre, describe the semicircle S, cutting the wheel centre and the pitch circle at its junction with R (as at A). From A, with compasses set to the length of one of the parts, as A 3, describe the arc B, cutting S at F, and F will be the centre from which one side of the tooth may be struck; hence from F as a centre, with the compasses set

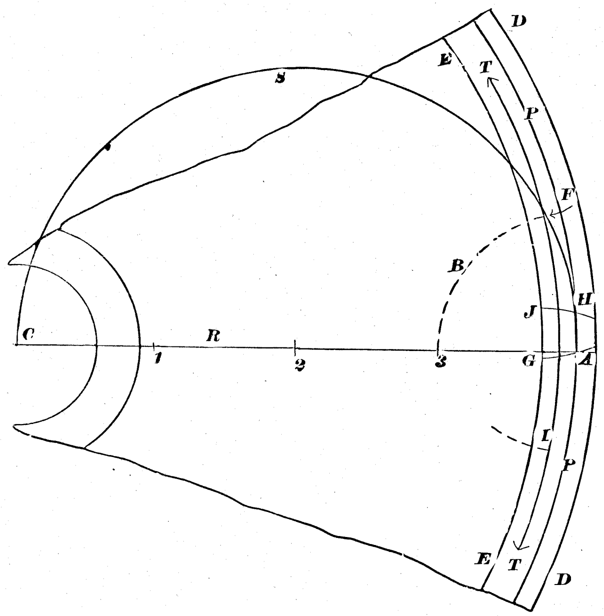

Fig. 93.

to the radius A B, mark the curve G. From the centre C strike, through F, a circle T T, and the centres wherefrom to strike all the teeth curves will fall on T T. Thus, to strike the other curve of the tooth, mark off from A the thickness of the tooth on the pitch circle P P, producing the point H. From H as a centre (with the same radius as before,) mark on T T the point I, and from I, as a centre, mark the curve J, forming the other side of the tooth.

In Fig. 94 the process is shown carried out for several teeth. On the pitch circle P P, divisions 1, 2, 3, 4, &c., for the thick-

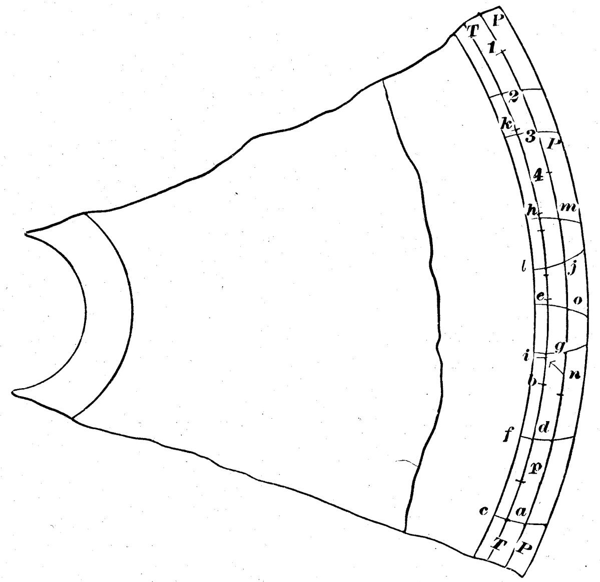

Fig. 94.

ness of teeth and the width of the spaces are marked. The compasses are set to the radius by the construction shown in Fig. 93, then from *a*, the point *b* on T is marked, and from *b* the curve *c* is struck.

In like manner, from *d*, *g*, *j*, the centres *e*, *h*, *k*, wherefrom to strike the respective curves, *f*, *i*, *l*, are obtained.

Then from *m* the point *n*, on T T, is marked, giving the centre wherefrom to strike the curve at *h m*, and from *o* is obtained the point *p*, on T T, serving as a centre for the curve *e o*.

A more simple method of finding point F is to make a sheet metal template, C, as in Fig. 95, its edges being at an angle one to the other of 75° and 30'. One of its edges is marked off in quarters of an inch, as 1, 2, 3, 4, &c. Place one of its edges coincident with the line R, its point touching the pitch circle at the side of a tooth, as at A, and the centre for marking the curve on that side of the tooth will be found on the graduated edge at a distance from A equal to one-fourth the length of R.

