SUMMARY OF ARITHMETIC.

The following abridgment of several of the rules of arithmetic, often referred to in elementary books on mechanical science, are here inserted for the convenience of reference. These rules and examples are given merely to refresh the memory, it being taken for granted that the reader has already acquainted himself with the principles of common arithmetic. They will, however, be found serviceable, both as a convenience of reference and to give some insight to the subjects on which they treat.

Arithmetic is the science of numbers, and numbers treat of magnitude or quantity. Whatever is capable of increase or diminution is a magnitude or quantity.

The processes of arithmetic are merely expedients for making easier the discovery of results which every man of ordinary ingenuity would find a means for discovering himself. Roger Bacon lived eight centuries ago ; in the great roll of modern scientists, his name stands first ; these are his

---

NOTE.—Calculation is the art, practice or manner of computing by numbers : the use of numbers by addition, subtraction, multiplication or division, for the purpose of arriving at a certain result.

Upon this art—of calculation—rest not only the mechanical arts, but the whole structure of modern civilization. Consider the solar system, a time-piece, a well-equipped modern factory—the characteristic of each is its "calculability." Everything comes at last to correct figuring for assured success.

words: "For he who knows not mathematics cannot know any other sciences; and, what is more, he cannot discover his own ignorance or find its proper remedies."

In every branch of science, our knowledge increases as the power of measurement becomes improved; it is very generally true that the one ignorant of useful numbers is the one who serves, while the leader in all departments is the one who calculates.

A glossary is a collection of words not in general use, especially of an art or science; the ordinary use of a glossary is to explain in some detail many of the more difficult words used in the text, hence the following—

SYMBOLS, ABBREVIATIONS AND DEFINITIONS.

= *Equal to.* The sign of equality; as 100 cts. = $1, signifies that one hundred cents are equal to one dollar.

— *Minus* or *Less.* The sign of subtraction; as 8 — 2 = 6; that is, 8 less 2, is equal to 6.

+ *Plus* or *More.* The sign of addition; as 6 + 8 = 14; that is, 6 added to 8 is equal to 14.

× *Multiplied by.* The sign of multiplication; as 7 × 7 = 49; that is, 7 multiplied by 7 is equal to 49.

÷ *Divided by.* The sign of division; as 16 ÷ 4 = 4; that is, 16 divided by 4 is equal to 4.

SYMBOLS, ABBREVIATIONS AND DEFINITIONS.

$\therefore$ Signifies then or therefore.

$\because$ Since or because.

d^2 = diameter squared, or is a number multiplied by itself, thus $2 \times 2 = 4$.

d^3 = diameter cubed, or is a number multiplied by itself twice, thus $2 \times 2 \times 2 = 8$.

d^4 = diameter to the fourth power, or is a number multiplied by itself thrice, thus $2 \times 2 \times 2 \times 2 = 16$.

A single accent (′) signifies feet; *a double accent* (″) inches; thus 3′ 6″ = 3 feet 6 inches.

Dia. = diameter. ° Degrees.

Revs. per min. = revolutions per minute.

Lbs. per sq. in. = pounds per square inch.

Brackets ( ) or [ ] are employed to denote that several numbers are to be taken collectively. Thus $4(a + b)$ signifies that the number represented by $a + b$ is to be multiplied by 4; again $(a + b) \times (c - d)$ denotes that the number represented by $a + b$ is to be multiplied by the number which is the result of subtracting d from c.

The Greek Letter π denotes the ratio of the circumference of a circle to its diameter. In the English alphabet, this letter stands in place of *p*, and is called *pi;* it is very frequently met with in mechanical literature.

The Decimal Point.—In both France and Germany, one-fourth (¼) reduced to a decimal is always written as 0,25; in England it is written 0·25, and in the United States in this way, 0.25.

SYMBOLS, ABBREVIATIONS AND DEFINITIONS.

A formula is an arithmetical rule in which all words are omitted, all the quantities represented by letters and figures, and all the operations indicated by signs, and by the position of the different characters; the word "formula" is another name for "form."

The following 10 formulas include the elementary operations of arithmetic and follow from the succeeding illustrations.

1. *The Sum = all the parts added.*
2. *The Difference = the Minuend — the Subtrahend.*
3. *The Minuend = the Subtrahend + the Difference.*
4. *The Subtrahend = the Minuend — the Difference.*
5. *The Product = the Multiplicand × the Multiplier.*
6. *The Multiplicand = the Product ÷ the Multiplier.*
7. *The Multiplier = the Product ÷ the Multiplicand.*
8. *The Quotient = the Dividend ÷ the Divisor.*
9. *The Dividend = the Quotient × the Divisor.*
10. *The Divisor = the Dividend ÷ the Quotient.*

A number is exactly divisible by—2, when the number ends in an even number or in 0; *3*, when the sum of the digits is exactly divisible by 3; *4*, when the number formed by the last two digits is exactly divisible by 4; *5*, when the number ends in 5 or 0.

Ratio is the relation of one number to another, as obtained by dividing one by the other; hence, ratio means the same as the word quotient.

SYMBOLS, ABBREVIATIONS AND DEFINITIONS.

Log. This is the abbreviation of the term *logarithm;* these are auxiliary numbers, by means of which the simple operations of addition and subtraction may be substituted for the more cumbrous operations of multiplication and division, and easy cases of multiplication and division for involution and evolution.

The use of logarithms reduces multiplication to addition, division to subtraction; raising powers or extracting roots to multiplication and division, respectively.

Logarithms of numbers are arranged in tables, running to four and six figures, beginning with one and going to so high as to fill entire books with the columns.

Algebra is that science which deals with formulas; it is a mathematical science which teaches the art of making calculations by letters and signs instead of figures. The name comes from two Arabic words, *al gabron*, reduction of parts to a whole. The letters and signs are called *Symbols. Quantities* in Algebra are expressed by *letters*, or by a combination of *letters* and *figures;* as a, b, c, $2x$, $3y$, $5z$, etc. The first letters of the alphabet are used to express *known* quantities; the last letters, those which are *unknown.*

The operations to be performed are expressed by the same signs as in Arithmetic; thus + means Addition, — expresses Subtraction, and × stands for Multiplication.

NOTE.—A machinist has little or no use for algebra in his every-day work; but if he wants to find out more about the how and why of things and study into general principles, it is the most important subject that he can take up, next to arithmetic and mechanical drawing.

SYMBOLS, ABBREVIATIONS AND DEFINITIONS.

