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Determinants of Food Consumption in Rural Sierra Leone: Application of the Quadratic Expenditure System to the Consumption-Leisure Component of a Household-Firm Model

By

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Government policies affect the nutritional status of different population groups, sometimes intentionally but far more often without forethought. The nutritional well being of people, particularly persons with low incomes, has become an important consideration for governments of less developed countries. However, it is rare that policy planners have much indication how different policies will affect food consumption and thereby nutritional well being. This is especially so for people who operate their own firms and who can adjust outputs and inputs as well as labor supplied and consumption of goods and services in response to price and other socio-economic variables.

The purpose of this paper is to report the estimation of a quadratic expenditure system for the household consumption-leisure choice component of a household-firm model. This is the first step in estimation of the complete model, which will provide information regarding determinants of food consumption. Then the nutritional consequences may be traced using nutrient composition tables. The data are a cross section survey of households in rural Sierra Leone.

I

In order to trace all the impacts of socio-economic variables on household food consumption it is necessary to account for those felt indirectly through influence on the production and labor supply activities of the household as well as directly on food consumption. This leads to modelling the household using so-called household-firm models.[1] Our unit of analysis is thus the household. We assume certainty and abstract from time. A household utility function is assumed with arguments being household consumption of various goods and of leisure. Goods may be bought or sold in the market and produced. Labor may be bought or sold in the market. Goods are produced using labor, land and fixed capital. Land is assumed fixed in total amount but must be distributed between uses. A time constraint exists equating household leisure plus labor time to total time available. Finally, a budget constraint exists equating the value of net product transactions plus exogenous income to the value of net labor transactions. Product prices and wage are taken exogenously by the household, markets are assumed to be perfectly competitive and family and hired labor are assumed perfect substitutes.

Formally, let the household maximize

$$U = U(\bar{L}, X_i^c) \quad , \quad \text{where } \bar{L} = \text{leisure}$$

$$X_i^c = \text{good i consumed, } i=1, \ldots, n$$

subject to: $G(X_i, L_T, D, \bar{K}) = 0$

$$X_i^c = X_i - S_i \qquad i=1, \ldots, n$$

$$S_L = L_H - L_T$$

$$\bar{L} = T - L_H$$

$$\sum_{i=1}^{n} P_i S_i + A + P_L S_L = 0$$

where $G(\cdot)$ = implicit production function

X_i = production of good $i=1, \ldots, n$

L_T = total labor demanded

D = land

$\bar{K}$ = fixed capital

S_i = net sales of good i (purchase if negative), $i=1, \ldots, n$

S_L = net sales of labor (purchase if negative)

A = exogenous income

T = total time available to household to allocate between labor and leisure

L_H = total household labor time worked

P_i = price of good i, $i=1, \ldots, n$

P_L = price of labor

Assume the utility function to be twice differentiable, increasing in its arguments and strictly quasi-concave. Assume the implicit production function to be twice differentiable increasing in outputs, decreasing in inputs and strictly quasi-convex. We will also assume interior solutions even though border solutions are easily handed algebraically (this is because estimation incorporating border conditions is very messy). We set up the Lagrangian function as

$$W = U(\bar{L}, X_i^c) + \lambda\left(\sum_{i=1}^{n} P_i(X_i - X_i^c) + A + P_L(T - \bar{L} - L_T)\right) + \mu\left(G(X_i, L_T, D, \bar{K})\right) \tag{1.1}$$

Our first order conditions are:

$$\partial W/\partial X_i^c = \partial U/\partial X_i^c - \lambda P_i = 0 \qquad i=1, \ldots, n$$

$$\partial W/\partial \bar{L} = \partial U/\partial \bar{L} - \lambda P_L = 0$$

$$\partial W/\partial X_i = \lambda P_i + \mu\, \partial G/\partial X_i = 0 \qquad i=1, \ldots, n \tag{1.2}$$

$$\partial W/\partial L_T = -\lambda P_L + \mu\, \partial G/\partial L_T = 0$$

$$\partial W/\partial \lambda = \sum_{i=1}^{n} P_i(X_i - X_i^c) + A + P_L(T - \bar{L} - L_T) = 0$$

$$\partial W/\partial \mu = G(X_i, L_T, D, K) = 0$$

These may be expressed in the more conventional way of equating marginal rates of substitution in consumption between goods to price ratios to marginal rates of transformation in production:

$$\frac{\partial U/\partial X_i^c}{\partial U/\partial X_j^c} = \frac{P_i}{P_j} = \frac{\partial G/\partial X_i}{\partial G/X_j} = \frac{-\partial X_j}{\partial X_i}, \quad i \neq j = 1, \ldots, n$$

$$\frac{\partial u/\partial \bar{L}}{\partial u/\partial X_i^c} = \frac{P_L}{P_i} = \frac{-\partial G/\partial L_T}{\partial G/\partial X_i} = \frac{\partial X_i}{\partial L_T}, \quad i=1, \ldots, n$$

Graphically, for outputs, the household produces on its transformation function between two goods at the point at which the slope of the transformation curve equals relative market prices. Consumption is at the point of tangency between the same market possibilities line and the household indifference curves. Net marketed surpluses are measured by the usual trade triangles. In this case

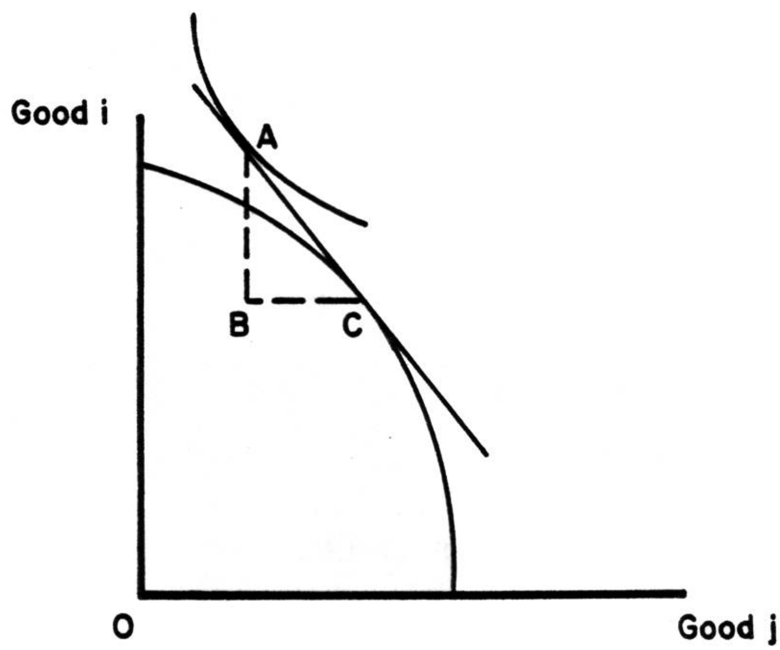

Figure 1

C-B of good j is sold and B-A of good i purchased. Between outputs and labor the same situation holds.

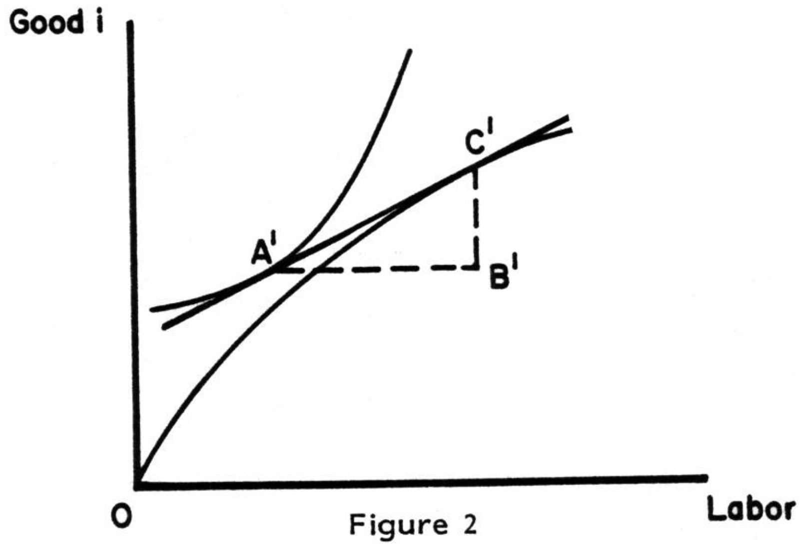

Figure 2

In the case pictured C'-B' of good i is sold and A'-B' of labor is hired.

