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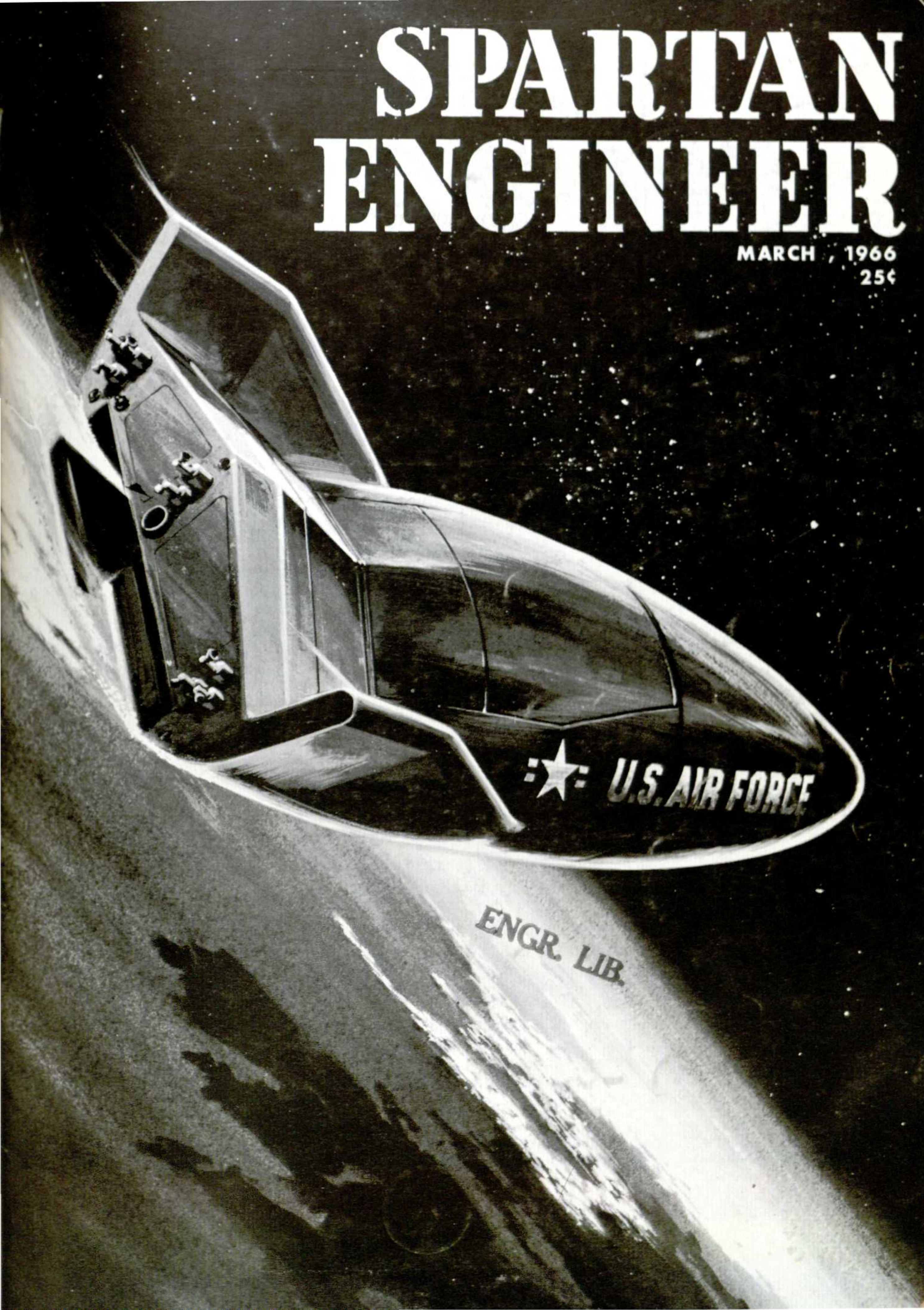

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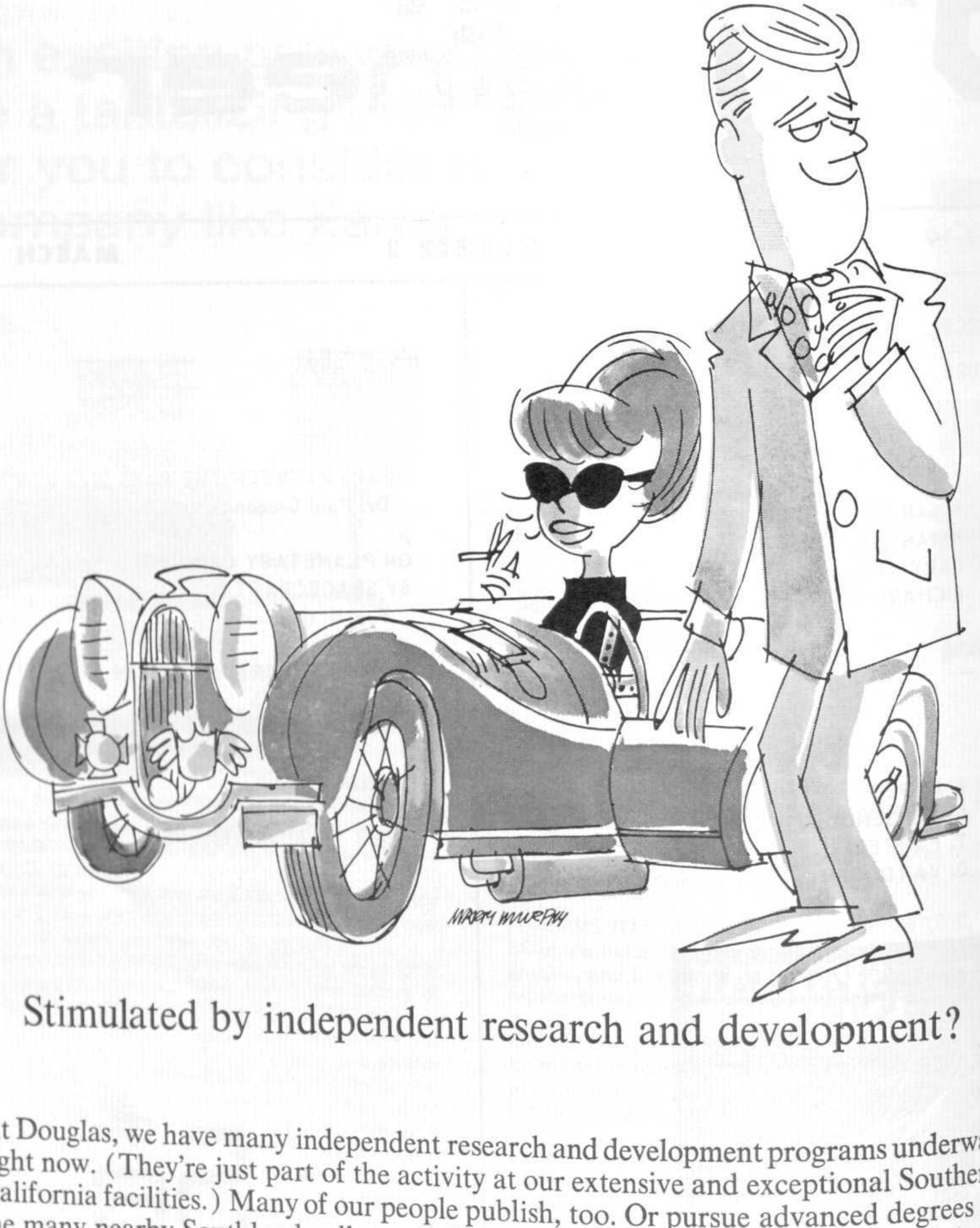