A NUMBER is a unit or collection of units; as two, five, six feet, etc.

An INTEGER is a number that represents whole things.

An ABSTRACT NUMBER is one which does not refer to any particular object.

A CONCRETE NUMBER is a number used to designate objects or quantities.

An ODD NUMBER is a number which cannot be divided by two.

An EVEN NUMBER can be exactly divided by two.

FACTORS of a number are those numbers which, when multiplied together, make that number.

A PRIME NUMBER is a number exactly divisible by one.

A COMPOSITE NUMBER is a number which can be divided by other integers besides itself and one.

An EXACT DIVISOR of a number is a whole number that will divide that number without a remainder.

The GREATEST COMMON DIVISOR of two or more numbers is the greatest number that will divide each of them exactly.

A MULTIPLE of a number is any number exactly divisible by that number.

The LEAST COMMON MULTIPLE of two or more numbers is the least number that is exactly divisible by each of them.

A PRIME FACTOR is any prime number used as a factor.

NOTE.—*Quantity* is the amount of anything considered, or of any commodity bought, or sold. *Price* is the value in money of one, or of a given unit of any commodity. *Cost* is the value in money of the entire quantity bought, or sold.

NOTATION AND NUMERATION.

NOTATION is a system of representing numbers by symbols. There are two methods of notation in use, the *Roman* and the *Arabic.* NUMERATION is a system of naming or reading numbers.

THE ARABIC METHOD OF NOTATION employs ten characters or figures, viz:

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*0*
One,	*Two,*	*Three,*	*Four,*	*Five,*	*Six,*	*Seven,*	*Eight,*	*Nine,*	*Zero.*

The nine figures are called *digits* or significant figures. The character 0 has no value when standing alone.

The nine digits have each a *simple* and a *local value.* The simple value of a figure is the one expressed by it when standing alone or in the units place. The local value of a figure is that which depends upon the place which the figure occupies in a number.

There must be three figures in every period, except the one at the left, which may have one, two or three. Every order of a number not occupied by a significant figure must be filled with a cipher, or 0.

NOTE.—By means of these ten figures or characters we can represent any number. When one of the figures stands by itself, it is called a unit; but if two of them stand together, the right-hand figure is still called a unit, but the left-hand figure is called tens; thus, 79 is a collection of 9 units and 7 sets of ten units each, or of 9 units and 70 units, or of 79 units, and is read as seventy-nine. If three of them stand together, then the left-hand figure is called hundreds; thus, 279 is read two hundred and seventy-nine.

NOTATION AND NUMERATION.

RULE FOR NOTATION.—*Beginning at the left, write the hundreds, tens and units of each successive period in their proper order, filling all vacant orders and periods with ciphers.*

NUMERATION TABLE.

Names of periods:	Billions.			Millions.			Thousands.			Units.						
Order of Units:	Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths
	8	7	6,	5	4	3,	2	0	1,	2	8	2	.	4	8	9

The number in the table is read "eight hundred and seventy-six billion, five hundred and forty-three million, two hundred and one thousand, two hundred and eighty-two, decimal point, four, eight, nine."

In the table given, it will be observed that the long row of figures is divided into groups of three figures, called periods. This is to aid in their ready reading. The first set is called *units,* the second *thousands,* the third *millions,* etc.

Beginning at units place, the orders on the right of the decimal point express tenths, hundredths, thousandths, etc.

THE READING OF DECIMALS.—In reading decimals, it is well to omit, even in thought, the idea of a denominator, and to say, thus—example, .25 ; to read, say "point, 2, 5"; in reading .48437, say "point, 4, 8, 4, 3, 7."

EXAMPLE.—Write sixty-four thousandths in decimals.

Since there are only two figures in the numerator 64, and the right-hand figure of the decimal must occupy the

NOTATION AND NUMERATION.

third decimal place to express thousandths, it is necessary to prefix a cipher to bring the right-hand figure into its proper place. Therefore write *point, oh, six, four* (.064), in the order named.

It is well also to say "oh" (this is the letter O).

THE ROMAN NOTATION is the method of notation by letters, and is illustrated as follows:

I,	V,	X,	L,	C,	D,	M,
1,	5,	10,	50	100,	500,	1,000.

Repeating a letter repeats its value; thus: I = 1, II = 2.

Placing a letter of less value before one of greater value diminishes the value of the greater by the less; thus, IV = 4, IX = 9, XL = 40.

Placing the less after the greater increases the value of the greater by that of the less; thus, VI = 6, XI = 11, LX = 60.

Placing a horizontal line over a letter increases its value a thousand times; thus, IV = 4,000 $\overline{\text{M}}$ = 1,000,000.

ROMAN TABLE.

I	denotes	One.
II	"	Two.
III	"	Three.
IV	"	Four.
V	"	Five.
VI	"	Six.
VII	"	Seven.
VIII	"	Eight.
IX	"	Nine.
X	"	Ten.
XI	"	Eleven.
XII	"	Twelve.
XIII	"	Thirteen.
XIV	"	Fourteen.
XV	"	Fifteen.
XVI	"	Sixteen.
XVII	denotes	Seventeen.
XVIII	"	Eighteen.
XIX	"	Nineteen.
XX	"	Twenty.
XXX	"	Thirty.
XL	"	Forty.
L	"	Fifty.
LX	"	Sixty.
LXX	"	Seventy.
LXXX	"	Eighty.
XC	"	Ninety.
C	"	One hundred.
D	"	Five hundred.
M	"	One thousand.
$\overline{\text{X}}$	"	Ten thousand.
$\overline{\text{M}}$	"	One million.

ADDITION.

Addition is uniting two or more numbers into one. The result of the addition is called the Sum or Amount. In addition, the only thing to be careful about except the correct doing of the sum, is to place the unit figures under the unit figure above it, the tens under the tens, etc.

RULE.

After writing the figures down so that units are under units, tens under tens, etc.:

1. *Begin at the right hand, up and down row, add the column and write the sum underneath if less than ten.*

2. *If, however, the sum is ten or more, write the right-hand figure underneath, and add the number expressed by the other figure or figures with the numbers of the next column.*

3. *Write the whole of the last column.*

EXAMPLES FOR PRACTICE.

7,060	248,124	13,579,802
9,420	4,321	83
1,743	889,876	478,652
4,004	457,902	87,547,289
22,227 Ans.		

Use care in placing the numbers in vertical lines; irregularity in writing them down is the cause of mistakes.