An extremely important property of this model is that it is recursive. The household's production decisions are first made and subsequently used in allocating available "total income" between consumption of goods and leisure. This result is wholly dependent on the existence of markets for goods and labor. Intuitively this allows the family to separate its decisions on goods demanded and household goods supplied, the difference being labor hired (or sold out). This can be seen graphically in Figures 1 and 2. More formally, in the first order conditions, the partial derivatives with respect to outputs yield n equations in n+2 unknowns (n good outputs, total labor demanded and the ratio of two multipliers). Two more equations are added by the partial derivative with respect to total labor demanded and with respect to the multiplier of the implicit production function. This system of n+2 equations in n+2 unknowns can be solved in terms of all prices, the wage rate, fixed land and capital, as the result of the quasi-convexity of the implicit production function, first order conditions and the implicit function theorem. Such solutions may then be substituted into the budget constraint. With the partial derivatives with respect to leisure and consumption of goods this yields an additional n+2 equations in n+2 unknowns (n good consumptions, leisure and a multiplier), which may also be solved in terms of prices, the wage rate and nonearned income, as the second order conditions are met.

Conditional on the production decisions this second set of n+2 equations is identical to the first order conditions of the labor-leisure choice problem. This, along with our assumptions about the utility function, implies that the usual constraints of economic theory apply: zero homogeneity of demand with respect to prices, wage rate and unearned income, and symmetry and negative semi-definiteness of the Slutsky substitution matrix. Likewise on the production side.

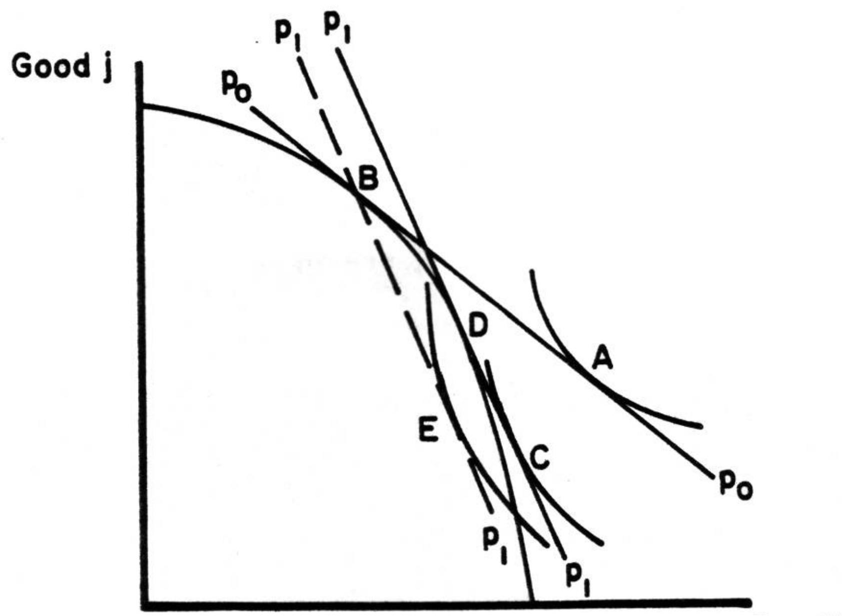

Figure 3

The remainder of this paper is concerned with the demand component of the household-firm model, that is the production decisions assumed fixed. When we later look at comparative static changes in prices, from p_0-p_0 to p_1-p_1 in Figure 3 we will not be examining the entire shift in consumption from point A to point C. Rather we will be looking at a shift from point A to some point, say E, with production held fixed at point B, rather than moving from B to D. The reader only interested in empirical results may wish to skip to section 5, in which the data are discussed, and then to sections 7 and 8, in which results are presented and summarized.

II

Systems of demand equations relate an exhaustive set of expenditures to all prices and total expenditure (or income). Two broad approaches are used in specifying functional form. First, one can specify a particular functional form. This can be done either for the direct or indirect utility function, in which case one works forward to derive the demand functions; or for the demand functions, in which case one derives a class of direct or indirect utility functions giving rise to that function. In doing so three restrictions are generally imposed: an adding up of expenditures criterion, zero degree homogeneity in prices and expenditures, and symmetry of the Slutsky substitution matrix. Negative semi-definiteness of the substitution matrix is not imposed but is usually tested with the data upon estimation. Alternatively, one can approximate an unknown direct or indirect utility function at a point to any desired degree of accuracy and derive the demand functions from the approximated utility function. Which approach one uses will depend on what relationships the research wants to highlight, the number of observations available to use in estimation and so forth. As a general rule approximating functions, when taken to the second degree of approximation as most have been thus far (e.g., translog or generalized Leontief), involve independent parameters to be estimated increasing as a multiple of the square of the number of commodities in the system. To decrease the number of parameters to be estimated additional constraints need to be placed on the system. Some specific functional forms have the number of parameters increasing as a multiple of the number of commodities included. This is achieved at the price of restrictions on the type of behavior admitted by that form. In general, the wider the range of behavior the functional form permits, the greater the number of parameters are.

The linear expenditure system is one example of a demand system having the number of parameters being a multiple of the number of commodities. For our purposes, it has two severe limitations. First, it restricts Engel curves to be linear and second it restricts all uncompensated cross price effects to be negative (that is, income effects always dominate substitution effects). Of lesser concern is the fact that it allows for no Hicks-Allen complementarity. An alternative system also parsimonious in parameters but which does not suffer from the first two defects is the quadratic expenditure system. Howe, Pollak and Wales (1979) have shown that any quadratic expenditure system (QES) consistent with Engel aggregation (summing up of expenditures), zero homogeneity in prices and total expenditure and symmetry of the substitution matrix is generated by an indirect utility function of the form $V(p,y) = -g(p)/(y-f(p))-a(p)/g(p)$, where $g(\cdot)$, $a(\cdot)$ and $f(\cdot)$ are homogeneous of degree one and $y \equiv$ total expenditure.

This function generates a class of quadratic expenditure systems of the form

$$p_i X_i^c = \frac{p_i}{g^2}\left(\frac{\partial a}{\partial p_i} - \frac{\partial g/\partial p_i}{g}a\right)(y-f)^2 + \frac{p_i \partial g/\partial p_i}{g}(y-f) + p_i\frac{\partial f}{\partial p_i} \qquad (2.1)$$

While existence of an indirect utility function implies existence of a direct utility function, no closed form for the direct function associated with the QES has been derived. Thus, to extend the class of QES to the household firm model we must work with indirect utility functions. Rewrite the budget constraint as $A+\pi^* + P_L T - \Sigma P_i X_i^c - P_L \bar{L} = 0$ where $\pi^* = \Sigma P_i X_i - P_L L_T$ can be interpreted as short run profits. Then as we have seen one may solve for X_i^c and $\bar{L}$ as functions of the P_i, P_L, and $A + \pi^* + P_L T$, where the latter sum which we might call total income replaces income in the indirect utility function. Hence, to use the indirect utility function in deriving demand curves in the household-firm model we need only Roy's identity. Then $X_i^c = \frac{-\partial V/\partial P_i}{\partial V/\partial (A+\pi^*+P_L T)}$ and $\bar{L} = \frac{-\partial V/\partial P_L}{\partial V/\partial (A+\pi^*+P_L T)}$. Setting $g(P) = \prod_{k=1}^{n+1} P_k^{a_k}$, $f(p) = \sum_{k=1}^{n+1} P_k C_k$ and $a(p) = -\prod_{k=1}^{n+1} P_k^{(2a_k-d_k)}$ we obtain the indirect utility function

$$V = -\prod_{k=1}^{n+1} P_k^{a_k} / (A+P_L T+\pi - \sum_{k=1}^{n+1} P_k C_k) + \prod_{k=1}^{n+1} P_k^{(a_k-d_k)} \qquad (2.2)$$

$\sum_{k=1}^{n+1} a_k = \sum_{k=1}^{n+1} d_k = 1$, where leisure is treated as the n+1 good. The C_k and d_k are constants to be determined from the data. The resulting expenditure equation is

$$P_i X_i^c = P_i C_i + a_i (P_L T + \pi + A - \sum_{k=1}^{n+1} P_k C_k) - (a_i - d_i) \prod_{k=1}^{n+1} P_k^{-d_k} \qquad (2.3)$$

$$(P_L T + \pi + A - \sum_{k=1}^{n+1} P_k C_k)^2 \qquad i=1, \ldots, n+1$$

This has as a special case the linear expenditure system provided $a_i = d_i$, $\forall_i$.