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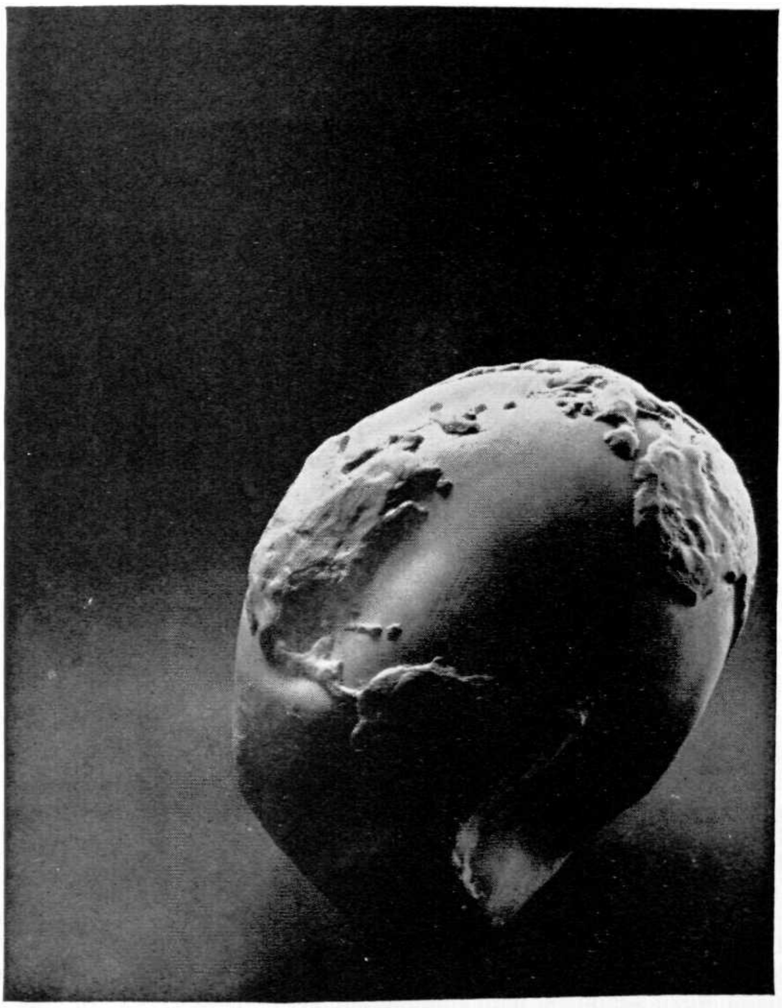

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PART THREE

FUN WITH TRIANGLES,

OR

"Pythagorean Pastimes"

Knowledge of the familiar 3-4-5 right triangle dates back through untold centuries to the ancient civilization of the Egyptians and the Chinese. It served the purpose of helping man lay out squarely his plots and tracts of land, his villages and towns, his highways and canals, and his buildings and monuments.

Although these ancient men commonly employed the 3-4-5 principle for the construction of a right angle, it is believed they were satisfied with the results so obtained and considered not why they were so. The knowledge was simply handed down along with the other arts and crafts necessary for the continuity of their civilizations.

Pythagoras, a frequent visitor to Egypt in the Sixth Century, B.C., picked up much of the mathematical learning and lore of that country and was the first man history records as having stated the proof that has become a classic in Euclidian geometry.

We recall a simple proof derived from the right triangle, abc, appearing in Figure 1, below: By similar triangles,

TRIANGLES

In all, several hundred separate proofs have been drived that state this remarkable property of the right triangle. The story is told that a humble Hindu mathematician of several centuries ago once etched in a stone wall a geometric figure similar to Figure 2, at right. Before passing on his way, he adorned his handiwork with the single word, BEHOLD! We leave it to you to discover the proof of the Pythagorean Theorem contained in the following construction.

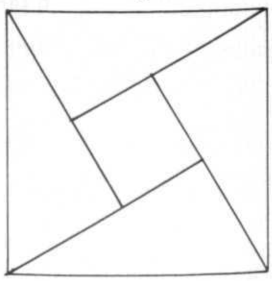
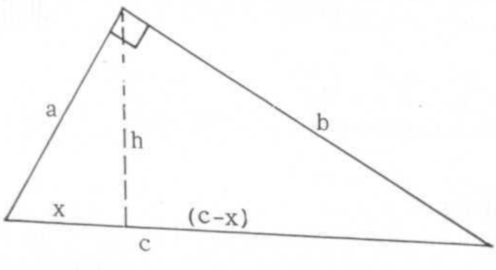


Figure 2. BEHOLD!



$$\frac{a}{x} = \frac{c}{a}, \text{ or } a^2 = cx \qquad (1)$$

$$\frac{b}{c-x} = \frac{c}{b}, \text{ or } b^2 = c(c-x) \qquad (2)$$

Adding (1) and (2)

$$a^2 + b^2 = cx + c(c-x) = c^2 \qquad (3)$$

Figure 1. Pythagorean Proof

CONTINUED ON PAGE 9

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Thus we have a general expression involving the three sides, a, b, and c, of a right triangle. The 3-4-5 right triangle is the age-old example of this relationship that is most often cited. What other such triangles are there with integer sides? Mention of 6-8-10 and 9-12-15 are simply the same old familiar 3-4-5 triangle wearing a transparent disguise.

Quite by accident, we may have chanced upon 5-12-13 and/or 7-24-25 as other integer combinations that form right triangles. From these discoveries, we may begin to suspect there are a great many more combinations of three integers that express a right triangle.

From a table of squares appearing in FUN WITH SQUARES, we recall:

$26^2 = 24^2 + 100 = 24^2 + 10^2$;

or 10-24-26 is a right triangle with integer sides.

and $29^2 = 21^2 + 400 = 21^2 + 20^2$;

or 20-21-29 is a right triangle with integer sides. Continuing with the aid of the identity we found useful in this earlier part,

$34^2 = 16^2 + 900 = 16^2 + 30^2$;

or 16-30-34, is a combination which reduces if we wish to 8-15-17,

and $41^2 = 9^2 + 1600 = 9^2 + 40^2$;

or 9-40-41 is a combination.

We recognize now that every odd number may be squared to produce another odd number. For instance, $13^2 = 169$. If we divide that square into two nearly equal parts, we get 84 and 85. Now we learned in FUN WITH SQUARES that the difference between 85^2 and 84^2 is 85 + 84. This sum, of course, is a perfect square. Thus we immediately recognize the right triangle combination 13-84-85. This tactic may be used with any odd number larger than unity to produce a combination of three integers that satisfy the right triangle requirement.

The square of any even number is "twice even," taking on the form (4n), and therefore is divisible by four. At the same time, we realize that the sum of any sequence of integers represented by

$((n-1) + n) + (n + (n + 1)) = (4n)$

expresses the differences between two pairs of consecutive squares

$((n^2) - (n - 1)^2)$ and $((n + 1)^2 - (n)^2)$

The difference in squares over the span of two integers is (4n), where (n) is the integer in the middle of the span.

We wish now to concern ourselves with how to make use of two seemingly unrelated conditions that revolve around a number of the form (4n). Take a number such as 12 and square it; then identify the square so obtained as being (4n).