The result obtained in this process is precisely the same as that by the construction in Fig. 93, as will be plainly seen, because there are marked on Fig. 93 all the circles by which point F was arrived at in Fig. 95; and line 3, which in Fig. 95 gives the centre wherefrom to strike curve *o*, is coincident with point F, as is shown in Fig. 95. By marking the graduated edge of C in quarter-inch divisions, as 1, 2, 3, &c., then every division will represent the distance from A for the centre for every inch of wheel radius. Suppose, for example, that a wheel has 3 inches radius, then with the scale C set to the radial line R, the centre therefrom to strike the curve *o* will be at 3; were the radius of the wheel 4 inches, then the scale being set the same as before (one edge coincident with R), the centre for the curve *o* would be at 4, and arc T would require to meet the edge of C at 4. Having found

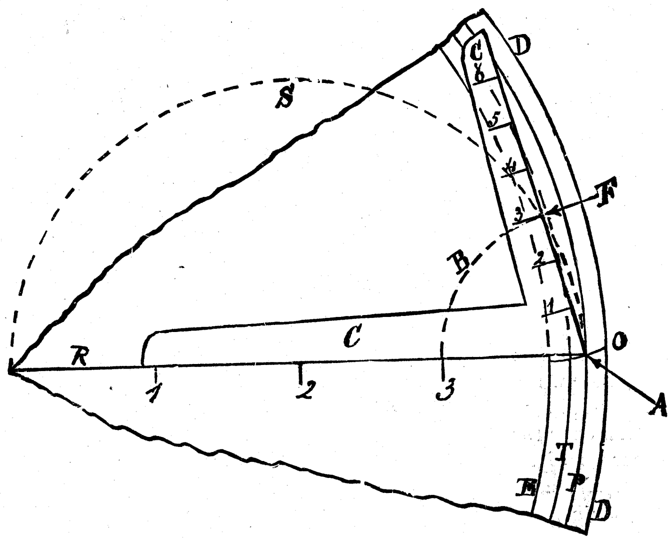

Fig. 95.

the radius from the centre of the wheel of point F for one tooth, we may mark circle T, cutting point F, and mark off all the teeth by setting one point of the compasses (set to radius A F) on one side of the tooth and marking on circle T the centre wherefrom to mark the curve (as *o*), continuing the process all around the wheel and on both sides of the tooth.

This operation of finding the location for the centre wherefrom to strike the tooth curves, must be performed separately for each wheel, because the distance or radius of the tooth curves varies with the radius of each wheel.

In Fig. 96 this template is shown with all the lines necessary to set it, those shown in Fig. 95 to show the identity of its results with those given in Fig. 93 being omitted.

The principles involved in the construction of a rack to work correctly with a wheel or pinion, having involute teeth, are as in Fig. 97, in which the pitch circle is shown by a dotted circle and the base circle by a full line circle. Now the diameter of the base circle has been shown to be arbitrary, but being assumed the radius B Q will be determined (since it extends from the centre B to the point of contact of D Q, with the base circle); B D is a straight line from the centre B of the pinion to the pitch line of the rack, and (whatever the angle of Q D to B D) the sides of the rack teeth must be straight lines inclined to the pitch line of the rack at an angle equal to that of B D Q.

Involute teeth possess four great advantages—1st, they are thickest at the roots, where they should be to have a maximum of strength, which is of great importance in pinions transmitting much power; 2nd, the action of the teeth will remain practically perfect, even though the wheels are spread apart so that the pitch circles do not meet on the line of centres; 3rd, they are much easier to mark, and truth in the marking is easier attained; and 4th, they are much easier to cut, because the full depth of the teeth can, on spur-wheels, in all cases be cut with one revolving cutter, and at one passage of the cutter, if there is sufficient power to drive it, which is not the case with epicycloidal teeth whenever the flank space is wider below than it is at the pitch