III

Since our unit of analysis is the household rather than the individual, we must decide how to incorporate household characteristics such as size and age distribution into our analysis.[2] The method we use is translation. It subtracts commodity specific indices from quantities in the direct utility function, $U(X) = U(x_1 - v_1, \ldots, x_n - v_n)$, where V_i is a function of household characteristics. One possible interpretation of the V_i's is as committed quantities of goods; however, there is no reason for the V_i's to be positive. Indeed, this modified utility function gives rise to a demand system meeting the negative semi-definiteness of the Slutsky substitution matrix (assuming the "untranslated" system met it) only for V_i sufficiently close to zero. Using this specification, the effects of demographic variables on quantities consumed comes through income effects. This may be seen by writing the indirect utility function associated with this specification, $V(p, P_L T+\pi^* A - \sum_{k=1}^{n+1} P_k V_k)$. That is everywhere total income appears one subtracts from it the sum of values of these commodity indices.

An alternative specification due to Barten (1964) is scaling. For this, one writes the direct utility function as $U(X_i/I_i, \ldots, X_n/I_n)$, where the I_i are functions of the demographic variables and have the interpretation of commodity specific consumer equivalence scales (for the QES the I_i must be positive). The resulting indirect utility function is of the form $V(p_1 I_1, \ldots, p_n I_n, P_L T+\pi+A)$; hence the influence of demographic variables will be felt through prices. The scaling specification was tried but discarded for reasons to be discussed later.

Another variable which should be dependent on household characteristics is total time available to the household. Using a linearly homogeneous specification for the translation parameters we write $V_i = \sum_{r=1}^{K} \sigma_{ir}\eta_r$, where η_r, $r=1, \ldots, K$ are household characteristics and the σ_{ir}'s are parameters. Likewise, for total time we may write $T = \sum_{r=1}^{q} \gamma_r m_r$, where m_r, $r=1, \ldots, L$ are household characteristics and the γ's are parameters. The resulting expenditure equation of the QES is

$$P_i X_i^c = P_i C_i + P_i \sum_{r=1}^{K} \sigma_{ir}\eta_r + a_i\Big(P_L \sum_{r=1}^{q} \gamma_r m_r + \pi + A - \sum_{k=1}^{n+1} P_k\big(C_k + \sum_{r=1}^{K} \sigma_{kr}\eta_r\big)\Big) \tag{3.1}$$

$$-(a_i - d_i) \prod_{k=1}^{n+1} P_k^{-d_k} \Big(P_L \sum_{r=1}^{q} \gamma_r m_r + \pi + A - \sum_{k=1}^{n+1} P_k\big(C_k + \sum_{r=1}^{K} \sigma_{kr}\eta_r\big)\Big)^2$$

Since leisure is not directly observed we subtract from both sides of the leisure expenditure equation the value of time available to the household. The left hand side becomes the negative of the value of household labor, which we do observe. Thus the leisure equation becomes

$$-P_L L_H = P_L C_L + P_L \sum_{r=1}^{K} \sigma_{ir} \eta_r - P_L \sum_{r=1}^{q} \gamma_r m_r + a_i \left(P_L \sum_{r=1}^{q} \gamma_r m_r + \pi + A - \sum_{k=1}^{n+1} P_k \left(C_k + \sum_{r=1}^{K} \sigma_{kr} \eta_r\right)\right)$$

$$-(a_i - d_i) \prod_{k=1}^{n+1} P_k^{-d_k} \left(P_L \sum_{r=1}^{q} \gamma_r m_r + \pi + A - \sum_{k=1}^{n+1} P_k \left(C_k + \sum_{r=1}^{K} \sigma_{kr} \eta_r\right)\right)^2 \quad (3.2)$$

This device avoids the need to impose values for T, such as a male having exactly sixteen hours per day available for work and leisure. With n+1 commodities, K translation demographic variables and q demographic variables for total time this system has at most (3+K) (n+1)-2+q parameters to estimate (fewer if some of the η_r's and m_r's are identical).

IV

Specifying the error structure of the household-firm model can proceed in two ways. We can specify an error structure within the utility and production (or profit) functions and derive the appropriate error structure for the expenditure equations. The more common approach has been to append an error structure onto the demand and supply equations with, perhaps, some attention to properties of the error structure.

In the first approach we could add a stochastic component to utility and production except that we are abstracting from uncertainty. Alternatively, we can assume randomness in parameters which reflects differences in household tastes. This has been pursued by Pollak and Wales (1969) and Wales and Woodland (1979). For this study randomness in demand parameters to account for differences in tastes makes sense only if we think important differences exist which are not due to demographic characteristics. Wales and Woodland append errors to first order conditions of utility maximization. Interpreting such errors as errors in allocation rather than deterministic components reflecting differences in tastes would lead to estimation of the structural first order conditions rather than the reduced form demand, expenditure or share equations. Deriving the likelihood function for the observed commodity and factor input demands and output supplies would be a straightforward (though messy) matter of taking the Jacobian of the transformation from errors to observed variables and multiplying that by the likelihood function of the error terms, which we would assume.

If we are to be more conventional we can add errors to the reduced form. Here the question arises which form of the reduced form should errors be added to. The choices are threefold: for the demand system they are demand equations, expenditure equations and share equations. The choice will depend on which form one expects the disturbances to have desirable properties. For household t let ε_t be an n vector error. Assume ε_t's to be iid $N(0,\Sigma)$ so that $\varepsilon = (\varepsilon_1', \varepsilon_2', \ldots, \varepsilon_T')' \sim N(0, I_T \otimes \Sigma)$. On which form of the reduced form is this most likely to hold? In particular for which form are the ε_ts identically distributed? Pollak and Wales in most of their work believe the share equations are the proper ones to which to add this error structure. Using experience from estimating Engel curves they feel the errors on expenditure equations have a heteroskedastic nature of the form $E(\varepsilon_{ti}\varepsilon_{tj}) = \sigma_{ij} y_t^2$, $y_t \equiv$ total expenditure. Hence, dividing each equation by y_t, resulting in share equations, is the appropriate solution. Alternatively, one might assume as did Pollak and Wales (1969) that errors on the demand equations have structure $E(\varepsilon_{ti}\varepsilon_{tj}) = \sigma_{ij}\hat{X}_{it}^c \hat{X}_{jt}^c$ where the hats indicate non-stochastic portions. Defining

$$F_t = \begin{pmatrix} \hat{X}_{1t}^c & & 0 \\ & \ddots & \\ 0 & & \hat{X}_{nt}^c \end{pmatrix}$$

we have $\varepsilon_t \sim N(0, F_t \Sigma F_t)$. However the error structure is specified, residuals may be examined for the appropriateness of the specification, and if heteroskedasticity is suspected statistical tests may be performed.