$12^2 = 144 = 4n$; $n = 36$,

from which

$(n - 1) = 35$ and $(n + 1) = 37$

The difference between 37^2 and 35^2 is the succession of numbers $35 + 36 + 36 + 37 = 4(36) = 144 = 12^2$! This strategem produces right triangle combinations derived from the succession of even numbers in the pattern of 12-35-37, 14-48-50, 16-63-65, etc.

At the expense of some repetition, we are now able to establish a particular right-angle combination to fit every small integer larger than two.

Odd Numbers	Even Numbers
3) 3- 4- 5	4) 4- 3- 5
5) 5-12-13	6) 6- 8-10
7) 7-24-25	8) 8-15-17
9) 9-40-41	10) 10-24-26

These in turn, eventually may be reduced to the form of identities, where a, b, and c are the three sides of the triangle and n is any integer.

Odd Numbers	Even Numbers
$a = (2n - 1)$	$a = 2n$
$b = \frac{(2n - 1)2 - 1}{2}$	$b = n2 - 1$
$c = \frac{(2n - 1)2 - 1}{2}$	$c = n2 + 1$

It is a simple matter to verify the identities by adding the squares of a and b to obtain the square of c. From these identities it also may be shown that 1-0-1 and 2-0-2 are the limiting cases of the right triangles that apply to the two integers left out heretofore. Thus we see there are simply as many of these combinations as there are integers, both even and odd. In other words, the number of right triangles with integer sides is truly infinite.

Continuing the tabular information of right triangles for each odd and even integer started above, we are now able to obtain new combinations in abundance.

11) 11- 60- 61	12) 12-35- 37
13) 13- 84- 85	14) 14-48- 50
15) 15-112-113	16) 16-63- 65
17) 17-144-145	18) 18-80- 82
19) 19-180-181	20) 20-99-101

Possibly we should stop searching for any more of these special triangles now that it has been established there is one that fits every positive integer. Yet we sometimes happen across an example that cannot be classified according to either of the two groups we have established heretofore. The integer 20 is a part of no fewer than five different combinations, 12-16-20, 15-20-25, 20-21-29, 20-48-52, and 20-99-101. Two of the examples may not be reduced to simpler form, and one of the examples lies outside of the defining identities we have derived. There must be more of these oddities than the one-for-one relationship with respect to numbers themselves, but a new defining relation for determing them is required.

There happens to be just such a relation which arises in the most roundabout way. Recalling the double-angle formula from trigonometry, $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan \theta}$

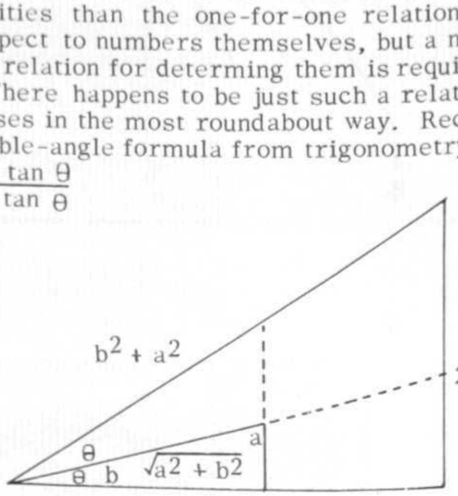

Figure 3. Role of Double-Angle Formula in Producing Pythagorean Triples

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Now, if $\tan\theta = a/b$, an integer fraction, then

$$\tan 2\theta = \frac{2a/b}{1 - a^2/b^2} = \frac{2a/b}{1 - a^2/b^2} \cdot \frac{b^2}{b^2} = \frac{2ab}{b^2 - a^2}$$

Thus, $\tan 2\theta$ is still an integer fraction, and $2ab$ along with $(b^2 - a^2)$ may be used to form the two legs of a larger right triangle as represented in Figure 3. Further, upon obtaining the square root of the sum of their squares we learn that the third side, or the hypotenuse, is $(b^2 + a^2)$, another integer! All three sides of any right triangle determined in this manner are integer values. Given __any__ integer fraction, a/b, less than unity, a right triangle follows automatically from these simple manipulations.

The following form, with a few examples, is suggested for the mass production of these special triangles which are now known to permeate the scene. It is suggested that the investigation proceed systematically with integer fractions such as 1/2, 1/3, 1/4, 1/5, etc., until it becomes evident just what further types will appear from that series. Then begin 2/3, 2/4, 2/5, etc., until a new series unfolds. Each numerator, when used in combination with the successive integers as a denominator, will produce a series of triangles which may be found by inspection after the first three or four examples.

TABLE I. Right Triangles with Integer Sides from Double-Angle Formula

a	b	a^2	b^2	2ab	$b^2 - a^2$	$b^2 - a^2$	Triangle
1	2	1	4	4	3	5	3- 4- 5
2	7	4	49	28	45	53	28-48-53
3	8	9	64	48	55	73	48-55-73

It may be well here to take a moment in trying to comprehend the infinite. We learned first that each integer may be used to generate a right triangle combination, yielding in the process __two new larger integers__ to spur us on. Now we see that each integer may be used with the infinite array of integers to produce Pythagorean triples. Moreover, this infinite array may be performed an infinite number of times. Finally, each of these infinity of infinities of operations produces __three new integers__ that form three possible combinations of a/b that feed new numbers to the formulation for our purposes faster than they can be consumed.

The preceding discussion is largely purely theoretical and may appear to have little practical application. Yet, it has been passed along to spur an interest in these phenomena and has been presented with the hope that you will come into closer working contact with the readily available network of facts and data which are number theory.

Long usage is what makes the old shoe fit comfortably, and it takes usage to make the information contained in this and the preceding parts worthwhile in everyday practice.

There are problems everywhere throughout studies in physics and engineering that involve the addition of vector quantities in a two-dimensional system. Every one of these problems can be reduced to the solution of one or more right triangles. Given the x and y components of two or more vector quantities, the situation often arises that the sum of the vector components along one of the axes will predominate. An example is an alternating-current electrical circuit analysis with some reactance, but largely resistance prevailing. Another problem is a turbine stage velocity diagram where either the x or the y component of the steam velocity may be small, depending upon whether inlet or exit conditions are in mind. There are countless tractive, friction and conveying problems where a small grade or incline is involved. Other problem solutions of this nature concern the boat crossing a river with a current or an airplane flying with a cross wind.

We know, of course, with precision that the solution of a right triangle involves solving an equation of the form

$$c^2 = a^2 + b^2.$$

for one of its parts, a, b, or c. We may elect to rearrange the above equation in the form

$$c^2 - a^2 = b^2.$$

The lefthand side begs of factoring

$$(c + a)(c - a) = b^2$$

Dividing through by the term, $(c + a)$, yields

$$(c - a) = b^2/(c+a)$$

$$c = a + b^2/(c + a) \cong a + b^2/2a.$$

Where c is unknown, we may get a first approximation for the hypotenuse of a right triangle by solving the approximate expression on the right above. Similarly, if a short side and the hypotenuse are known, we may approximate the long side as follows:

$$a \cong c - b^2/2c.$$

We will see later how we can improve upon the results of these approximation methods.

The ancient civilizations used these methods in their approximate solutions of right triangles before methods of extracting square roots were invented. It is interesting to observe how closely interrelated are the problems of square roots and right triangle solutions. We learned to approximate square roots in FUN WITH RECIPROCALS by using a method that is virtually synonymous with the derivation above.