RULE FOR PROVING THE CORRECTNESS OF THE SUMS.—*Add the columns from the top downward, and if the sum is the same as when added up, the answer is right.*

Add and prove the following numbers:

684 32 257 20. Ans. 993.

SUBTRACTION.

Subtraction is taking a lesser sum from a greater one.

As in addition, care must be used in placing the units under the units, the tens under the tens, etc.

The answer is called the remainder or the difference.

The sign of subtraction is (—) Example: 98—22=76.

Subtraction is the opposite of addition: one "takes from," while the other "adds to."

RULE.

1. *Write down the greater number first, and then under it the lesser number, so that the units stand under the units, the tens under the tens, etc., etc.*

2. *Begin with the units, and take the under from the upper figure, and put the remainder beneath the line.*

3. *But if the lower figure is the larger, add ten to the upper figure, and then subtract and put the remainder down: this borrowed ten must be deducted from the next column of figures where it is represented by* 1.

EXAMPLES FOR PRACTICE.

892	89,672	89,642,706
46	46,379	48,765,421
846 remainder.		

NOTE.—In the first example, 892—46, the 6 is larger than 2; borrow 10, which makes it 12, and then deduct the 6; the answer is 6. The borrowed 10 reduces the 9 to 8, so the next deduction is 4 from 8=4 is the answer.

SUBTRACTION.

RULE FOR PROVING THE CORRECTNESS OF SUBTRACTION.—*Add the remainder, or difference, to the smaller amount of the two sums, and if the two are equal to the larger, then the subtraction has been correctly done.*

EXAMPLE.	898		246
	246	Now then,	652
	652		898 Ans.

MULTIPLICATION.

MULTIPLICATION is finding the amount of one number increased as many times as there are units in another.

The number to be multiplied or increased is called the MULTIPLICAND.

The MULTIPLIER is the number by which we multiply. It shows how many times the multiplicand is to be increased.

The answer is called the PRODUCT.

The multiplier and multiplicand which produce the product are called its FACTORS. This is a word frequently used in mathematical works, and its meaning should be remembered.

The sign of multiplication is $\times$ and is read "times," or multiplied by; thus, 6×8 is read, 6 times 8 is 48, or, 6 multiplied by 8 is 48.

The principle of multiplication is the same as addition; thus, $3\times8=24$ is the same as $8+8+8=24$.

MULTIPLICATION.

RULE FOR MUTIPLYING.

1. *Place the unit figure of the multiplier under the unit figure of the multiplicand, and proceed as in the following:*

EXAMPLES. Multiply 846 by 8, and 487,692 by 143. Arrange them thus:

```
                        487,692
                            143
   846                  -------
     8                  1463076
  ----                 1950768
 6,768                 487692
                     ----------
                     69,739,956
```

2. *But if the multiplier has ciphers at its end, then place it as in the following:*

Multiply 83,567 by 50, and 898 by 2,800.

```
                           898
  83567                     2800
      50                  ------
  --------                718400
  4,178,350              1796
                         --------
                         2,514,400
```

The product and the multiplicand must be in like numbers. Thus, 10 times 8 gallons of *oil* must be 80 gallons of *oil*; 4 times 5 *dollars* must be 20 *dollars*; hence, the multiplier must be the *number* and not the *thing* to be multiplied.

In finding the cost of 6 tons of coal at 7 dollars per ton, the 7 *dollars* are taken 6 times, and not multiplied by 6 tons.

MULTIPLICATION.

When the multiplier is 10, 100, 1000, etc., the product may be obtained at once by annexing to the multiplicand as many ciphers as there are in the multiplier.

EXAMPLE.

1. Multiply 486 by 100. Now 486 with 00 added = 48,600.

2. 6,842 × 10,000 = how many? Ans. 68,420,000.

TO PROVE THE RESULT IN MULTIPLICATION.

RULE.—*Multiply the multiplier by the multiplicand, and if the product is the same in both cases, then the answer is right.*

DIVISION.

Division is a word derived from the Latin, *divido* meaning to separate into parts. In arithmetic, it may be defined as the dividing of a number or quantity into any number of parts assigned.

When one number has to be divided by another number, the first one is called the DIVIDEND, and the second one the DIVISOR, and the result is the QUOTIENT.

1. TO DIVIDE BY ANY NUMBER UP TO 12.

RULE.—*Put the dividend down with the divisor to the left of it, with a small curved line separating it, as in the following*

EXAMPLE.—Divide 7,865,432 by 6.

```
6)7,865,432
  ---------
  1,310,905—2
```

DIVISION.

Here at the last we have to say, "6 into 32 goes 5 times and 2 over"; always place the number that is over as above, separated from the quotient by a small line, or else put it as a fraction, thus, $\frac{2}{6}$, the top figure being the remainder, and the bottom figure the divisor, when it should be put close to the quotient; thus, 1,310,905$\frac{2}{6}$.

2. TO DIVIDE BY ANY NUMBER UP TO 12, WITH A CIPHER OR CIPHERS AFTER IT, as 20, 70, 90, 500, 7,000, etc.

RULE.—*Place the sum down as in the last example, then mark off from the right of the dividend as many figures as there are ciphers in the divisor; also mark off the ciphers in the divisor; then divide the remaining figures by the number remaining in the divisor; thus:—*

EXAMPLE.—Divide 9,876,804 by 40.

```
40)9,876,804
   ---------
    246,920—4.
```

The 4 cut off from the dividend is put down as a remainder, or it might have been put down as $\frac{4}{40}$ or $\frac{1}{10}$.

3. TO DIVIDE BY ANY NUMBER NOT INCLUDED IN THE LAST TWO CASES.

RULE.—*Write the divisor at the left of the dividend and proceed as in the following*

EXAMPLE.

Divide 726,981 by 7,645.

```
7,645)726981(95
      68805
      -----
       38931
       38225
       -----
         706   Ans. 95 706/7645.
```

DIVISION.

EXAMPLES FOR PRACTICE.

1.—Divide 76,298,764,833 by 9.
2.— " 120,047,629,817 " 20.
3.— " 9,876,548,210 " 48.
4.— " 3,247,617,219 " 63.

Multiplying the dividend, or dividing the divisor by any number, multiplies the quotient by the same number.

Dividing the dividend, or multiplying the divisor by any number, divides the quotient by the same number.

Dividing or multiplying both the dividend and divisor by the same number does not change the quotient.

RULE FOR PROVING DIVISION.

Division may be proved by multiplying the quotient by the integral part of the Divisor, and adding to the product the remainder, if there is any. The result will be equal to the dividend if the work is correct.