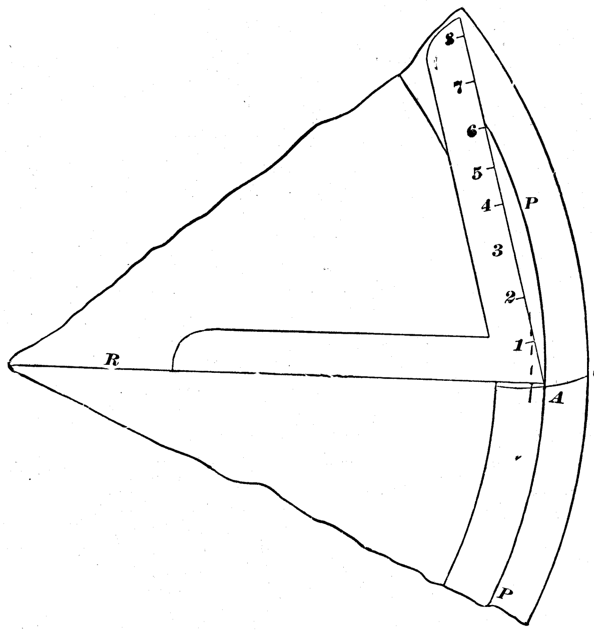

Fig. 96.

circle. On account of the first-named advantage, they are largely employed upon small gears, having their teeth cut true in a gear-cutting machine; while on account of the second advantage, interchangeable wheels, which are merely required to transmit motion, may be put in gear without a fine adjustment of the pitch circle, in which case the wear of the teeth will not prove destructive to the curves of the teeth. Another advantage is, that a greater number of teeth of equal strength may be given to a wheel than in the epicycloidal form, for with the latter the space must at least equal the thickness of the tooth, while in involute the space may be considerably less in width than the tooth, both measured,

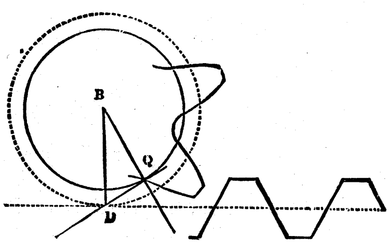

Fig. 97.

of course, at the pitch circle. There are also more teeth in contact at the same time; hence, the strain is distributed over more teeth.

These advantages assume increased value from the following considerations.

In a train of epicycloidal gearing in which the pinion or smallest wheel has radial flanks, the flanks of the teeth will become spread as the diameters of the wheels in the train increase. Coincident with spread at the roots is the thrust shown with reference to Fig. 39, hence under the most favorable conditions the wear on the journals of the wheel axles and the bearings containing them will take place, and the pitch circles will separate. Now so soon as this separation takes place, the motion of the wheels will not be as uniformly equal as when the pitch circles were in contact on the line of centres, because the conditions under which the tooth curves, necessary to produce a uniform velocity of motion, were formed, will have become altered, and the value of those curves to produce constant regularity of motion will have become impaired in proportion as the pitch circles have separated.

In a single pair of epicycloidal wheels in which the flanks of the teeth are radial, the conditions are more favorable, but in this case the pinion teeth will be weaker than if of involute form, while the wear of the journals and bearings (which will take place to some extent) will have the injurious effect already stated, whereas in involute teeth, as has been noted, the separation of the pitch circles does not affect the uniformity of the motion or the correct working of the teeth.

If the teeth of wheels are to be cut to shape in a gear-cutting machine, either the cutters employed determine from their shapes the shapes or curves of the teeth, or else the cutting tool is so guided to the work that the curves are determined by the operations of the machine. In either case nothing is left to the machine operator but to select the proper tools and set them, and the work in proper position in the machine. But when the teeth are to be cast upon the wheel the pattern wherefrom the wheel is to be moulded must have the teeth proportioned and shaped to proper curve and form.

Wheels that require to run without noise or jar, and to have uniformity of motion, must be finished in gear-cutting machines, because it is impracticable to cast true wheels.

When the teeth are to be cast upon the wheels the pattern-maker makes templates of the tooth curves (by some one of the methods to be hereafter described), and carefully cuts the teeth to shape. But the production of these templates is a tedious and costly operation, and one which is very liable to error unless much experience has been had. The Pratt and Whitney Company have, however, produced a machine that will produce templates of far greater accuracy than can be made by hand work. These templates are in metal, and for epicycloidal teeth from 15 to a rack, and having a diametral pitch ranging from $1\frac{1}{2}$ to 32.

The principles of action of the machine are that a segment of a ring (representing a portion of the pitch circle of the wheel for whose teeth a template is to be produced) is fixed to the frame of the machine. Upon this ring rolls a disk representing the rolling, generating, or describing circle, this disk being carried by a frame mounted upon an arm representing the radius of the wheel, and therefore pivoted at a point central to the ring. The describing disk is rolled upon the ring describing the epicycloidal curve, and by suitable mechanical devices this curve is cut upon a piece of steel, thus producing a template by actually rolling the generating upon the base circle, and the rolling motion being produced by positive mechanical motion, there cannot possibly be any slip, hence the curves so produced are true epicycloids.