As an example assume a resistance, $R_x = 110$ ohms, and the net reactance, $R_y = 15$ ohms. Applying methods learned in FUN WITH SQUARES, we recognize immediately that $R_y^2 = 15^2 = 225$ units. We also know that the interval between 110^2 and 111^2 comprises some 221 "square units." We adopt the convention of putting quotes around the 221 "square units" so that it is possible to work in terms of a, b^2, and c at the same time without becoming confused. We may therefore simply state that if,

$$Z^2 = R_x^2 + R_y^2 \text{ ohms},$$

then,

$$Z \cong 110 + \frac{225}{221}$$

$$\cong 111 + \frac{4}{222} = 111 + 1/55.5.$$

Now, we should recall from FUN WITH RECIPROCALS that

$1/18 = (1/2)(1/9) = (0.5)(0.1111.....) = 0.05555.....$

Turn about being fair play, $1/55.5 = 0.018$. We may

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now state Z to a greater degree of precision than any laboratory measurements of resistances allow:

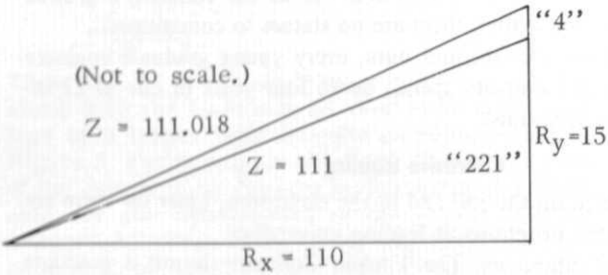

Figure 4. Solution of 15-110∽ 111.018 Right Triangle

(The curved line, ∽, used above implies that the following term in the triple of numbers forming a right triangle does not yield an exact solution but does yield a solution that is correct through the number of significant figures furnished.)

Or, in a turbine problem, given a vectorial equation

$$V_4^2 = V_x^2 + V_y^2 \text{ feet per second.}$$

Data are supplied, V_x = 50 feet per second and V_y = 410 feet.

We quickly ascertain V_x^2 = 2,500 "square units." Applying these units to the task of increasing the square of V_y to include the square of the next and succeeding integers beyond 410, we immediately see that hardly more than three additional integers can be included. It takes 821 units to get from 410^2 to 411^2, an additional 823 units to get from 411^2 to 412^2, and lastly 825 units are required to include the difference between 412^2 and 413^2. Including three significant figures, the solution of V_4 = 413 feet per second is complete.

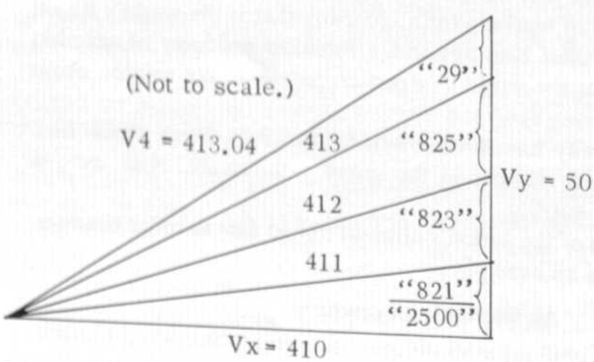

Figure 5. Solution of a 50-410-413 Right Triangle

It is within our capability to carry the above solution out to 413.04 if we dared introduce the implied significance and accuracy in the velocity determinations.

Assume a new circumstance in which the hypotenuse or a right triangle is known, along with a proportionately small side, and determine the other leg. For example, given:

$$F^2 = F_x^2 + F_y^2,$$

where F = 2,000 pounds,
F_y = 600 pounds; find F_x.

Nearly 4000 "square units" in terms of F_y^2 are required to yield F_x a single pound less than F, itself. As it happens, F_y^2 = 360,000 "square units," or enough for 90 such intervals. Now we may state cautiously that F_x = 1,910 pounds. We nevertheless realize that we have applied an approximation method, which is only nearly exact for the <u>first</u> interval, though <u>89</u> additional increments. Yet, if we perform the actual solution by longhand methods, the four-place answer is F_x = 1,908 pounds. The accuracy of the short-cut method is 99.9 percent in this instance whereas the angle involved is the arcsin 0.3000, or approximately 17.5 degrees. We mention this because the accuracy of this method depends upon the magnitude of the acute angle involved, and not upon the length of the sides. You may verify that the method is approximately 99 percent correct where the angle is 30 degrees. Insofar as either one acute angle or the other in a right triangle has a good chance of being smaller than 30 degrees, the method may be employed over a wide range of application.

In assuming above that $F_x = F - F_y^2/2F$, an error of a single unit was made in the first application of the approximation, three units error appeared in the second approximation, then five units error, seven, nine, etc., units in each succeeding approximation until 179 units error was introduced in the 90th instance. In all the accumulative error is:

Units error = 1 + 3 + 5+..... + (2n-1), where n = 90.

Recall now FUN WITH SQUARES and recognize that the sum of the accumulated error represented by the series above is nothing more than the square of 90! That is, the sum of the first n odd numbers is n^2, itself! Therefore, the accumulated error after 90 intervals totals 8,100 units. These, in turn, suggest that two additional intervals may be included, of F = 1,908 pounds. This solution now agrees with the calculated answer referred to above.

As a matter of fact, we may tabulate the problem as below and eventually arrive at any desired degree of accuracy.

$$\begin{aligned} F_x &= 1{,}910 - \frac{8{,}100}{3{,}819} \\ &= 1{,}909 - \frac{4{,}281}{3{,}817} \\ &= 1{,}908 - \frac{464}{3{,}815} \\ &= 1{,}908 - 0.12 \\ &= 1{,}907.88. \end{aligned}$$

Although we started with 3-4-5 as a right triangle, we now also recognize <u>600∽1907.88-2000</u> as being another, where the 1907.88 is as accurate as we please to make it!

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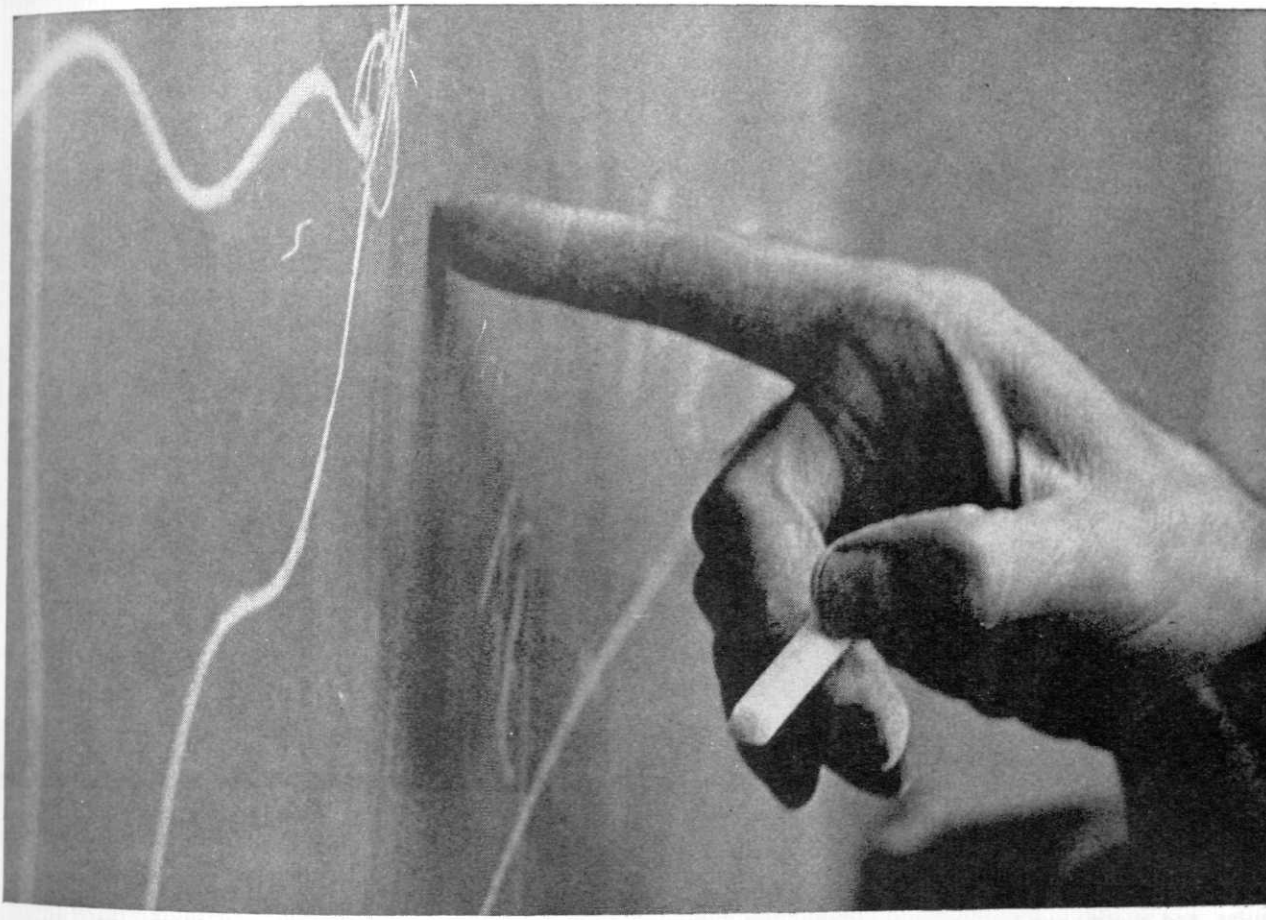
Madison, Wisconsin
March 13, 1964