EXAMPLE. 12)48679

4056—7
12

48679 Proof.

QUOTATION.—"As long ago as the days of ancient Greece, Aristotle said: 'I find the young men who study mathematics quick and intelligent at other studies.' But, apart from the value of mathematical studies as a mental training, the modern engineer, whatever branch of the science he may pursue, will find mathematics one of the necessary tools of his profession."

REDUCTION.

A DENOMINATE NUMBER is a number applied to an object; thus, 40 inches and 3 feet 5 inches are denominate numbers; the first is a *simple* and the latter a *compound denominate number*.

REDUCTION is changing these numbers from one denomination to another without altering their values. It is of two kinds, DESCENDING and ASCENDING.

Reduction Descending is changing higher denominations to lower, as tons to pounds. Reduction Ascending is changing lower to higher denominations, as cents to dollars.

Reduction of Denominate Numbers is the process of changing the denomination of a number without changing the value. Thus, 3 yards may be expressed as 9 feet, or 108 inches.

TO CHANGE DENOMINATE NUMBERS TO LOWER DENOMINATIONS is done by multiplication and by the following

RULE.—1. *Multiply the number of the highest denomination given by the number of units of the next lower denomination required to make one of that higher, and to the product add the given number of the lower denomination, if any.*

2. *Proceed in like manner with this result and each successive denomination obtained, until the given number is reduced to units of the required denomination.*

NOTE.—*A simple number* is one which expresses one or more units of the same denomination. *A compound number* expresses units of two or more denominations of the same kind, as 5 yards, 1 foot, 4 inches—or example, page 36, 6 T., 8 cwt., 3 qrs.—these are compound numbers; but *ten oxen*, or *five dollars*, are simple numbers.

REDUCTION.

EXAMPLE.

Reduce six tons, eight hundred weight, three quarters, to lbs.

```
 6  T. 8 cwt. 3 qrs.
 20
----
120
  8 add above.
----
128
  4
----
512
  3 add above.
----
515 qrs.
 25
----
2575
1030
-----
12875 lbs.   Answer.
```

TO REDUCE LOWER DEMOMINATIONS TO HIGHER IS DONE BY DIVISION.

RULE.—1. *Divide the given number by the number of units of the given denomination required to make a unit of the next higher denomination.*

2. *In the same manner, divide this and each successive quotient until the required denomination is reached. The last quotient, with the remainders annexed, will be the required result.*

Ex.—Bring 98,704,623 lbs. to tons and lbs.

```
2000)98704623
     --------
      49352 Tons, 623 lbs.
```

REDUCTION.

Ex.—76,245 gills to gallons, etc.

```
4)76245
 ------
2)19061—1 gill
 ------
 4)9530—1 pint.
   ----
   2382—2 quarts.
```

Ans., 2382 gallons, 2 quarts, 1 pint and 1 gill.

PROOF.—Reduction Ascending and Descending *prove* each other; for one is the reverse of the other.

FRACTIONS.

A fraction means a part of anything. A vulgar fraction is always represented by two numbers (at least), one over the other and separated by a small horizontal line. The one above the line is always called the NUMERATOR, and the one below the line the DENOMINATOR.

The denominator tells us how many parts the whole thing has been divided into, and the numerator tells us how many of those parts we have. Thus, in the fraction $\frac{3}{8}$ the eight is the denominator, and shows that the object has been divided into eight equal parts; and three is the numerator, and shows that we have three of those pieces or parts of the object.

A PROPER FRACTION is one whose numerator is less than the denominator, as $\frac{3}{8}$ or $\frac{2}{5}$.

AN IMPROPER FRACTION is one whose numerator is more than its denominator, $\frac{8}{3}$ or $\frac{5}{2}$.

NOTE.—$\frac{8}{3}$ means more than a whole one, because $\frac{3}{3}$ must be a whole one. Thus $\frac{8}{3}$ will be three-thirds+three-thirds+two-thirds, or $2\frac{2}{3}$, and this form of fraction is called *a mixed number*.

REDUCTION OF FRACTIONS.

TO REDUCE AN IMPROPER FRACTION TO A MIXED NUMBER.

RULE.—*Divide the numerator by the denominator; the quotient is the whole number part, and the remainder is the numerator of the fractional part.*

EXAMPLES: $\frac{16}{7}=2\frac{2}{7}$. $\frac{15}{3}=5$. $\frac{27}{8}=3\frac{3}{8}$.

TO REDUCE A MIXED NUMBER TO AN IMPROPER FRACTION.

RULE.—*Multiply the whole number part by the denominator, and add on the numerator; the result is the numerator of the improper fraction.*

EXAMPLES: $2\frac{2}{7}=\frac{16}{7}$. $5\frac{1}{3}=\frac{16}{3}$. $3\frac{3}{8}=\frac{27}{8}$.

TO REDUCE A FRACTION TO ITS LOWEST TERMS.

RULE.—*Divide both numerator and denominator by the same number; if by so doing there is no remainder.*

EXAMPLE.—Reduce $\frac{8}{12}$. Here 4 will divide both top and bottom without a remainder. Divide by 4.

$$4)\frac{8}{12}=\frac{2}{3}.$$

The meaning of this is, that if you divide a thing into 12 equal parts, and take 8 of them, you will have the same as if the thing had been divided into 3 equal parts and you had two of them.

TO REVERSE THE LAST RULE; TO BRING A FRACTION OF ANY DENOMINATOR TO A FRACTION HAVING A GREATER DENOMINATOR.

RULE.—*See how often the less will go into the greater denominator and multiply both numerator and denominator by it. The result is the required fraction.*

REDUCTION OF FRACTIONS.

EXAMPLES.

Bring $\frac{1}{2}$ to a fraction whose denominator is 8.

Here 2 goes in 8 four times; then multiply the numerator and denominator of $\frac{1}{2}$ by $4=\frac{4}{8}$, which is the required fraction.

Bring $\frac{2}{3}$ to a fraction whose denominator is 15.

Here 3 goes into 15 five times; then $\frac{2}{3}$ becomes $\frac{10}{15}$.

In case of a fraction of a fraction, as $\frac{1}{2}$ of $\frac{1}{4}$, it is called a compound fraction, and should always be reduced to a simple fraction *by multiplying all the numerators together for a new numerator, and all the denominators together for a new denominator; then, if necessary, reduce this fraction to its lowest terms.*

EXAMPLE.—$\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{4}{9}$. Reduce to a single fraction: $3\times2\times4=24$; and $4\times3\times9=108$.