The general construction of the machine is shown in the side view, Fig. 98 (Plate I.), and top view, Fig. 99 (Plate I.), details of construction being shown in Figs. 100, 101 (Plate I.), 102, 103, 104, 105, and 106. A A is the segment of a ring whose outer edge represents a part of the pitch circle. B is a disk representing the rolling or generating circle carried by the frame C, which is attached to a rod pivoted at D. The axis of pivot D represents the axis of the base circle or pitch circle of the wheel, and D is adjustable along the rod to suit the radius of A A, or what is the same thing, to equal the radius of the wheel for whose teeth a template is to be produced.

When the frame C is moved its centre or axis of motion is therefore at D and its path of motion is around the circumference of A A, upon the edge of which it rolls. To prevent B from slipping instead of rolling upon A A, a flexible steel ribbon is fastened at one end upon A A, passes around the edge of A A and thence around the circumference of B, where its other end is fastened; due allowance for the thickness of this ribbon being made in adjusting the radii of A A and of B.

E′ is a tubular pivot or stud fixed on the centre line of pivots E and D, and distant from the edge of A A to the same amount that E is. These two studs E and E′ carry two worm-wheels F and F′ in Fig. 102, which stand above A and B, so that the axis of the worm G

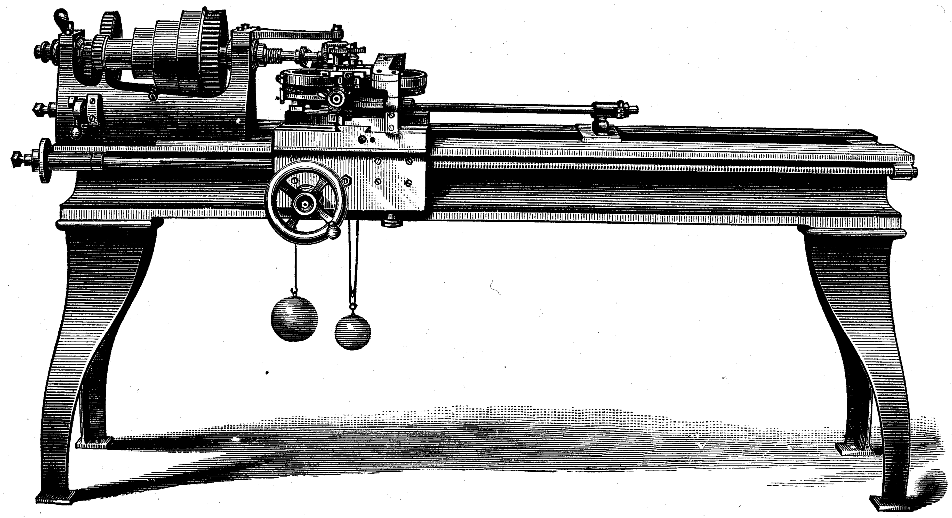

Fig. 98.