Professor Paul J. Grogan, Chairman
Department of Engineering
University Extension Division
The University of Wisconsin



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ON PLANETARY LANDINGS BY SPACECRAFT

by Tom Heppenheimer

PART TWO

In Part 1 of this discussion, pulished in the January issue of the magazine, we derived and discussed certain preliminary results. These results are extremely important; the entire discussion of the specifics of a planetary landing, as presented in this article, can be carried on within the framework of these resulta and their implications. Therefore, we will restate them at this point.

(1) Variation of atmospheric density with altitude:

$$\rho = \rho_o e^{-(m/RT)gy}$$

where m is molecular weight of the atmosphere, R the universal gas constant, T the Kelvin temperature, g the acceleration due to gravity, y the altitude.

(2) Shape of a Keplerian orbit:

$$r = r_{min} \frac{1 + E}{1 + E \cos \varnothing}$$

where r is radial distance from the planet's center, E the orbital eccentricity, r_{min} the perigee, and ø the angular distance from the perigee.

(3) Eccentricity of orbit, when ship's motion is approximately parallel to the planet's surface:

$$E = \pm (v^2 r/gR^2 - 1)$$

where v is the ship's velocity relative to the planet's center and R the planet's radius.

(4) Velocity of a ship falling toward a planet:

$$v = v_o + \sqrt{2gR^2/r}$$

where v_o is the velocity relative to the planet's center of the ship when it is very far (at least 250 R) from the planet.

(5) Drag force of an atmosphere on a ship at any time:

$$F_d = 1/2\ \rho U^2 S C_D$$

U is the ship's airspeed, S the surface area exposed to the airstream, and C_D the ship's coefficient of drag.

(6) Reduction in airspeed as a result of drag in a constant-density atmosphere:

$$U = \frac{U_i}{(1/2 U_i \rho S C_D/M)t + 1}$$

where U_i is the ship's initital airspeed, M the mass of the ship, and t is time.

(7) Angular distance traveled by the ship while decelerating under atmospheric drag:

$$\varnothing = \varnothing_o + \left[\frac{M}{1/2 \rho S C_D r_{eff}}\right] \times \left[\ln (1 + 1/2 U_i \rho S C_D t/M)\right]$$

where øo is the angular position of the spacecraft on entering the atmosphere (which is of constant density), and r_{eff} is half the distance between R and the upper limit of the atmosphere.

(8) Radial velocity of a ship which is under the effect of gravity, "centrifugal force", and a radial drag force which obeys Eq. (5):

$$\dot{r} = \frac{\sqrt{G}}{c} \tanh c\sqrt{G}\, t'$$

where we have made the following changes of variable:

$$G = (g - v^2 r_{eff});\ c^2 = 1/2 \rho S C_D/M;$$

$$t' = t + \frac{1}{2c\sqrt{G}} \ln \frac{\sqrt{G} - c\dot{r}_o}{\sqrt{G} - c\dot{r}_o},$$

$\dot{r}_o$ being the initial radial velocity.

(9) Radial distance traveled by the ship whose radial drag force obeys Eq. (8):

$$r = r_o - \frac{1}{c^2} \ln \cosh c\sqrt{G}\, t'.$$

(10) Velocity of a ship which is moving vertically and is firing rockets:

$$v = \frac{T}{\mu} \ln \frac{M_o}{M_o - \mu t} - gt - v_o,$$

where T is rocket thrust (and is not to be confused with the T in Eq. (1), M_o and v_o are the ship's initital mass and velocity, and μ is the rate of mass ejection by the rockets.

Having restated these results, we may now go on to discuss the specifics of a planetary landing.

We need consider only those portions of a landing approach which lie in the atmosphere, for when a ship is outside the atmosphere its orbital motion is governed explicitly by the laws of Kepler. In particular, if the ship's velocity and direction of motion are known at a given point, then the eccentricity may be found from Eq. (3) or from its generalization which was discussed in Part I of this paper. Further, the perigee may be found, by differentiating Eq. (2) with respect to ø, setting the derivative dr/dø equal to the direction of motion of the ship at the given point, and proceeding to solve for r_{min}.

If a ship is passing through an atmosphere there are two cases to consider. The first is that the ship is undergoing a series of braking ellipses and will there-

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fore leave the atmosphere again; the second is that the ship will remain in the atmosphere and will land directly. These cases correspond respectively to Steps 1 and 2 and to Step 3 of the generalized landing approach discussed in Part I of this paper.

Suppose a ship is passing through the atmosphere in a braking ellipse (as opposed to a terminal descent). It is a well-known fact that the ship will not be strongly braked along the entire length of its flight, but will undergo a rather short period of intensive deceleration about midway through its atmospheric flight, this short period being bracketed by much longer periods in which the deceleration is rather small. Thus we may idealize the situation as one of flight down to perigee with velocity unchanged, with a sudden decrease in velocity in the immediate vicinity of the perigee, followed by flight up from the perigee to the atmospheric limit at constant velocity (the reduced velocity).

A physical system described by this model is that of a stone skipping over water: it flies at nearly constant velocity, but its velocity is reduced very suddenly during the time of contact with the water. Now it is a remarkable fact that if a spacecraft has the proper shape and enters the atmosphere at the proper angle, it will glance off in a manner precisely analogous to that of the skipping stone; this was first shown by Sanger and Bredt in 1944. This analogy lends credence to the model.

The corresponding idealization of the orbit is a "piecewise" Keplerian conic, the first part, extending down to perigee, having the eccentricity associated with the ship when it enters the atmosphere and the second part, extending up from the perigee, having the eccentricity associated with the ship when it leaves the atmosphere.