Thus, $\frac{24}{108}$ is the fraction. Reduce this $12)\frac{24}{108}=\frac{2}{9}$.

TO REDUCE TWO OR MORE FRACTIONS TO EQUIVALENT FRACTIONS HAVING THEIR LEAST COMMON DENOMINATOR.

RULE.—*Find the least common multiple of the given denominators for the least common denominator, and reduce the given fractions to this denominator.*

EXAMPLE.

Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{9}{10}$ to equivalent fractions having their least common denominator; then $\frac{2}{3}=\frac{40}{60}$, $\frac{3}{4}=\frac{45}{60}$, $\frac{5}{6}=\frac{50}{60}$, $\frac{9}{10}=\frac{54}{60}$.

CANCELLATION.

This is a method of shortening problems by rejecting equal factors from the divisor and dividend.

The sign of cancellation is an oblique mark drawn across the face of a figure, as $\not 4$, $\not 6$, $\not 2$.

Cancellation means to leave out; if there are the same numbers in the numerator and the denominator they are to be left out.

Ex.—$\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{4}{9}$. Here the 3 in the first numerator and the 3 in the second denominator are left out; also 4 of the first denominator and the last numerator, thus:

$$\text{Ans.}\ \frac{\not 3}{\not 4}\times\frac{2}{\not 3}\times\frac{\not 4}{9}=\frac{2}{9}$$

Ex.—$\frac{2}{9}$ of $\frac{5}{8}$ of $\frac{14}{18}$ of $\frac{90}{170}$=by cancellation thus:

$$\frac{\not 2}{\underset{3}{\not 9}}\text{ of }\frac{\not 3}{\underset{\underset{2}{\not 4}}{\not 8}}\text{ of }\frac{\overset{7}{\not{14}}}{\not{18}}\text{ of }\frac{\overset{\not 5}{\not{90}}}{\underset{34}{\not{170}}}=\frac{7}{3\times2\times34}=\frac{7}{204}$$

See note.

NOTE.—The process is as follows: The first numerator, 2, will go into 8, the denominator of the second fraction, 4 times; the denominator of the third fraction, 18, will go into 90, the numerator of the last quantity, 5 times. The numerator of the second fraction, 3, will go into the denominator of the first fraction 3 times; 5 will go into 170, 34 times; 2 will go into 4 twice, and 2 into 14, 7 times, and as we cannot find any more figures that can be divided without leaving a remainder, we are at the end, and the quantities left must be collected into one expression. On examination, we have 7 left on the top row; this is put down at the end as the final numerator; on the bottom we have 3, 2 and 34; these multiplied together give us 204, which is the final denominator.

USEFUL DEFINITIONS.

RULES FOR CANCELLING.

1. *Any numerator may be divided into any denominator, provided no remainder is left, and vice versa, thus:*

$$\frac{\not{3}}{5} \text{ of } \frac{4}{\underset{3}{\not{9}}} = \frac{4}{15} \qquad \Big| \qquad \frac{\not{3}}{\not{5}} \text{ of } \frac{\overset{\not{3}}{\not{15}}}{\underset{\underset{2}{\not{6}}}{\not{18}}} = \frac{1}{2}$$

2. *Any numerator and denominator may be divided by the same number, provided no remainder is left, and the decreased value of such numerator and denominator be inserted in the place of those cancelled.*

$$\frac{\not{3}}{\underset{2}{\not{8}}} \text{ of } \frac{\overset{5}{\not{20}}}{31}$$

Here 8 is divided by 4, and 20 can also be divided by the same number without leaving any remainder. Answer, $\frac{15}{62}$.

Ex.—

$$\frac{\not{8}}{\underset{3}{\not{15}}} \text{ of } \frac{\not{5}}{\underset{\underset{2}{\not{4}}}{\not{32}}} \text{ of } \frac{\overset{7}{\not{14}}}{17} = \frac{7}{3 \times 2 \times 17} = \frac{7}{102}$$

DEFS.—A COMMON DENOMINATOR of two or more fractions is a denominator to which they can all be reduced, and is the common multiple of their denominators.

THE LEAST COMMON DENOMINATOR of two or more fractions is the least denominator to which they can be reduced, and is the least common multiple of their denominators.

A MULTIPLE of a number is a number that is exactly divisible by it; or it is any product of which the given number is a factor.

Thus, 12 is a multiple of 6; 15 of 5, etc.

A COMMON MULTIPLE of two or more numbers is a number that is exactly divisible by each of them.

Thus, 12, 24, 36 and 48 are multiples of 4 and 6.

THE LEAST COMMON MULTIPLE of two or more numbers is the *least* number that is exactly divisible by each of them.

Thus, 12 is the least common multiple of 4 and 6.

ADDITION OF FRACTIONS.

Addition of fractions is the process of finding the sum of two or more fractions. In order that fractions may be added, they must have like denominators and be parts of like units.

RULE.—*Bring all the fractions to the same common denominator, add their numerators together for the new numerator, and reduce the resulting fraction to its simplest form.*

EXAMPLES.

What is the sum of $\frac{1}{4}+\frac{1}{2}=\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$. Ans.

What is the sum of $\frac{3}{4}+\frac{1}{2}+\frac{3}{8}+\frac{6}{8}=\frac{19}{8}=2\frac{3}{8}$. Ans.

SUBTRACTION OF FRACTIONS.

Bring the fractions to others having a common denominator, as in addition, and subtract their numerators.

EXAMPLES.

From $\frac{7}{8}$ subtract $\frac{3}{8}=\frac{4}{8}=\frac{1}{2}$.

From $\frac{1}{2}$ take $\frac{1}{6}$. $\frac{3-1}{6}=\frac{2}{6}=\frac{1}{3}$.

$\frac{7}{16}-\frac{3}{8}=\frac{7-6}{16}=\frac{1}{16}$.

What is the difference between $\frac{1}{2}$ of $\frac{3}{4}$ and $\frac{1}{4}$ of $1\frac{1}{2}$?

$\frac{1}{2}$ of $\frac{3}{4}=\frac{3}{8}$; and $\frac{1}{4}$ of $1\frac{1}{2}=\frac{1}{4}$ of $\frac{3}{2}=\frac{3}{8}$.

Therefore, it is $\frac{3}{8}-\frac{3}{8}=0$.

MULTIPLICATION OF FRACTIONS.

First bring each fraction to its simplest form; then multiply the numerators together for the new numerator, and the denominators together for the new denominator. Reduce the fraction to its simplest form.