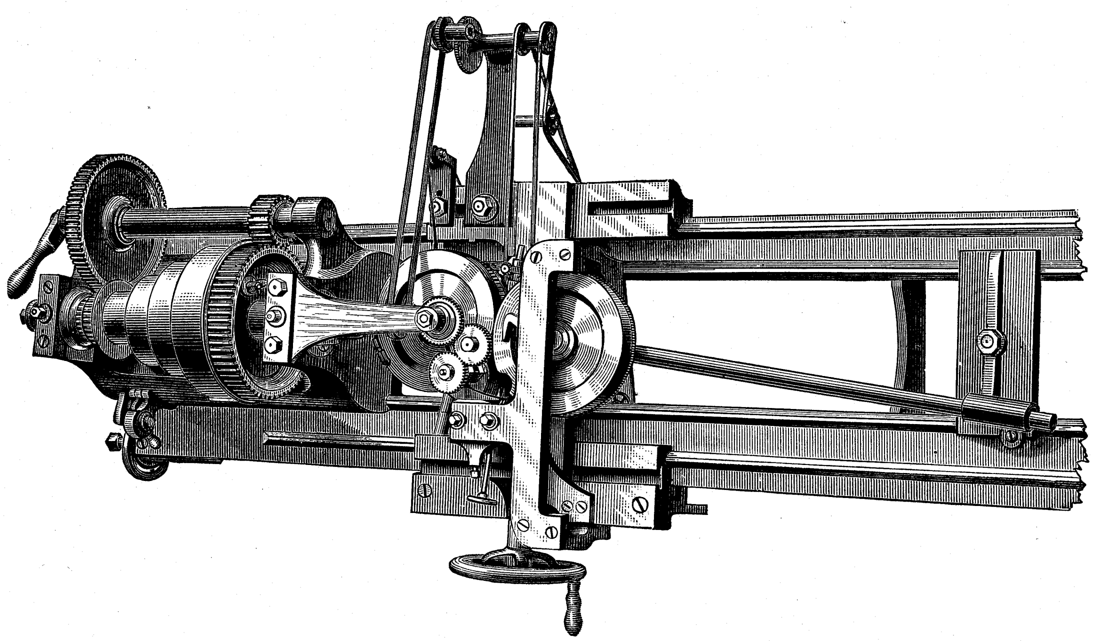

Fig. 99.

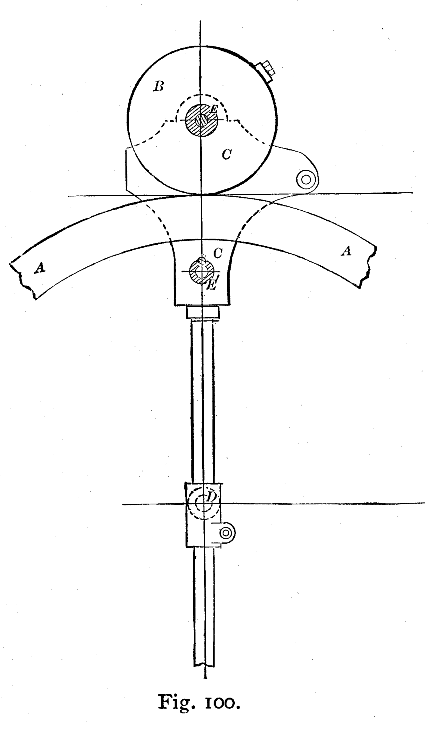

Fig. 100.

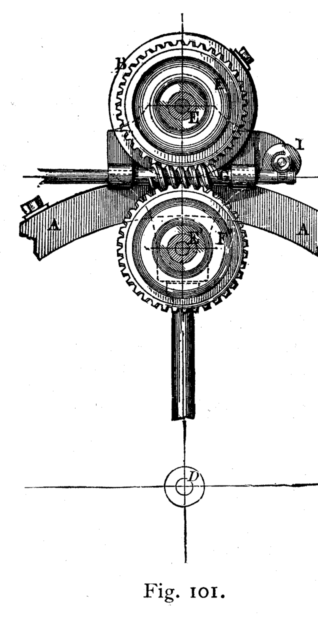

Fig. 101.

is vertically over the common tangent of the pitch and describing circles.

The relative positions of these and other parts will be most clearly seen by a study of the vertical section, Fig. 102.* The worm G is supported in bearings secured to the carrier C and is driven by another small worm turned by the pulley I, as seen in Fig. 101 (Plate I.); the driving cord, passing through suitable

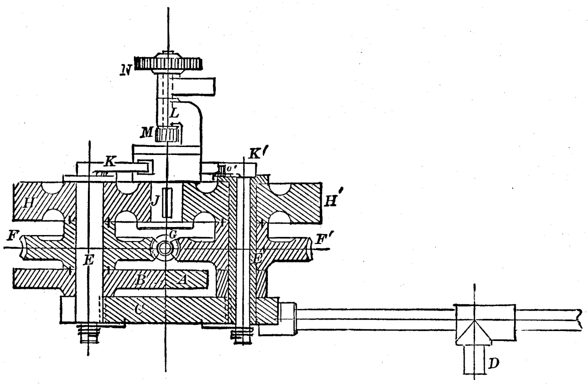

Fig. 102.

guiding pulleys, is kept at uniform tension by a weight, however C moves; this is shown in Figs. 98 and 99 (Plate I.).