The key to analysis of the problem, then, lies in determining the proper perigee. There is no explicit solution to this problem in the form of an equation, but it is possible to set up an iterated process which will lead to the desired solution. This process rests upon two theorems, which we will now consider.

Theorem I is motivated by the fact that the idealized orbit discussed above is, ordinarily, (i.e. for moderate reductions in velocity) very nearly symmetrical, with the perigee lying near the middle of the flight path. This theorem states: Let one spaceship follow the idealized orbit, so as to enter the atmosphere at a point ϕ_o and leave at a point ϕ_e, $\phi_o \approx \phi_e$, reducing its velocity in the process from v_o to v_e. Let an identical spaceship fly at a constant altitude which has been so chosen as to insure that this ship will also reduce its velocity from v_o to v_e while traveling from position ϕ_o to ϕ_e. Then the average velocities for the two flights will be very nearly equal, and furthermore will be given approximately by:

$$v_{av} = \frac{2kv_o}{k+1} \text{ where } k = v_e/v_o.$$

To prove this, it is necessary to consider the two situations. In the idealized orbit the ship travels half the flight path at velocity v_o and the other half at velocity v_e. Let s be the length of the flight path. The time for traversing the first half $= s/2v_o$. Likewise, the time for traversing the second half is $s/2v_e$ or $s/2kv_o$. Thus the total time is $(s/2v_o + s/2kv_o) = s(k+1)/2kv_o$. Then the average velocity is as given in the theorem, $2kv_o/k+1$.

For the constant-altitude situation the mathematics is not so simple. The relevant equations are Eqs. (6) and (7). These, however, are written for airspeed, or velocity relative to the surface of the planet. The airspeed U and velocity v differ by a factor $\vec{\omega} \times \vec{r}_{eff}$, where $\vec{\omega}$ is the planet's angular velocity and X represents the cross product. However, the difference between U and v is ordinarily quite small, amounting in the case of Earth to about 5% at most, in the case of Mars to no more than 8%, and in the case of Venus to very nearly zero. Thus we may modify Eq. (6) slightly and the modification will still be quite accurate. The modified equation will read:

$$v = \frac{v_i}{(1/2\, U_i \rho S C_D/M)t + 1}.$$

This equation is not exact, as was the unmodified (6), but for small values of t such as we will be concerned with it is accurate enough. Call the term in parentheses Q; then we have that $v \approx \frac{v_o}{1 + Qt}$.

Let T be the time required to reduce v from v_o to v_e; then this equation implies that $Q = \frac{1-k}{kT}$.

Then we have:

$$v = \frac{v_o}{1 + [(1-k)/kT]t}$$

To find v_{av} we must integrate this expression from 0 to T and divide by T:

$$v_{av} = \frac{v_o}{T}\int_0^T \frac{dt}{1 + [(1-k)/kT]t}$$

$$= \frac{v_o k}{1-k}\int_0^T \frac{[(1-k)/kT]dt}{1 + [(1-k)/kT]t}$$

$$= \frac{v_o k}{1-k} \ln(1/k)$$

Consider now $\ln(1/k)$. Let $\frac{1}{k} = \frac{1+x}{1-x}$ so that $x = \frac{1-k}{1+k}$. If we expand in a Taylor series about the point $x - 0$ we have: $\ln\frac{1+x}{1-x} = 2(x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots)$ But for $k > 1/2$ we can easily neglect higher powers of x. Then, making appropriate substitutions, we have: $\ln(1/k) = 2(\frac{1-k}{1+k})$, or, multiplying by the coefficient $\frac{v_o k}{1-k}$, $v_{av} = \frac{2v_o k}{1+k}$.

But this is precisely the expression for v_{av} found for the idealized orbit. Thus the theorem is proved.

From this theorem we have two extremely useful equations:

$$v_{av} \approx \frac{2v_o k}{1+k} = \frac{2v_o v_e}{v_o + v_e} \tag{11}$$

$$\frac{v_e}{v_o} \approx \frac{1}{1 + 1/2\,\rho U_i S C_D T/M} \tag{12}$$

Theorem II states: If two identical spaceships reduce their velocity from v_o to v_e solely by atmospheric drag, then they must encounter the same mass of gas.

The proof stems from the fact that if the two ships reduce their velocity from v_o to v_e then they must lose the same amount of momentum. The only way they can lose momentum is to transfer it to the gas encountered. Let the mass of gas encountered by one ship be m_1 and the mass of gas encountered by the other be m_2. Then we require that $m_1(v_o - v_e) = m_2(v_o - v_e)$, from which we see immediately that $m_1 = m_2$.

Now we may find the perigee. The process is as follows: Guess a value of the perigee. Since v_o

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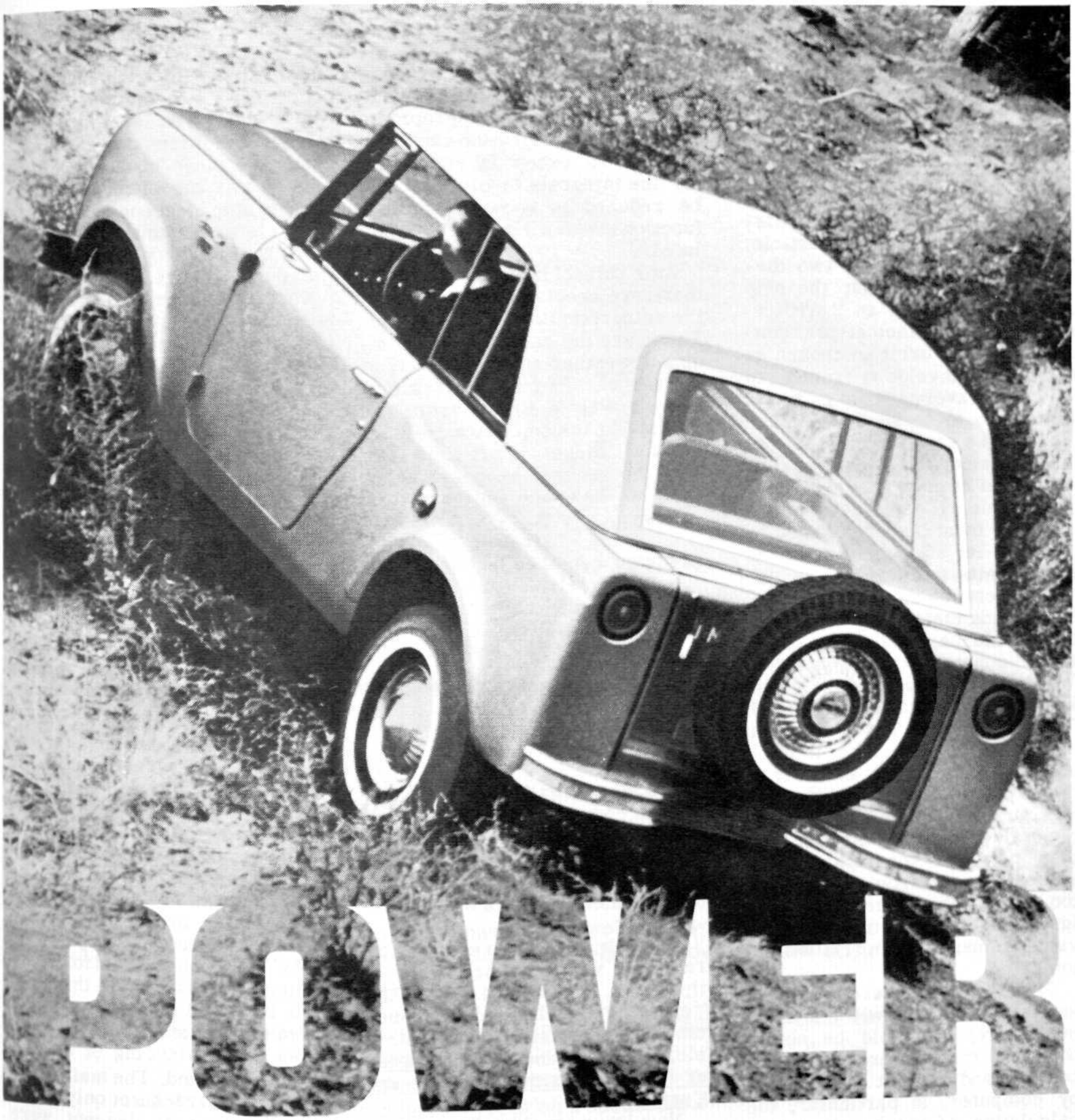
POWER