MULTIPLICATION OF FRACTIONS.

EXAMPLES.

1. Multiply $\frac{4}{7} \times 1\frac{5}{16}$; that is, $\frac{4}{7} \times \frac{21}{16} = \frac{84}{112} = \frac{21}{28} = \frac{3}{4}$, or by canceling

$$\overset{1}{\underset{1}{\frac{\cancel{4}}{\cancel{7}}}} \times \overset{3}{\underset{4}{\frac{\cancel{21}}{\cancel{16}}}} = \frac{3}{4}$$

The 4 cancels into the 16 four times, and the 7 into the 21 three times. Thus $1 \times 3 = 3$, and $1 \times 4 = 4$. Answer $\frac{3}{4}$.

2. $2\frac{1}{10}$ of $3\frac{4}{7} \times 6\frac{1}{8}$ of $\frac{8}{21}$.

$$\overset{3}{\underset{2}{\frac{\cancel{21}}{\cancel{10}}}} \text{ of } \overset{5}{\underset{1}{\frac{\cancel{25}}{\cancel{7}}}} \times \overset{7}{\underset{1}{\frac{\cancel{49}}{\cancel{8}}}} \text{ of } \overset{1}{\underset{3}{\frac{\cancel{8}}{\cancel{21}}}}$$

$$\overset{5}{\frac{\cancel{15}}{2}} \times \underset{1}{\frac{7}{\cancel{3}}} = \frac{35}{2} = 17\tfrac{1}{2} \text{ Answer.}$$

DIVISION OF FRACTIONS.

Reverse the divisor and proceed as in multiplication.

The object of inverting the divisor is convenience in multiplying.

After inverting the divisor, cancel the common factors.

EXAMPLES.

$\frac{3}{4} \div 1\frac{1}{8}$, that is, $\frac{3}{4} \div \frac{9}{8}$, reverse the $\frac{9}{8}$ and it becomes $\frac{8}{9}$; then the question is $\frac{3}{4} \times \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$ Ans.

$4\frac{2}{7}$ of $\frac{14}{15} \div 3\frac{3}{4}$ of $3\frac{1}{5}$, that is, $\frac{30}{7}$ of $\frac{14}{15} \div \frac{15}{4}$ of $\frac{16}{5}$; canceling reduces the dividend to $\frac{4}{1}$ and the divisor to $\frac{12}{1}$ and we have $\frac{4}{1} \div \frac{12}{1}$, that is, $\frac{4}{1} \times \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$ Ans.

DECIMALS.

A decimal fraction derives its name from the Latin *decem*, "ten," which denotes the nature of its numbers. It has for its denominator a UNIT, or whole thing, as a pound, a yard, etc., and is supposed to be divided into ten equal parts, called tenths; those tenths into ten equal parts, called hundredths, and so on.

The denominator of a decimal being always known to consist of a unit, with as many ciphers annexed as the numerator has places, is never expressed, being understood to be 10, 100, 1000, etc., according as the numerator consists of 1, 2, 3 or more figures. Thus: $\frac{2}{10}$, $\frac{24}{100}$, $\frac{125}{1000}$, etc., the numerators only are written with a dot or comma before them, thus: .2, .24, .125.

The use of the dot (.) is to separate the decimal from the whole numbers.

The first figure on the right of the decimal point is in the place of tenths, the second in the place of hundredths, the third in the place of thousandths, etc., always decreasing from the left towards the right in a tenfold ratio, as in the following

TABLE.

Etc., Etc.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal point.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths.	Millionths.	Ten Millionths.	Etc., Etc.
	5	5	5	5	5	5	5	5	.	5	5	5	5	5	5	5	

Ascending. Descending.

DECIMALS.

A cipher placed on the left hand of a decimal decreases its value in a tenfold ratio by removing it farther from the decimal point. But annexing a cipher to any decimal does not alter its value at all. Thus 0.4 is ten times the value of 0.04, and a hundred times 0.004. But 0.7=0.70=0.700 =0.7000, etc., as above remarked.

0.2	is equal to			two-tenths.
0.25	"	"	"	twenty-five hundredths.
0.1876	"	"	"	one thousand eight hundred and seventy-six ten thousandths, and so on.

Mixed numbers consist of a whole number and a decimal, as 4.25 and 3.875.

TO REDUCE A FRACTION TO A DECIMAL.

RULE.—*Annex decimal ciphers to the numerator, and divide by the denominator, pointing off as many decimal places in the quotient as there are ciphers annexed.*

EX.—Reduce $\frac{3}{4}$ to a decimal.

EX.—4) 3.00
.75

TO REDUCE A DECIMAL TO A FRACTION.

RULES.—1, *Omit the decimal point*; 2, *Supply the proper denominator*; 3, *Reduce the fraction to its lowest terms.*

EX.—Reduce .075 to an equivalent fraction.

$.075=\frac{75}{1000}=\frac{3}{40}$.

NOTE.—"It is not merely the ability to calculate that constitutes the utility of mathematical knowledge to the engineer; it is also the increased capacity for understanding the natural phenomena on which the engineering practice is based."

ADDITION OF DECIMALS.

RULE.—*Place the quantities down in such a manner that the decimal point of one line shall be exactly under that of every other line; then add up as in simple addition.*

EXAMPLE.

Thus:—Add together 36.74, 2.98046, 176.4, 31.0071 and .08647.

```
 36.74
  2.98046
176.4
 31 0071
   .08647
---------
247.21403
```

SUBTRACTION OF DECIMALS.

RULE.—*Place the lines with decimal point under decimal point, as in addition. If one line has more decimal figures than another, put naughts under the one that is deficient till they are equal, then subtract as in simple subtraction.*

EXAMPLES.

From 146.2004 take 98.9876.

```
146.2004
 98.9876
--------
 47.2128 Answer.
```

From 4.17 take 1.984625.

```
4.170000
1.984625
--------
2.185375 Ans.
```

MULTIPLICATION OF DECIMALS.

RULE.—*Place the factors under each other, and multiply them together as in whole numbers; then point off as many figures from the right hand of the product as there are decimal places in both factors, observing, if there be not enough, to annex as many ciphers to the left hand of the product as will supply the deficiency.*

EXAMPLE.—Multiply 3.625 by 2.75.

$3.625 \times 2.75 = 9.96875$ Ans.

DIVISION OF DECIMALS.