As usual for complete systems $\sum_{i=1}^{n+1} \varepsilon_{ti}=0$, $\forall_t$ since the value of expenditures on goods and leisure adds to total income at all sample points. Hence, the full covariance matrix is singular and we drop one equation for estimation (see Barton (1969)).[3] Doing that we can write the likelihood function as

$$L = (2\pi)^{-nT/2}|\Sigma|^{-T/2} \prod_{t=1}^{T} ||F_t||^{-1} \exp\{-\tfrac{1}{2}\varepsilon_t' F_t^{-1} \Sigma^{-1} F_t^{-1} \varepsilon_t\}^{4} \qquad (4.1)$$

This function is nonlinear in parameters. Barnett (1976) and Gallant and Holly (1980) have shown that under suitable regularity conditions maximum likelihood estimators are consistent and asymptotically efficient with the asymptotic distribution of $\sqrt{T}\,(\hat{B}-B)$ being $\lim_{T\to\infty}(I/T)^{-1}$, where I=information matrix.

One remaining question is the independence of π^* and ε_t. If π^* is assumed given as is usually done for total expenditure in demand systems, then there is no problem. However, this system is derived from a household firm model hence if π^* has a stochastic component it might be correlated with ε_t in which case there is an endogeneity problem and the demand system cannot be consistently estimated apart from the production system. If, however, the disturbances on the demand and production equations of the household firm model are independent then the system is block recursive[5] and indeed separate maximum likelihood estimation is identical to maximum likelihood estimation of the larger system. At this time we assume such independence. However, it is a testable assumption and in later work will be tested.[6]

V

The data are from a cross section survey of households in rural Sierra Leone taken during the 1974-75 cropping year (May-April). Sierra Leone was divided into eight geographical regions chosen to conform with agro-climatic zones and those were used to stratify the sample. Within these regions three enumeration areas were randomly picked and households sampled within these. Households were visited twice weekly to obtain information on production, sales and labor use, among other variables. Half the households were visited twice during one week per month to obtain market purchase information. Estimates of amounts apparently consumed out of home production were derived by subtracting sales, wages in kind paid out (and seed use for rice, the major crop) from production and adding wages in kind received. These were further adjusted for processing to avoid double counting, and for storage losses. Market purchases were then added to obtain total apparent consumption.[7] Our sample size is 138 households.

Labor supplied data were formed by summing hours worked for agricultural and nonagricultural enterprises and for labor sold out. Units are in terms of male equivalents with weights 1 for males over 15, .75 for females over 15 and .5 for children aged 10-15. The weights are derived from relative wage rates in the sample as reported by Spencer and Byerlee (1977).

Prices were formed by the eight geographical regions. Annual sales prices were formed using the larger sample of 328 households for which reliable production and labor use data were available. Value of regional sales was divided by sales quantity for each of 195 commodities. Likewise, regional purchase prices were formed for 113 commodities. A concordance between commodities purchased and sold was established and a commodity price for each region was then formed by taking a weighted average of sales and purchase prices with region specific weights being the share of total expenditure for a commodity coming from either purchases or home production. Commodities were then aggregated into six groups with values consumed being used as weights to form arithmetically weighted prices. Wage is in terms of male equivalents.

Table I

Actual Average Total Expenditure Shares
By Expenditure Group[1]

Commodity	Expenditure Group			
	Low	Middle	High	Mean
Rice	.25	.24	.24	.24
Root crops and other cereals (other than rice)	.05	.06	.14	.10
Oils and fats	.08	.07	.11	.10
Fish and animal products	.13	.12	.11	.12
Miscellaneous foods	.12	.13	.09	.11
Nonfoods	.38	.37	.30	.33

[1] See Table A.II for definitions.

Data on household characteristics were available for total size, age composition by 0-5 years, 5-10 years, 11-15 years, 16-65 years and over 65 years. In addition, data on number of wives, years of English and Arabic education by the household head, age of household head, ethnic group (there are three major ones in our sample) and region lived in are available. Since ethnic groups tend to live in contiguous areas this information is also regional in character (though not identical to the eight survey regions).

The commodity classification is given in Appendix Table A.I. As one can see by the average total expenditure shares, reported in Table I, rice is the major staple with cassava (included in "root crops and other cereals") the main substitute. Rice tends to be eaten with a sauce and boiled cassava with a stew, both cooked with palm oil. Both sauce and stew are made with vegetables (onions, peppers, tomatoes and leafy greens) and some meats. Sauces tend to include dried fish and stews fresh fish.

Sample characteristics are shown in Table A.II in the appendix. The sample is divided into three expenditure groups when computing the averages as it is for much of the later analysis. These groups are total expenditure under 350 Leones, between 350 and 700 Leones, and greater than 700 Leones. To get an idea of how poor these households are, the annual per capita expenditures in 1974-75 U.S. dollars are $54, $88, and $136 respectively for the low, middle and high expenditure groups. For the capital city, Freetown (which was sampled for a migration component of this study) when divided into three groups, the average income of the middle group is $153. Hence, even our "high" expenditure households are quite poor both compared to urban Sierra Leone as well as compared to other countries.

VI

Specification of prices and of the deterministic part of total income ($A+\pi^*$) is dictated by the commodity classification; however, specification of the translation parameters and of household total time is not. All of the potential household characteristic variables could not be included in the QES estimation because too many parameters would be involved (remember each demographic variable has n+1 parameters associated with it in a system of n commodities plus leisure). In order to choose which characteristic variables should enter the system single equation demand regressions were run using all of the potential variables. Functional form was chosen to mimic the QES and the equations were estimated in share form.[8] All possible subsets of independent variables were examined and ranked by $\bar{R}^2$. In general equations with maximum $\bar{R}^2$ included the relevant price and expenditure variables. When this was not so equations having the highest $\bar{R}^2$ and including these variables were chosen. From this exercise several household characteristic variables did well in the sense of being included in the chosen equations for several commodities. Moreover, some variables had coefficients which were fairly consistently close in magnitude; hence, they could be combined. The final set of chosen demographic variables for translation parameters was household size, children under 10 and either an ethnic dummy set to one if the household was Temne or Limba (Mende is the other major group), or a regional dummy set to one if the household lived in the northern region.[9] For total time available to the household the variables chosen were persons over 10, females over 15 and children aged 11-15. Since adding a child under 10 also increases household size by one the total effect of adding a child under 10 on the translation parameters will be the sum of the children under 10 and household size coefficients. The children under 10 coefficient may be interpreted as being the differential effect of children under 10 from persons over 10.

From equation 3.1 or 3.2 we can see that the household characteristic variables are multiplied by prices when they enter the QES. An identification problem arises from our choice of demographic variables because wage times household size equals wage times persons over 10 plus wage times persons under 10. Hence, one of these variables must be dropped to avoid perfect multicollinearity. We drop the household size variable and rewrite equation 3.1.

$$P_iX_i^c = P_iC_i+P_i \sum_{r=1}^{3} \sigma_{ir} \eta_r+a_i(P_L m_1(\gamma_1-\sigma_{71})+P_L \sum_{r=2}^{3} \gamma_r m_r+\pi+A- \sum_{k=1}^{6} P_k(C_k+\sum_{r=1}^{3}\sigma_{kr}\eta_r)$$

$$-P_L(C_L+(\sigma_{72}+\sigma_{71})\eta_2+\sigma_{73}\eta_3))-(a_i-d_i)\prod_{k=1}^{7} P_k^{-d_k}(P_L m_1(\gamma_1-\sigma_{71}) \quad (6.1)$$

$$+P_L \sum_{r=2}^{3} \gamma_r m_r+\pi+A-\sum_{k=1}^{6} P_k(C_k+\sum_{r=1}^{3}\sigma_{kr}\eta_r)-P_L(C_L+(\sigma_{72}+\sigma_{71})\eta_2+\sigma_{73}\eta_3))^2$$

where we have used the fact that n+1=7, K=q=3. It is apparent from equation 6.1 that the coefficient of wage times persons over 10 $(\gamma_1-\sigma_{71})$ is identified, but not its components. Likewise, for the coefficient of wage times children under 10 $(\sigma_{72}+\sigma_{71})$.[10] In consequence total time, $T=\sum_{r=1}^{3}\gamma_r m_r$ is not identified. For the major questions in which we are interested this is not troublesome.