CONTINUED FROM PAGE 16

and v_e are known, it is possible to calculate the idealized orbit. Now calculate the mass of gas which lies along this orbit; to do this it will be necessary to compute the value of the line integral of Eq. (1) along the path of this orbit; this computation is discussed further on in this article. Now under the above two theorems it follows that the ship could travel along an "artificial", purely mathematical, constant-altitude orbit so chosen as to reduce its velocity from v_o to v_e while traveling from ϕ_o to ϕ_e; if it were to do so it would encounter the same mass of gas as it would along the idealized orbit, and would require very nearly the same time, $\mathcal{T}$. Let the mass of gas that must be encountered be m_g, and let s be the distance in linear units (not radians or degrees) between ϕ_o and ϕ_e, measured along the orbit. Find mg/s; this would be the density of gas in the constant-altitude orbit, ρ. We find v_{av} from Eq. (11) and knowing s, we can easily find $\mathcal{T}$: $\mathcal{T} = s/v_{av}$. This $\mathcal{T}$ and ρ can be put into Eq. (12); the implied equality will hold if and only if the proper value of the perigee was guessed at the start. For if the value guessed was too low, then ρ will be too high, and the value computed from (12) will be too low; if the value of the perigee was guessed too high, the value computed from (12) will be too high.

This therefore makes possible an iteration which will converge eventually. It should be noted, however, that the process is laborious and is best carried out by computer. In particular, the calculation of mg by computing the line integral of Eq. (1) along the path of the orbit will proceed in the following manner:

Suppose the orbit has been calculated; it is in two "branches", either of which is given by Eq. (2): $r = r_{min} \dfrac{1 + E_i}{1 + E_i \cos \phi}$ where E_i is the eccentricity corresponding to either v_o or v_e, as the case may be. It will be necessary to find the variation of altitude with ϕ; this is r - R, i.e. $y(\phi) = \dfrac{(1+E_i)r_{min}}{1 + E_i \cos \phi} - R$ or $y(\phi) = y_{min} \dfrac{1 + E_i}{1+E_i \cos \phi} + \dfrac{RE_i(1-\cos \phi)}{1 + E \cos \phi}$ where y_{min} is the altitude at perigee. This is the expression that must be put in the exponent of (1) in the place of y, the resulting expression being integrated between appropriate limits. This integration cannot be carried out except by computer, for the integrals involved cannot be reduced to known functions, functions which have been tabulated.

As a point of interest, however, there are special cases for which the integration is simple indeed. These are the cases for which ϕ_o or ϕ_e is rather small -- less than 0.4 radian. For such cases $(1 + E \cos \phi) \approx (1 + E)$, and from the Taylor expansion, $\cos \phi \approx 1 - 1/2\, \phi^2$. Putting these approximations into the general expression for $y(\phi)$, we have the approximation $y(\phi) \approx y_{min} + \dfrac{RE\phi^2}{2(1+E)}$. This expression can then be substituted for y in Eq. (1). Then we may set up the line integral for m_g:

$$m_g = \rho_o \int^{\phi_i} e^{-(mg/KT)\left(y_{min} + \frac{RE\phi^2}{2(1+E)}\right)} d\phi = \rho_o e^{-\frac{mgy_{min}}{KT}} \int_0^{\phi_i} e^{-\frac{mgRE}{2KT(1+E)}\phi^2} d\phi$$

where we are calling the universal gas constant K, not R, to avoid confusion with the R that represents planetary radius.

But this integral can easily be evaluated, for it is of the form $\int_0^x e^{-t^2} dt$. This integral, or rather a variant which is multiplied by $2/\sqrt{\pi}$, is called the "error integral" and has been tabulated extensively, for example, in Jahnke and Emde's <u>Tables of Functions with Formulae and Curves.</u>

Now let us consider the terminal landing phase, in which the ship enters the atmosphere and does not leave but spirals to a landing. The solution to this problem will yield only to a computer, but the key lies in a relatively simple iterated process. Let the atmosphere be divided into a large number of equally-spaced layers, each layer being only a mile or so thick so that density in each lyer is very nearly constant. As a ship traverses an individual layer its tangential velocity will not change by very much; hence the "centrifugal force" will change little, and G of Eqs. (8) and (9) may be taken as constant for a given layer. Then, remembering that $\vec{v} = \vec{U} + \vec{\omega} \times \vec{R}$, Eqs. (6), (7), (8), and (9) may be applied successively to each layer, the conditions of the ship at the bottom of any given layer furnishing initital conditions for the application of these equations beginning at the top of the next layer as the ship descends. It is even possible to account for the use of dive brakes or parachutes, for these will change the values of S, the exposed surface area (and perhaps also the value of C_D), but it is a trivial matter to arrange the boundaries of layers so that these changes will occur at such a boundary. In this fashion the ship may mathematically be tracked on down until it is rather near the surface of the planet, for near the surface it may well be desirable to fire retrorockets for the final touchdown. This is especially true if the planet in question is Mars; it is fairly certain that the final descent must be made by rocket, rather in the manner of the Russians' recent Luna 9.

It may reasonably be assumed that through the use of appropriate aerodynamic braking devices the ship has greatly reduced its airspeed so that it is moving only slightly with respect to the surface. Further, it is reasonable to suggest that there are two possible cases. In the first case the rocket would have been slowed by parachute to just over a safe touchdown speed, and only a slight "kick" would be needed for a safe landing. An example of this case is to be found in a series of experiments recently carried out in Texas on the landing of Gemini capsules on land. The main braking was by parachute; only in the last few feet of descent were rockets needed, and these were quite small.* For this case the mass of the system is very nearly constant, and the problem of the altitude at which retrofire must begin for there to occur a desired velocity reduction can be solved quite easily by a single application of Newton's laws.

In the second case the ship must be slowed a great deal-- 100 feet per second, or more, as a reasonable estimate. Here the ship will have to cut loose from its parachute and effect the remaining descent solely on

*See Time, August 13, 1965, p. 62 for an account of these tests.