Upon the same studs, in a plane still higher than the worm-wheels, turn the two disks H, H′, Figs. 103, 104, 105. The diameters of these are equal, and precisely the same as those of the describing circles which they represent, with due allowance, again, for the thickness of a steel ribbon, by which these also are connected. It will be understood that each of these disks is secured to the worm-wheel below it, and the outer one of these, to the disk B, so that as the worm G turns, H and H′ are rotated in opposite directions, the motion of H being identical with that of B; this last is a rolling one upon the edge of A, the carrier C with all its attached mechanism moving around D at the same time.

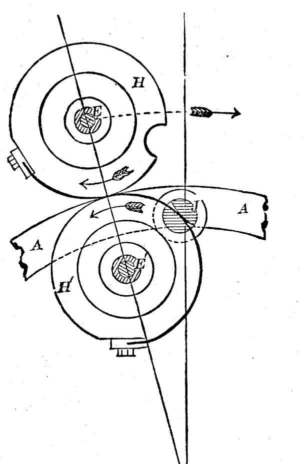

Fig. 103.

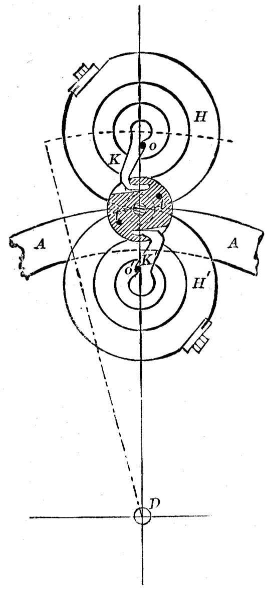

Fig. 104.

Ultimately, then, the motions of H, H′, are those of two equal describing circles rolling in external and internal contact with a fixed pitch circle.

In the edge of each disk a semicircular recess is formed, into which is accurately fitted a cylinder J, provided with flanges, between which the disks fit so as to prevent end play. This cylinder is perforated for the passage of the steel ribbon, the sides of the opening, as shown in Fig. 103, having the same curvature as the rims of the disks. Thus when these recesses are opposite each other, as in Fig. 104, the cylinder J fills them both, and the tendency of the steel ribbon is to carry it along with H when C moves to one side of this position, as in Fig. 105, and along with H′ when C moves to the other side, as in Fig. 103.

This action is made positively certain by means of the hooks K, K′, which catch into recesses formed in the upper flange of J, as seen in Fig. 104. The spindles, with which these hooks turn, extend through the hollow studs, and the coiled springs attached to their lower ends, as seen in Fig. 102, urge the hooks in the directions of their points; their motions being limited by stops *o, o′*, fixed, not in the disks H, H′, but in projecting collars on the upper ends of the tubular studs. The action will be readily traced by comparing Fig. 104 with Fig. 105; as C goes to the left, the hook K′ is left behind, but the other one, K, cannot escape from its engagement with the flange of J; which, accordingly, is carried along with H by the combined action of the hook and the steel ribbon.

On the top of the upper flange of J, is secured a bracket, carrying the bearing of a vertical spindle L, whose centre line is a prolongation of that of J itself. This spindle is driven by the spur-wheel N, keyed on its upper end, through a flexible train of gearing seen in Fig. 99; at its lower end it carries a small milling cutter M, which shapes the edge of the template T, Fig. 105, firmly clamped to the framing.

When the machine is in operation, a heavy weight, seen in Fig. 98 (Plate I.), acts to move C about the pivot D, being attached to the

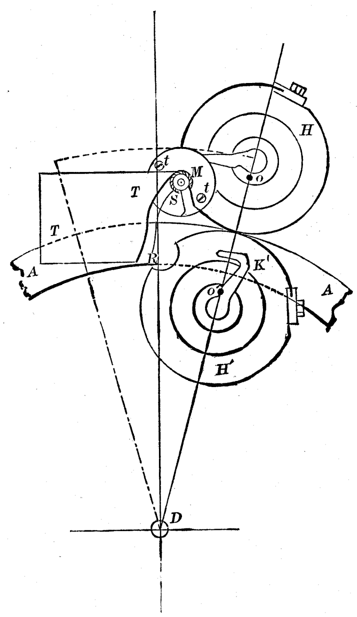

Fig. 105.

carrier by a cord guided by suitably arranged pulleys; this keeps the cutter M up to its work, while the spindle L is independently driven, and the duty left for the worm G to perform is merely that of controlling the motions of the cutter by the means above described, and regulating their speed.