The final QES specifications which we estimate have seven commodities, three translation demographic variables and three total time demographic variables. The number of parameters is 42.[11] These systems in their expenditure form were estimated using the Davidon—Fletcher-Powell algorithm as available on the GQOPT package of numerical optimization routines.[12] Since there was question a priori whether the disturbances on the expenditure equations were identically distributed we took squared residuals from these equations and regressed them on variables to which the variances were hypothesized to be proportionate. In particular, they were regressed on a constant and the square of fitted value (i.e., $V_{ar}(\varepsilon_{ti})=\hat{X}_{ti}^2\sigma_{ii}$), and a constant and the square of the observed part of total income ($V_{ar}(\varepsilon_{ti})=\pi^2\sigma_{ii}$). The results of the latter were mixed, in three out of six regressions the constant term being significant and not squared profits and vice versa. As can be seen from Table A.IV squared fitted values were very significant in five out of six regressions and significant at the .10 level in the sixth. Moreover, regression standard errors for the regression using squared fitted values were uniformly lower than for the regressions using squared profits. The error specification giving rise to this result is $\varepsilon_t \sim N(O\ F_t \Sigma\ F_t)$ where F_t = diagonal $(|p_i \hat{X}^c_{ti}|)$. Alternatively, this amounts to weighting each equation for observation t and good i by $1/|p_i \hat{X}^c_{ti}|$. Clearly then the function is not defined for $|p_i \hat{X}^c_{ti}| = 0$.

The error specification using absolute fitted values was used and maximum likelihood estimation tried. Unfortunately, the algorithms kept stopping at a point at which $|p_i \hat{X}^c_{ti}|$ was nearly zero for some i and some t, but which were clearly not local optima.[13] Different starting values for parameters were tried, unsuccessfully. It was then decided to use for $p_i \hat{X}^c_{ti}$ the values from estimation of the expenditure form equations, and to treat these as constants.[14] This is an extension to regressions nonlinear in parameters of Amemiya's (1973) suggested two step procedure for the linear regression case. He showed such two-step estimators to be consistent with a known distribution, but not asymptotically efficient. Halbert White (1980) has shown (theorem 2.4) that an unweighted nonlinear least squares estimator is a strongly consistent estimator when error terms are not identically distributed, under some fairly weak regularity assumptions. What we have is a system of nonlinear seemingly unrelated regressions. Since estimating such equations jointly affects only efficiency, not consistency (assuming no misspecification), White's result is applicable to our first round estimators. In particular our estimates of fitted values are consistent. That in turn means our second stage estimates are consistent. These estimates are not unrestricted maximum likelihood and so are presumably not asymptotically efficient. Conditional on the first round estimates of fitted values they are mle and $\sqrt{T}\ (B-\hat{B})$ should be asymptotically distributed as $N(O, \lim_{T\to\infty} (I/T)^{-1})$, with the information matrix calculated treating F_t as being fixed.

The second stage conditional maximum likelihood estimates were obtained with resulting parameters and their asymptotic standard errors shown in Table A.III.

Use of the ethnic group dummy resulted in a lower log-likelihood value, -3577.1 as against -3487.4 for the estimation using the regional dummy. Regularity conditions were tested by computing eigenvalues of the Slutsky substitution matrix.[15] For the system using the regional dummy regularity conditions held at 113 out of 138 sample points[16] as against none when using the ethnic group dummy.

The reason for this failure was a small negative (i.e., -.2) compensated own price elasticity for labor supply. The other compensated own elasticities were of the expected signs and somewhat higher in absolute value than those of the system using the regional dummy. For these two reasons the regional dummy variable seems preferable and results from that estimation will be used in the ensuing discussion.

Using the regional dummy, twenty-two out of forty-two parameters have the absolute value of their coefficients greater than 1.96 times their standard errors, twenty-six have absolute values of coefficients more than 1.65 times their standard errors, and thirty have standard errors less than their coefficients' absolute value. The heteroskedasticity problem has nearly disappeared. Table A.IV shows a significant constant term and insignificant coefficient for squared fitted values on four out of six regressions of squared weighted residuals on those variables. For one regression both constant and squared fitted value are significant and for the other the constant term is significant and the squared fitted value term borderline.[17]

A series of Wald tests were run on different hypotheses and are reported in Table 2. First we test $H_o: a_i = c_i, \forall_i = 1, \ldots, 6$.[18] The value of the statistic

Table II

Chi-Square Statistics from Wald Tests[1]

Test of	Statistic	Degrees of Freedom
1. LES as special case of QES	19.0	6
2. Household size coefficients	29.1	6
3. Children under 10 years coefficients	70.1	7
4. Equality with opposite signs of household size and children under 10 coefficients	100.1	6
5. Price coefficients	38.9	7
6. Ethnic group dummy coefficients	50.1	7
7. Equality with opposite signs of price and ethnic group dummy coefficients	18.1	7

[1]From QES with regional dummy.

is 19.0 which is asymptotically distributed as a chi-square variable with six degrees of freedom. This is significant at somewhat less than the .005 level; hence we can reject the hypothesis that we should have estimated a linear expenditure system. The coefficients on household size, which is the effect of a unit change in persons over 10 on the commodity specific translation parameters, are jointly significant as are the coefficients for children under 10. Hence, children under 10 affect the translation parameters in a way different from household members over 10. Since the total effect of children under 10 on translation parameters is the sum of their coefficients plus household size coefficients it is interesting to test whether the sum of these is jointly significantly different from zero. As can be seen the statistic is 100.1 which with six degrees of freedom is highly significant. The price coefficients, the c_i's, are jointly significant as are the regional coefficients. This means that the price coefficients for southern households (for when the dummy is zero) are significant and significantly different from the price coefficients for northern households. Since the price coefficients for the latter are the sum of the southern price coefficients and the dummy coefficients we test whether this sum is jointly significantly different from zero, which it turns out to be between the .025 and the .01 levels.

VII

Shares of marginal expenditure, price elasticities of demand and marginal effects of household characteristic variables are functions, using the QES, not only of parameters but also of data.[19] Hence, one has to choose at which sample points to evaluate these. We have chosen to divide the sample into three groups based on total expenditure for this purpose. The dividing lines chosen are less than 350 Leones annual expenditure,[20] between 350 and 750 Leones inclusive, and greater than 750 Leones. The sample sizes for these groups are 44, 51 and 43 respectively. The main justification for such a division is that many observers are concerned with responses of people in different income groups, particularly the lower ones.

One can see from Table A.II that the lower expenditure group faces relatively lower prices for root crops and other cereals and for nonfoods, but higher prices for oils and fats and fish and animal products.[21] Household size tends to be smaller for the lower expenditure group as does the proportion of family members under ten years.

Shares of marginal total expenditure[22] are reported in Table 3. They generally seem to be plausible. The share for rice declines with higher total expenditure as one would expect although the .02 share for high expenditure households seems a little low. The low share for root crops and other cereals is not surprising, though one would not have expected the marginal share to rise with expenditure.[23] Note that in both the low and the high expenditure groups the marginal share is less than the average. In particular, the share is not negative at our mean evaluation points. This is interesting because many observers have hypothesized that cassava may be an inferior good for higher income groups in West Africa. This may still be the case, however, since the group, root crops and other cereals, contains expenditures on sorghum roughly equal to those on cassava, and sorghum is not usually hypothesized to be an inferior good.

Uncompensated price elasticities of demand are reported in Table 4. For rice the own price elasticity declines in absolute value with expenditure group. Part, but not all, of this is due to an income effect declining with expenditure group. This is certainly not surprising. Root crops seem not to be price responsive. The higher expenditure group is slightly more responsive to price, partly due to an increasing income effect. The relative unresponsiveness

Table III

Share of Marginal Total Expenditure[1]
by Expenditure Group[2]

Commodity	Low	Middle	High	Mean
	Expenditure Group			
Rice	.22	.16	.02	.13
Root crops and other cereals	.03	.06	.12	.07
Oils and fats	.13	.20	.36	.23
Fish and animal products	.13	.11	.07	.11
Miscellaneous foods	.09	.07	.04	.07
Nonfood	.40	.40	.39	.39

[1]Partial derivative of commodity expenditure with respect to total income divided by partial derivative of total expenditure with respect to total income. Evaluated at expenditure group means using QES with regional dummy.