RULE.—*Prepare the decimal as directed for multiplication; divide as in whole numbers; cut off as many figures for decimals in the quotient as the number of decimals in the dividend exceeds the number in the divisor; and if the places in the quotient be not so many as the rule requires, supply the deficiency by annexing ciphers to the left hand of the quotient.*

EXAMPLE.—Divide 173.5425 by 3.75.

```
3.75)173.5425(46.27+
     1500
     ----
      2354
      2250
      ----
       1042
        750
       ----
        2925
        2625
        ----
         300
```

RATIO, PROPORTION, RULE OF THREE.

THE RULE OF THREE, so called because there are always *three* numbers to find a fourth.

The solving of this problem, *i. e.*, having three numbers, to find the fourth, is the most important part of proportion. On account of its great utility and extensive application, it has been called *the golden rule.*

RATIO is the relation of two numbers as expressed by the quotient of the first divided by the second. Thus, the ratio of 6 to 3 is 6÷3, or 2.

THE RATIO BETWEEN TWO NUMBERS *is expressed by placing a colon between them*; *thus, the ratio of* 8 *to* 4 *is expressed* 8 : 4.

A SIMPE RATIO IS A RATIO BETWEEN TWO NUMBERS, as 4 : 5.

A COMPOUND RATIO is a ratio formed by the combination of two or more simple ratios.

Thus, $\begin{matrix} 4 : 5 \\ 3 : 2 \end{matrix}$ is a compound ratio, and is equivalent to 4×3 : 5 × 2, or 12 : 10.

The numbers whose ratio is expressed are *the terms of the ratio.* The two terms of a ratio form *a couplet*, the first of which is *the antecedent* and the second *the consequent.*

PROPORTION IS AN EQUALITY OF RATIOS. The first and fourth terms of a proportion are called *the extremes*, and the second and third *the means.*

The product of the means is equal to the product of the extremes.

RATIO AND PROPORTION.

A missing mean may be found by dividing the product of the extremes by the given mean.

A missing extreme may be found by dividing the product of the means by the given extreme.

SIMPLE PROPORTION is an equality of two simple ratios, as,

9 lb. : 18 lb. : : 27 cents : 54 cents.

Ex.—If 24 wrenches cost $27, what will 32 wrenches cost?

ANS.—36 dollars. See note.

RULE.—*For convenience, take for the third term the number that may form a ratio with, or is of the same denomination as, the answer. If, from the nature of the example, the answer is to be greater than the third term, make the greater of the two remaining terms (which must be of the same denomination) the second term; when not, make the smaller the second term. Then multiply the means (the second and third) together, and divide their product by the given extreme (the first term).*

EXS.—The missing term, x, in the examples below, can be found by applying the principles given on page 48).

$16 : x :: 24 : 18$. Ans. 12.

$x : 27 :: 18 : 54$. Ans. 9.

$32 : 27 :: x : 135$. Ans. 160.

$16 : 12 :: 24 : x$. Ans. 18.

NOTE.—For convenience in working this example make the fourth term the missing term, or the required answer. Since the third and fourth terms must be of the same denomination and the denomination of the answer will be dollars, take $27 as the third term. From the nature of the example the answer will be more than $27, the third term; therefore, make 32 wrenches the second term and 24 wrenches the first term. The proportion will then be stated as follows: 24 wrenches : 32 wrenches : : $27 : x (Let x represent the unknown term). Multiplying 32 by 27, and dividing the product by 24, the fourth or missing term will be $36.

EVOLUTION OR SQUARE ROOT.

The SQUARE ROOT of a number is one of the two equal factors of a number. Thus, the square root of 25 is 5. $5 \times 5 = 25$.

TO FIND THE SQUARE ROOT OF A NUMBER.

RULE.—*Beginning at units' place, separate the given number into periods of two figures each.*

Find the greatest square in the left-hand period, and write its root at the right in the form of a quotient in division. Subtract this square from the left-hand period, and to the remainder annex the next period to form a dividend.

Double the part of the root already found for a trial divisor. Find how many times this divisor is contained in the dividend, exclusive of the right-hand figure, and write the quotient as the next figure of the root. Annex this quotient to the right of the trial divisor to form the complete divisor. Multiply the complete divisor by the last figure of the root, and subtract the product from the dividend.

To the remainder annex the next period, and proceed as before.

When the given number is a decimal, separate the number into periods of two figures each, by proceeding in both directions from the decimal point.

EXAMPLE.

Find the square root of 186624.

```
      18,66,24(432
      16
 83  | 266
     | 249
 862 | 1724
     | 1724
```

Proof

```
   432
   432
   ---
   864
  1296
 1728
 ------
 186624
```

EXAMPLE.

Find the square root of 735.

7,35(27.11 etc.	Proof 2711
4	2711
47 | 335	
| 329	2711
541 | 600	2711
| 541	18977
5421 | 5900	5422
| 5421	
etc.	734.9521

We proceed as before till we get the remainder 6, and we see it is not a perfect square; we wish the root to be taken to two or three places of decimals; there are no more figures to bring down, therefore bring down two ciphers and proceed as in the first example; to the remainder attach two more ciphers and proceed as before, and by attaching two ciphers to the remainder you may carry it to any number of decimal places you please. In the above example the answer is 27.11, etc.

The following important note is to be studied in connection with example at the bottom of the opposite page.

NOTE.—Begin at the last figure 4, count two figures, and mark the second as shown in the example; count two more, and mark the figure, and so on till there are no more figures; take the figures to the left of the last dot, 18, and find what number multiplied by itself will give 18. There is no number that will do so, for $4\times4=16$, is too small, and $5\times5=25$, is too large; we take the one that is too small, viz., 4, and place it in the quotient, and place its square, 16, under the 18, subtract and bring down the next two figures, 66. To get the divisor, multiply the quotient 4 by $2=8$, place the 8 in the divisor, and say 8 into 26 goes 3 times, place the 3 after the 4 in the quotient and also after the 8 in the divisor; multiply the 83 by the 3 in the quotient, and place the product under the 266 and subtract, then bring down the next two figures, 24. To get the next divisor, multiply the quotient 43 by $2=86$; see how often 8 goes into 17, twice; place the 2 after the 43 of the quotient, and also after the 86 of the divisor; multiply the 862 by the 2, and put it under the 1724, then subtract. Answer, 432.

EVOLUTION.

In expressing the square root it is customary to use simply the mark ($\sqrt{}$), the 2 being understood.

All roots as well as powers of *one* are 1, as $\sqrt{1} = 1$.

EXAMPLE.