The centre line of the cutter is thus automatically compelled to travel in the path R S, Fig. 105, composed of an epicycloid and a hypocycloid if A A be the segment of a circle as here shown; or of two cycloids, if A A be a straight bar. The radius of the cutter being constant, the edge of the template T is cut to an outline also composed of two curves; since the radius M is small, this outline closely resembles R S, but particular attention is called to the fact that it is *not identical with it, nor yet composed of truly epicycloidal curves of any generation whatever:* the result of which will be subsequently explained.

Number and Sizes of Templates.

With a given pitch every additional tooth increases the diameter of the wheel, and changes the form of the epicycloid; so that it would appear necessary to have as many different cutters, as there are wheels to be made, of any one pitch.

* From "The Teeth of Spur Wheels," by Professor McCord.

But the proportional increment, and the actual change of form, due to the addition of one tooth, becomes less as the wheel becomes larger; and the alteration in the outline soon becomes imperceptible. Going still farther, we can presently add more teeth without producing a sensible variation in the contour. That is to say, several wheels can be cut with the same cutter, without introducing a perceptible error. It is obvious that this variation in the form is least near the pitch circle, which is the only part of the epicycloid made use of; and Prof. Willis many years ago deduced theoretically, what has since been abundantly

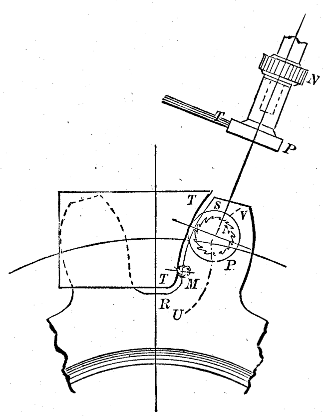

Fig. 106.

proved by practice, that instead of an infinite number of cutters, 24 are sufficient of one pitch, for making all wheels, from one with 12 teeth up to a rack.

Accordingly, in using the epicycloidal milling engine, for forming the template, segments of pitch circles are provided of the following diameters (in inches):

12,	16,	20,	27,	43,	100,
13,	17,	21.	30,	50,	150,
14,	18,	23.	34,	60,	300.
15,	19,	25,	38,	75,	

In Fig. 106, the edge T T is shaped by the cutter M, whose centre travels in the path R S, therefore these two lines are at a constant normal distance from each other. Let a roller P, of any reasonable diameter, be run along T T, its centre will trace the line U V, which is at a constant normal distance from T T, and therefore from R S. Let the normal distance between U V and R S be the radius of another milling cutter N, having the same axis as the roller P, and carried by it, but in a different plane as shown in the side view; then whatever N cuts will have R S for its contour, if it lie upon the same side of the cutter as the template.

The diameter of the disks which act as describing circles is $7\frac{1}{2}$ inches, and that of the milling cutter which shapes the edge of the template is $\frac{1}{8}$ of an inch.

Now if we make a set of 1-pitch wheels with the diameters above given, the smallest will have twelve teeth, and the one with fifteen teeth will have radial flanks. The curves will be the same whatever the pitch; but as shown in Fig. 106, the blank should be adjusted in the epicycloidal engine, so that its lower edge shall be $\frac{1}{16}$th of an inch (the radius of the cutter M) above the bottom of the space; also its relation to the side of the proposed tooth should be as here shown. As previously explained, the depth of the space depends upon the pitch. In the system adopted by the Pratt & Whitney Company, the whole height of the tooth is $2\frac{1}{8}$ times the diametral pitch, the projection outside the pitch circle being just equal to the pitch, so that diameter of blank = diameter of pitch circle + 2 × diametral pitch.

We have now to show how, from a single set of what may be called 1-pitch templates, complete sets of cutters of the true epicycloidal contour may be made of the same or any less pitch.

Now if T T be a 1-pitch template as above mentioned, it is clear that N will correctly shape a cutting edge of a gear cutter for a 1-pitch wheel. The same figure, reduced to half size, would correctly represent the formation of a cutter for a 2-pitch wheel of the same number of teeth; if to quarter size, that of a cutter for a 4-pitch wheel, and so on.

But since the actual size and curvature of the contour thus determined depend upon the dimensions and motion of the cutter N, it will be seen that the same result will practically be accomplished, if these only be reduced; the size of the template, the diameter and the path of the roller remaining unchanged.

The nature of the mechanism by which this is effected in the Pratt & Whitney system of producing epicycloidal cutters will be hereafter explained in connection with cutters.