[2]See Table A.II for definitions of expenditure groups.

Table IV

Uncompensated Quantity Elasticities with Respect to Price[1] by Expenditure Group[2]

With Respect to Price of	For Expenditure Group OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	-1.26	-.16	-.23	.02	.03	-.01	.01E-1
	Middle	-.78	-.13	-.31	.02	.02	-.02	.04E-1
	High	-.45	-.12	-.38	.05	.07	-.04	.01
	Mean	-.74	-.10	-.29	.03	.03	-.03	.01
Root Crops and Other Cereals	Low	-.02	-.15	-.02	-.02	-.02	-.02	.01
	Middle	-.02	-.26	-.04	-.02	-.01	-.02	.01
	High	-.01	-.31	-.02	-.01	-.01	-.01	.01
	Mean	-.01	-.22	-.02	-.02	-.01	-.02	.01
Oils and Fats	Low	.04	.04	-.82	.05	.03	.05	-.02
	Middle	.01E-1	.04E-1	-1.10	.02E-1	.01E-1	.04E-1	-.02E-1
	High	-.01E-1	.05E-1	-1.25	.02E-1	.01E-1	.01	-.03E-1
	Mean	.04E-1	.01	-.97	.01	.01	.01	-.01
Fish and Animal Products	Low	.02	-.08	-.12	-1.29	.01	-.01	.01E-1
	Middle	.03	-.06	-.15	-.92	.01	-.01	.03E-1
	High	.06	-.05	-.15	-.81	.04	-.03E-1	-.04E-1
	Mean	.04	-.04	-.12	-.95	.02	-.01	.01E-3
Miscellaneous Foods	Low	.01	-.06	-.10	-.03E-1	-.99	-.01	.04E-1
	Middle	.01	-.06	-.14	-.03E-1	-.60	-.02	.01
	High	.04	-.04	-.14	.02	-.63	-.02	.01
	Mean	.02	-.04	-.11	.03E-1	-.71	-.02	.01
Nonfoods	Low	.10	-.16	-.21	.06	.06	-1.17	-.01
	Middle	.07	-.16	-.36	.02	.03	-.90	.01
	High	.14	-.12	-.38	.07	.08	-1.05	-.04E-1
	Mean	.09	-.11	-.30	.04	.05	-1.01	-.04E-1
Labor	Low	1.30	.72	1.81	1.38	1.03	1.39	-.06
	Middle	.56	.48	1.53	.71	.44	.74	.09
	High	.20	.31	1.16	.43	.31	.65	.28
	Mean	.47	.38	1.25	.67	.47	.78	.14

[1] Calculated at mean for each expenditure group. Uses QES with regional dummy.

[2] See Table A.II for definitions.

of total household labor supplied to wage rate changes (-.06 to .28) is not really surprising since this is measuring total supply, not its allocation between uses, and because for farm activities labor supplied is likely to be far more influenced by the cropping pattern and technology used, than by price, and the latter are held fixed in the demand component of the household-firm model (assuming annual labor supply adjusts to labor demand). The negative sign for the low expenditure group is due to the income effect (see below) and gives some slight evidence for a backward bending supply curve.

The cross price effects with respect to rice price are negative except for fish and miscellaneous foods. This is not surprising due to the large budget share of rice leading to a relatively large income effect. The fact that this is not as true for effects with respect to nonfood price is somewhat surprising since one would expect substitution effects of food commodities and rice to be larger than between food commodities and nonfood. This does not seem to be the case for our sample. Another cross price effect of some interest is between rice and root crops. One can see that root crop demand is more responsive to changes in price of rice than rice demand is to changes in price of root crops. Since rice represents a larger budget share its income effect is likely to be greater.

Income compensated price elasticities of demand are reported in Table 5. At the sample average and for all three expenditure group averages the substitution matrix was negative semi-definite.

As with the uncompensated elasticities there is a tendency for price responsiveness of rice to decline with total expenditure. All goods are Hicks-Allen substitutes except for root crops and rice at high expenditure levels. This is unlikely; however, the magnitude is small, -.01. Perhaps, then, it should be interpreted as suggesting independence. Also note that the substitution

Table V

Income Compensated Quantity Elasticities with Respect to Price[1] by Expendíture Group[2]

With Respect to Price of	For Expenditure Group OF	Rice	Root Crops and Other Cereals	Oils, and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	-1.05	.02E-1	.14	.26	.20	.23	-.08
	Middle	-.68	.01	.12	.17	.11	.15	-.09
	High	-.44	-.01	.01	.13	.11	.12	-.09
	Mean	-.65	.01	.09	.17	.13	.16	-.10
Root crops and Other cereals	Low	.03E-2	-.13	.02	.03E-1	.01E-1	.05E-1	-.01E-1
	Middle	.03E-2	-.23	.05	.01	.04E-1	.01	-.01
	High	-.01	-.27	.11	.02	.01	.04	-.02
	Mean	.03E-1	-.20	.05	.01	.01	.18	-.01
Oils and Fats	Low	.04	.04	-.81	.05	.04	.05	-.02
	Middle	.04	.05	-.95	.05	.03	.06	-.03
	High	.01	.09	-.93	.07	.04	.13	-.08
	Mean	.04	.05	-.84	.06	.04	.08	-.04
Fish and Animal Products	Low	.13	.01	.08	-1.17	.10	.12	-.04
	Middle	.08	.02	.09	-.84	.06	.08	-.05
	High	.06	.01	.07	-.76	.06	.08	-.06
	Mean	.08	.01	.07	-.88	.07	.09	-.05
Miscellaneous Foods	Low	.09	.04E-1	.05	.09	-.92	.08	-.03
	Middle	.06	.01	.06	.06	-.56	.06	-.03
	High	.04	.05E-1	.03	.05	-.61	.05	-.04
	Mean	.06	.01	.04	.06	-.67	.06	-.04
Nonfoods	Low	.36	.04	.24	.35	.27	-.87	-.11
	Middle	.23	.07	.30	.25	.16	-.64	-.15
	High	.15	.08	.35	.23	.18	-.75	-.19
	Mean	.22	.06	.28	.26	.19	-.73	-.16
Labor	Low	.42	.04	.28	.42	.32	.38	.28
	Middle	.27	.07	.34	.30	.19	.28	.36
	High	.18	.08	.36	.26	.21	.32	.49
	Mean	.26	.06	.30	.31	.23	.31	.40

[1] Calculated at mean values for each group. Uses QES with regional dummy.

[2] See Table A.II for definitions.

effects with respect to wage are small so that the compensated wage effects are largely income in nature due to changes in wage changing nominal total income as well as real income. Also the response of household labor supply to wage rates, while small, does increase with expenditure group part of which may be due to wage rates increasing slightly with higher expenditure group.

The foregoing results were evaluated at expenditure group averages; in particular, the regional dummy variable was also averaged. Of course, there is no household that lives partly in the north and partly in the south. Therefore we also calculated marginal budget shares and price elasticities by expenditure level and region. The marginal budget shares by expenditure group are nearly identical across regions. For own uncompensated price elasticities the differences are small. In general, southern households tend to be a little less price responsive than northern households; however, the differences shrink with higher expenditure groups and for the high expenditure group are negligible. Since differences due to expenditure group are far greater than because of region group the latter results are not reported, although they are available.

Changes in expenditure due to a unit change in household composition variables are shown in Table 6. These changes are evaluated at the sample average except for the regional dummy variable which is set to one for northern households and to zero for southern households. One can see that the largest marginal expenditures are for rice, nonfoods, and oils and fats (except for changes in children under 10). For males over 15 the value of household labor supply is also affected importantly. One can see that total expenditures increase for increases in each age, sex group. Also, region makes no real difference.[24]

As persons under 10 do not affect total household time the change in the value of household labor is the negative of the marginal change in expenditure on leisure.