Find the square root of 588.0625.

```
          5,88.06,25(24.25
          4
         ------
   44  |  188
       |  176
         ------
   482 |  1206
       |   964
         -------
  4845 |  24225
       |  24225
```

In a decimal quantity like the above, the marking off differs from the former examples. Instead of counting twos from right to left, we begin at the decimal point and count twos toward the left and toward the right. The rest of the work is similar to the other examples.

Notice, that when the .06 is brought down, the figure for a quotient is a decimal.

To familiarize oneself with the extracting of the square root, it is well first to square a number and then work backward according to the examples here given, and by long and frequent practice become expert in the calculation. But in first working square root, it is undoubtedly better to secure the services of a teacher.

INVOLUTION

Is the raising a number (called the root) to any power. The powers of a number are its square, cube, 4th power, 5th power, etc.

$2 \times 2 = 4$	4 is the square or 2nd power of 2.
$2 \times 2 \times 2 = 8$	8 is the cube or 3d power of 2.
$2 \times 2 \times 2 \times 2 = 16$	16 is the 4th power of 2.
Etc.	Etc.

RULE.—*To square a number multiply it by itself.*

EXAMPLE.

What is the square of 27 (written 27^2)?

```
 27
 27
----
 189
 54
----
 729 Answer.
```

RULE.—*To cube a number, multiply the square of the number by the number again.*

EXAMPLE.—What is the cube of 50 (written 50^3)?

```
   50
   50
 ----
 2500 the square
   50
 ------
125000 the cube.
```

A *power* of a quantity, is the product arising from multiplying the quantity by itself one or more times. When the quantity is taken *twice* as a factor, the product is called the *second* power; when taken *three* times, the *third* power, and so on.

INVOLUTION.

Signs that Represent the Roots of Numbers.

The sign common to all roots is $\sqrt{}$ or $\sqrt{}$ and is known as the Radical Sign. If we require to express the square root of a number we simply put this sign before it, as $\sqrt{}16$, but if the number is made up of two or more terms, then we express the square root by the same in front, but with a line as far as the square root extends, as $\sqrt{9+7}$ or $\sqrt{4\ (19+6)}$.

The cube root is expressed by the same sign, with a 3 in the elbow, as $\sqrt[3]{8}$ or $\sqrt[3]{7\ (100-51.)}$

All other roots in the same manner, the number of the root being put instead of the 3. As fifth root $\sqrt[5]{}$, and sixth root $\sqrt[6]{}$, etc.

In the above examples, $9+7=16$, and the square root of 16 is 4.

The $4\ (19+6)=4\times25=100$, and the square root of 100 is 10.

The other way of expressing that the root is required, is by putting a fraction after and above the quantity, as $16^{\frac{1}{2}}$, which means the square root of 16, $(19+17)^{\frac{1}{2}}$, or $\{4\ (19+6)\}^{\frac{1}{2}}$ all of which means the square root of the quantities to which they are attached.

The cube root, 4th root, 5th root, etc., are written in the same way, as $729^{\frac{1}{3}}=9$; $256^{\frac{1}{4}}=4$; $3125^{\frac{1}{5}}=5$, etc.

THE POWERS OF NUMBERS.

SIGNS REPRESENTING THE POWER OF NUMBERS.

6^2 is equal to $6\times6=36$; that is, 36 is the square of 6.

5^3 is equal to $5\times5\times5=125$: that is, 125 is the cube of 5.

4^4 is equal to $4\times4\times4\times4=256$; that is, 256 is the fourth power of 4.

The power and the root are often combined, as $4^{\frac{3}{2}}$; this is read as the square root of 4 cubed. So the numerator figure represents the power, and the denominator figure represents the root. In this case the square root of 4 is 2, and the cube of 2 is $2\times2\times2=8$ Answer.

Perhaps the most common form that the student will meet with this sign is in the following:

$8^{\frac{2}{3}}$, which is read the cube root of 8 squared. Now, 8 squared=64, and the cube root of 64 is 4 Answer.

Find the value of $20^{\frac{3}{2}}$.

20 cubed=8000, and square root of 8000=89.4, etc.

EXAMPLE.

What is the value of $\dfrac{8^{\frac{2}{3}}+81}{3^{\frac{3}{2}}}$?

$8^{\frac{2}{3}}=\sqrt[3]{8^2}=\sqrt[3]{64}=4$; $81^{\frac{1}{2}}=9$; $3^{\frac{3}{2}}=\sqrt{3^3}=\sqrt{27}=5.2$ nearly.

Hence, $\dfrac{4+9}{5.2}=\dfrac{13}{5.2}=2.5$ or $2\frac{1}{2}$ Answer.

( ) are called *brackets*, and mean that all the quantities within them are to be put together first; thus, $7\,(8-6+4\times3)$ means that 6 must be subtracted from 8=2, and 4 times 3=12 added to this 2=14; and then this 14 is to be multiplied by 7=98.

THE METRIC SYSTEM.

In the Metric or French system of weights and measures, the *Meter* is the basis of all the units which it employs. The *Meter* is the unit of length, and is equal to one ten-millionth part of the distance measured on a meridian of the earth from the equator to the pole, and equals about 39.37 inches, or $39\frac{1}{3}$ inches nearly.

The standard meter is a bar of platinum carefully preserved at Paris. Exact copies of the meter and the other units have been procured by the several nations (including the United States) that have legalized the system.

In this system, weights and measures are increased or decreased by the following words prefixed to them:

Milli	expresses	the	1,000th	part.	
Centi	"	"	100th	"	
Deci	"	"	10th	"	
Deka	"	10 times	the value.		
Hecto	"	100	"	"	"
Kilo	"	1,000	"	"	"

TABLE.

		1 Millimeter......($\frac{1}{1000}$ of a meter)	=	.03937	*in.*
10 *mm.*	=	1 Centimeter .....($\frac{1}{100}$ of a meter)	=	.3937	*in.*
10 *cm.*	=	1 Decimeter......($\frac{1}{10}$ of a meter)	=	3.937	*in.*
10 *dm.*	=	1 METER.........(1 meter)	=	39.37	*in.*
10 *m.*	=	1 Dekameter......(10 meters)	=	32.8	*ft.*
10 *Dm.*	=	1 Hectometer.....(100 meters)	=	328.09	*ft.*
10 *Hm.*	=	1 Kilometer......(1000 meters)	=	.62137	*mile.*

NOTE.—A gramme is the weight of a cubic centimeter of distilled water; a decigramme contains $\frac{1}{10}$ of a gramme; a dekagramme contains 10 grammes.