Table VI

Marginal Expenditure by Commodity Due to Unit Change in Age-Group Variables by Region[1]
(in Leones)

Commodity	Region	Age Group Under 10	11-15	Males over 15	Females over 15
Rice	North	10.1	6.8	17.6	9.2
	South	9.7	7.0	18.4	9.5
Root crops and other cereals	North	4.3	-2.5	3.7	-1.2
	South	4.5	-2.7	3.4	-1.3
Oils and fats	North	-5.9	8.7	28.9	13.2
	South	-5.4	8.4	28.0	12.8
Fish and animal products	North	-1.8	2.0	10.9	4.0
	South	-1.9	2.1	11.1	4.1
Miscellaneous foods	North	10.1	-2.5	3.0	-1.2
	South	10.0	-2.4	3.2	-1.2
Nonfoods	North	8.7	5.6	39.2	13.0
	South	8.7	5.6	39.1	13.0
Household labor	North	25.5	18.1	103.3	37.0
	South	25.6	18.0	103.2	37.0

[1]Calculated at sample averages except for regional dummy variable.

In each age bracket the marginal changes in goods expenditure less the change in value of labor supplied equals zero since the sum of total expenditures minus the value of labor supplied always equals the "profits" part of total income, which is constant. For persons over 10 total income changes for their time constitutes the time available to the household.

VIII

Clearly, there are many interesting results in these tables to which we cannot do justice in this paper. Of significance for development efforts is the general proposition that food demand is reasonably responsive to price (except for root crops and other cereals). Price as an important short run allocator of food consumption and hence caloric consumption has been stressed in recent years by such people as Mellor (1975) and Timmer (1978). Mellor has focused on the real income effect of price, which is supported here. However, we find own price substitution effects also to be important, contrary to previous expectations. Partly this is due to the commodity disaggregation we have used (five food groups including two of staples). Our results also supply information important to the nutritional planner. For example, the negative uncompensated effects on root crops with respect to rice price mean that decreases in rice consumption due to increases in rice price are not likely to be compensated by increases in cassava consumption, rather the opposite. Of course, in the longer run, people will shift their production and sales patterns when confronted by relative price changes. This points to the need to estimate the production side of this household-firm model, something which will be done in future work. With even more time, investment in fixed production and human capital variables as well as changes in household size and composition will take place, but these are outside the focus of this research.

APPENDIX

Table A.I

Components of Commodities

Commodity	No.	Components
Rice	1	
Root crops and other cereals	2	Raw cassava, cassava products, other root crops, sorghum, millet, maize, fundi, benniseed.
Oils and fats	3	Palm oil, groundnut oil, cocoa butter, other oils and fats.
Fish and animal products	4	Fresh fish, dried fish, game, other meat, other animal products.
Miscellaneous foods	5	Groundnuts, other legumes, vegetables, fruits, sugar, salt and other condiments, kolanuts, beverages.
Nonfoods	6	Clothing, cloth, fuel and light, metal work, wood work, other household and personal goods, transport, services and ceremonial, education, local saving, miscellaneous.
Household labor	7	All farm and nonfarm production and marketing activities, labor sold out. Excludes household activities such as food preparation, child care and ceremonies.

Table A.II

Mean Values of Data by Expenditure Group[1]

Variable	Expenditure Group			
	Low	Middle	High	Mean
Expenditures[2]				
Rice	58.2	125.2	262.9	146.7
Root crops & other cereals	10.7	32.4	147.4	61.3
Oils and fats	19.2	37.2	122.8	58.1
Fish and animal products	30.6	61.9	118.3	69.5
Miscellaneous foods	28.0	65.8	99.0	64.1
Nonfoods	90.0	190.1	324.0	199.9
Value of household labor supplied	306.4	361.8	530.1	396.5
Prices[2]				
Rice	.25	.23	.27	.25
Root crops & other cereals	.36	.66	.63	.55
Oils and fats	.73	.62	.66	.67
Fish and animal products	.62	.60	.39	.54
Miscellaneous foods	.56	.58	.60	.58
Nonfoods	.62	.64	.75	.66
Household labor	.08	.08	.09	.08
Household characteristics[3]				
Total size	4.8	6.4	8.7	6.7
Members under 10 years	1.2	2.1	2.7	2.0
Members, 11-15 years	.5	.7	1.1	.8
Males over 15 years	1.7	1.8	2.6	2.1
Females over 15 years	1.4	1.8	2.3	1.8
Number of households	44	51	43	138

[1]Households in low expenditure group are those with total expenditure less than 350 Leones. Households in middle expenditure group are those with total expenditure between 350 and 750 Leones. Households in high expenditure group are those with total expenditure greater than 750 Leones.

[2]In Leones per kilogram for foods and per hour of male equivalent for labor. One Leone = U.S. $1.1 in 1974/75.

[3]In numbers.

Table A.III

Coefficients and Asymptotic Standard Errors of Quadratic Expenditure Systems

Type of dummy variable:	Regional		Ethnic Group	
Parameter	Coefficient[1]	Standard Error[2]	Coefficient[1]	Standard Error[2]
c_1	-189.1	79.0	-167.8	53.2
c_2	42.4	16.4	-180.8	19.0
c_3	-12.2	23.3	-128.4	41.3
c_4	9.3	21.9	10.9	15.5
c_5	6.8	13.9	10.7	29.1
c_6	-.4	54.5	-1,907.4	690.7
c_7	-1,522.3	500.8	-1,309.3	1,579.5
σ_{11}	7.3	15.0	8.7	10.7
σ_{12}	61.5	23.5	8.4	15.5
σ_{13}	214.0	73.1	102.1	52.2
σ_{21}	-9.8	5.6	40.2	3.8
σ_{22}	24.9	8.8	4.0	9.0
σ_{23}	-30.8	28.2	153.9	28.4
σ_{31}	-.6	5.0	-1.3	6.6
σ_{32}	11.4	8.4	6.9	7.5
σ_{33}	-47.1	19.9	19.6	14.7
σ_{41}	-3.7	2.9	-1.9	1.9
σ_{42}	11.0	4.3	1.5	2.9
σ_{43}	-4.2	19.9	-18.2	15.1
σ_{51}	-8.5	3.2	-5.1	2.8
σ_{52}	32.0	5.6	22.3	4.8
σ_{53}	20.8	20.2	-27.5	21.8
σ_{61}	-14.6	8.2	-27.2	22.6
σ_{62}	60.3	13.2	25.0	34.4
σ_{63}	-37.7	37.9	97.1	115.4
$\sigma_{72}+\sigma_{71}$	-20.5	103.9	-396.5	208.0
σ_{73}	-152.1	371.1	-2,129.3	993.4
$\gamma_1-\sigma_{71}$	1,846.6	143.3	2,174.4	158.9
γ_2	-1,437.3	152.5	-1,461.6	229.7
γ_3	-1,117.7	167.7	-1,628.5	251.8
a_1	.23162	.35E-1	.55362E÷1	.20E-1
a_2	-.1405E-1	.11E-1	.13175	.42E-1
a_3	-.2803E-2	.36E-1	.420258E-1	.94E-2
a_4	.109989	.20E-1	.16796E-1	.90E-2
a_5	.7929E-1	.24E-1	-.2092E-2	.17E-1
a_6	.269242	.68E-1	1.0045	.58E-1
d_1	.23160	.35E-1	.55360E-1	.20E-1
d_2	-.1404E-1	.11E-1	.13170	.42E-1
d_3	-.2774E-2	.36E-1	.420263E-1	.94E-2
d_4	.109983	.20E-1	.16801E-1	.90E-2
d_5	.7928E-1	.24E-1	-.2086E-2	.17E-1
d_6	.269243	.68E-1	1.0044	.58E-1
Value of log-likelihood	-3,487.7		-3,577.1	

[1]Single subscripts refer to commodity number as given in Table A.I and double to commodity and demographic variable numbers. Demographic variable numbers for the σs are 1-household size, 2-under 10 years, 3 regional or ethnic group dummy=1 if northern or Limba-Temne household. For the γs the numbers are 1-over 10 years, 2-11 to 15 years, 3-females over 15.

[2]From information matrix calculated from second derivatives of log likelihood function.

Table A.IV

Regression Coefficients and Standard Errors for Regression of Squared Unweighted and Weighted QES Residuals on Squared Fitted Values[1]

Commodity	Equation	Constant	Squared Fitted Value	R^2 [2]
Rice	Unweighted	4,657.5	.78E-1	
		(2,130.8)	(.45E-1)	.02
	Weighted	.54	-.33E-5	
		(.11)	(.39E-5)	.01
Root crops and other cereals	Unweighted	7,032.8	.57	
		(4,478.3)	(.44E-1)	.55
	Weighted	2.0	.11E-4	
		(.96)	(.88E-4)	---
Oils and fats	Unweighted	1,928.3	.31	
		(875.2)	(.22E-1)	.58
	Weighted	9.3	-.22E-4	
		(2.51)	(.45E-4)	---
Fish and animal products	Unweighted	831.4	.24	
		(528.5)	(.59E-1)	.11
	Weighted	1.1	-.80E-4	
		(.29)	(.46E-4)	.02
Miscellaneous foods	Unweighted	1,428.4	.24	
		(594.2)	(.69E-1)	.08
	Weighted	1.9	-.12E-3	
		(.35)	(.61E-4)	.03
Nonfoods	Unweighted	5,107.1	.15	
		(2,580.8)	(.30E-1)	.15
	Weighted	.64	-.16E-5	
		(.21)	(.20E-5)	---

[1]Unweighted residuals are residuals from initial unweighted QES estimates, using regional dummy. Weighted residuals from the second stage QES estimates, which were weighted by fitted values from the initial estimates.

[2]-- indicates R^2 less than .005.

Notes

[1]For an introduction to this literature see Nakajima (1969) and Jorgenson and Lau (1974).

[2]For a much more complete discussion of entering demographic variables into systems of demand equations see Pollak and Wales (1978b, 1980).

[3]The fact that we have a production block in the household firm model makes no difference. If we estimate the latter as a system of value of input demands, output supplies and profit equation then summing error terms also results in a singular covariance matrix since profits equals the value of supply less the value of inputs. We then have a large system with two blocks each having one redundant equation. Barten's result on a single system applies to this situation also, so that maximum likelihood estimates are invariant to which equation is dropped in each of the subsystems.

[4]The Jacobian of the transformation of disturbances into dependent variables is one.

[5]Remembering that production parameters enter into the demand system through π^* but not vice versa.

[6]One can use a Lagrange multiplier test allowing testing using restricted parameter estimates.

[7]Net changes in storage were assumed to be zero.

[8]The equation estimated was $\frac{P_i X_i^c}{y} = b_0 + b_1 \frac{y}{P_i} + \sum_{j=1}^{n} \gamma_j \frac{P_j}{y} + \sum_{k=1}^{K} \sigma_k \frac{P_i \eta_k}{y}$ where $y \equiv$ total expenditure, $P_i \equiv$ price of good i, $\eta_k \equiv$ household characteristic k. This equation is homogeneous of degree zero in prices and total expenditure and has a quadratic term in total expenditure. Subsequent to the estimation of these single equations a data error was discovered. Seven households were mistakenly classified as Mende rather than Temne. Rerunning several of the regressions showed no major changes in

coefficients except for the ethnic dummy coefficient (that is, the others were generally within one standard deviation of the estimates using the corrected data). The mistake was corrected before obtaining the systems estimates.

[9]The ethnic and regional dummies are closely correlated. The northern region is predominantly Limba and Temne and the southern region predominantly Mende.

[10]Note that the effect of the ethnic dummy variable, η_3, is to add σ_{k3} to the price of coefficient C_k.

[11]That is, (3+3) 7-2+3 or 43 parameters, less one due to the identification problem. The ethnic and regional dummy variables were included separately.

[12]At first estimation was attempted of a QES with demographic variables entering through scaling. In the QES this involves raising the I_i scaling parameters to the $-d_i$ power. As the d_i are not integers this requires the I_i's to be positive for the function to exist. The I_is were specified as $I_i = \sum_{r=1}^{3} \sigma_{ir} \eta_r$, hence they had to be constrained to be positive. Unfortunately, the DFP algorithm kept getting "stuck" on an edge of the function where it was undefined (i.e., where I_i was almost zero for some i and some observation) and was unable to converge to a local optimum. Much effort was spent trying to obtain convergence, including use of several starting values for parameters and use of alternative algorithms. Finally, the translation specification was chosen because it has no undefined region. Alternatively, we might have specified the I_i as $I_i = \prod_{r=1}^{2} \eta_r^{\sigma_{ir}} e^{\sigma_{i3} \eta_3}$, which is necessarily positive and always defined since the η_rs are positive. Since we are not so interested in comparing the translation and scaling specifications this was not pursued.

[13]Eigenvalues of the information matrix were used to check for local optima. At a function maximum these should all be positive.

[14]In an unrestricted maximum likelihood estimation these values will change every iteration as parameter values, and hence fitted values, change.

[15]The substitution matrix was computed as $\partial X_i^c/\partial P_j|_{du=0} = \partial X_i^c/\partial P_j + \hat{X}_j^c \partial X_i^c/\partial(P_L T+\pi+A)$, where $\hat{X}_j^c$ represents fitted value so that the matrix will be symmetric as imposed by the QES.

[16]One might have improved this situation in several ways. First, using the QES, the demographic specification could have been changed by changing the variables used and/or the way in which they enter the system (i.e., use scaling). Alternatively, another system could be tried. Finally, it is conceivable that all or some of our sample simply do not behave as demand theory postulates.

[17]There were a few negative fitted values for all 138 observations. This is troublesome, but so are the solutions. We might have constrained fitted values to be positive. In our estimation, however, judging from the experience of estimating the unconstrained maximum likelihood version weighting by fitted values (actually their absolute values), we would have gotten caught on an edge of the illegal negative space. Alternatively, we might have used a Tobit procedure. However, this involves numerically evaluating multiple integrals, a very expensive procedure which would have necessitated aggregating commodities a good deal more than we did. In the raw data there are a very few zero values for expenditures, the most being five for oils and fats, and some small negative values reflecting either errors in the data or net withdrawal from storage over the year.

[18]Which implies $a_7=C_7$ since $\sum_{i=1}^{7} a_i = \sum_{i=1}^{7} C_i=1$. In this case the QES simplifies into a linear expenditure system.

[19]The share of marginal total expenditure for good i is the change in expenditure on good i when total expenditure changes by one Leone but all prices and household characteristics remain constant. The uncompensated price elasticity

of good i with respect to price j is the percent change in quantity consumed of good i when price of good j changes by one percent and all other prices, "total income" and household characteristics remain constant. The income compensated price elasticity of good i with respect to price j is the percent change in quantity consumed of good i when the price of good i changes by one percent and when the household is constrained to be at the same level of utility after the price change as before it. A price change alters the amount a household can consume given some total "nominal" income, thus the income compensated elasticity holds total "real" income constant. Changes in quantities consumed holding real income constant are referred to as the <u>substitution effects</u>. The change in quantities consumed due to the change in real total income is referred to as the <u>income effect</u>.

[20]In 1974-75 one Leone = U.S. $1.1.

[21]A relatively large number of low-expenditure households are found in areas in which cassava constitutes a large proportion of "root crops and other cereals." A relatively large number of high-expenditure households are found in areas that produce fish.

[22]We can write $\partial P_i X_i^c / \partial(P_L T + \pi + A) = \partial P_i X_i^c / \partial(\sum_{i=1}^{n} P_i X_i^c) \cdot \partial(\sum_{i=1}^{n} P_i X_i^c) / \partial(P_L T + \pi + A)$ from which we solve for $\partial P_i X_i^c / \partial(\sum_{i=1}^{n} P_i X_i^c)$, the marginal total expenditure for good i.

[23]Middle and high expenditure households tend to be in areas for which the root crops and other cereals group contains a relatively high proportion of cereals.

[24]Differences due to expenditure group are larger, which is not surprising since household characteristic variables affect expenditure through an income effect when entered into the demand system by translation. The differential effects at different expenditure levels are available, but not reported here.

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