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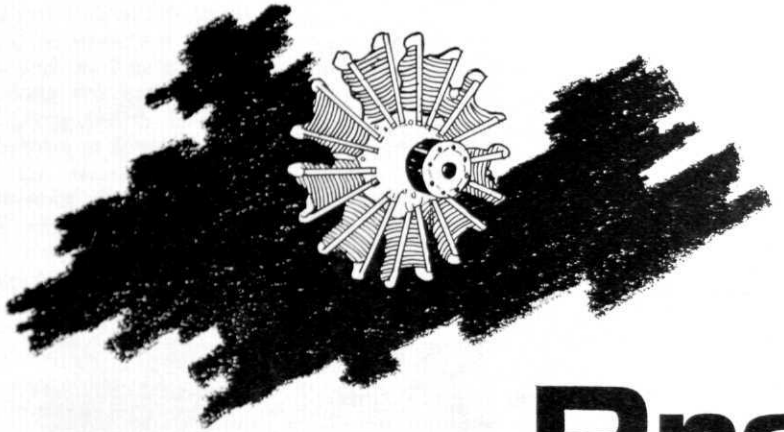
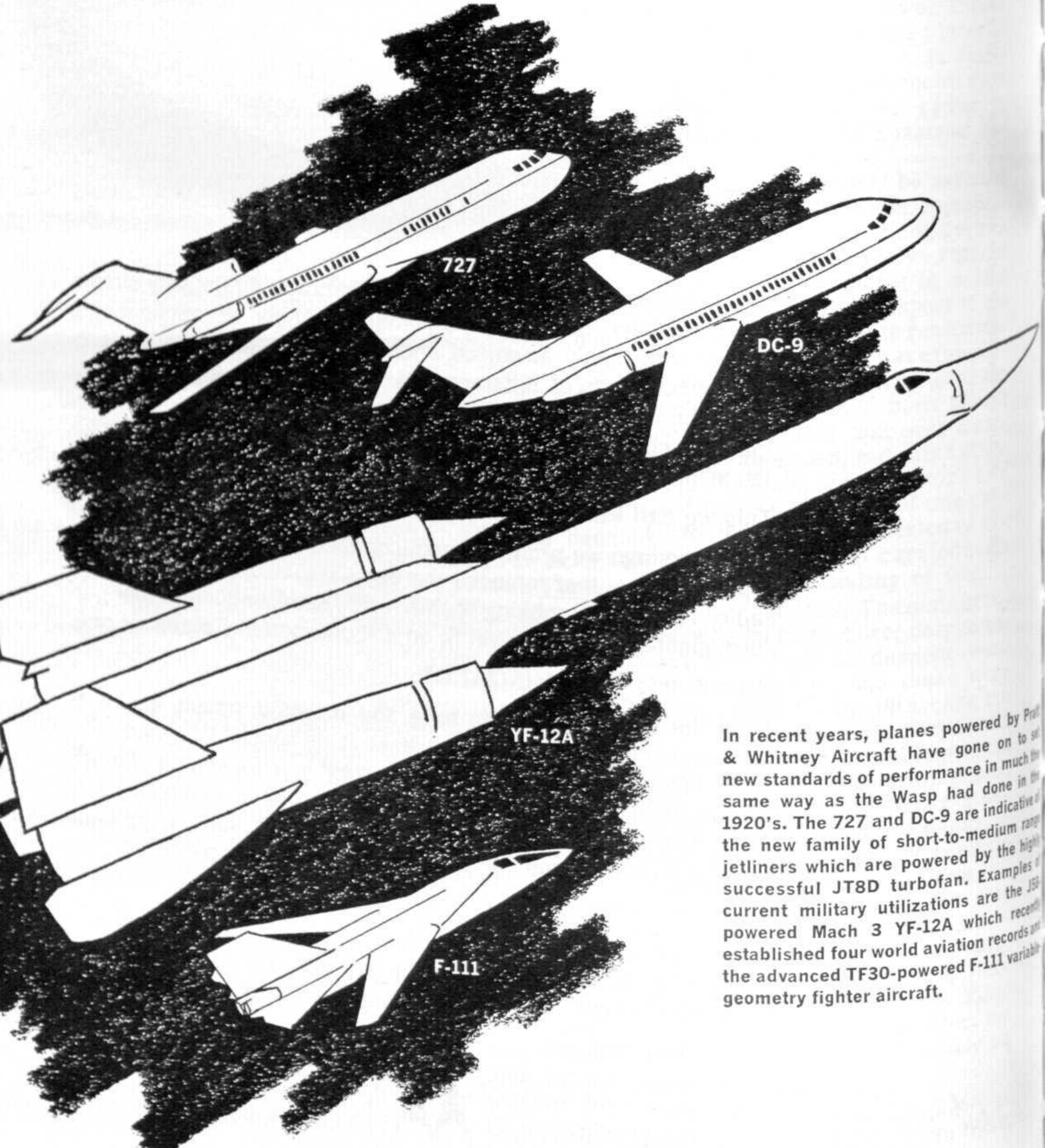
Past



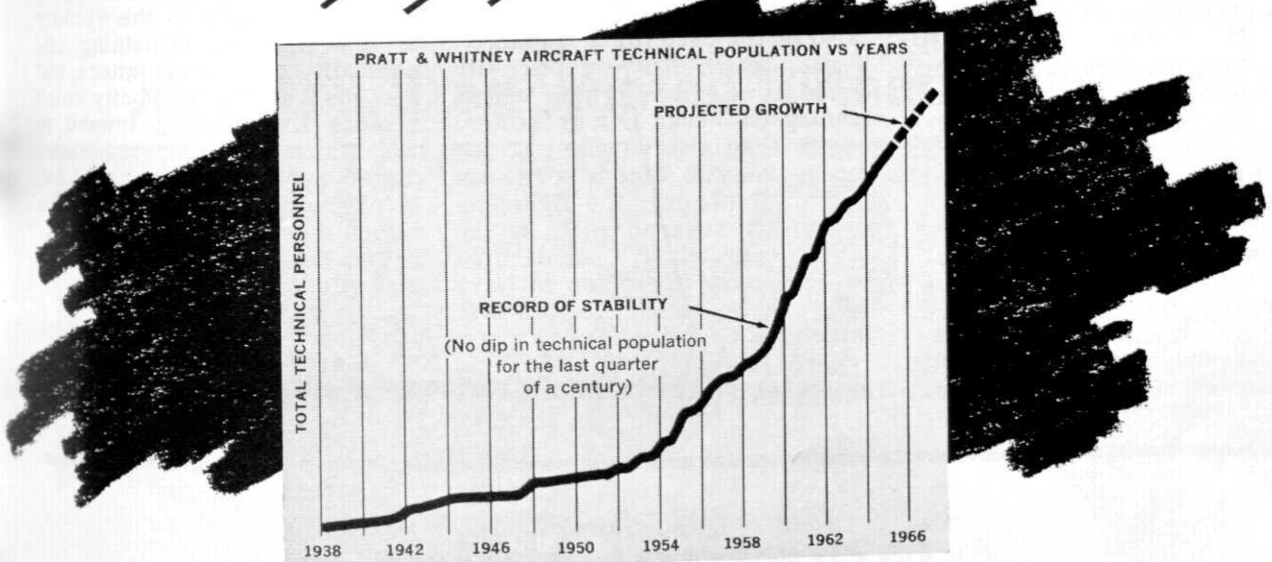
The Company's first engine, the Wasp, took to the air on May 5, 1926. Within a year the Wasp set its first world record and went on to smash existing records and set standards for both land and seaplanes for years to come, carrying airframes and pilots higher, farther, and faster than they had ever gone before.

Present



In recent years, planes powered by Pratt & Whitney Aircraft have gone on to set new standards of performance in much the same way as the Wasp had done in the 1920's. The 727 and DC-9 are indicative of the new family of short-to-medium range jetliners which are powered by the highly successful JT8D turbofan. Examples of current military utilizations are the J58-powered Mach 3 YF-12A which recently established four world aviation records and the advanced TF30-powered F-111 variable-geometry fighter aircraft.

Your and ~~the~~ Future



Take a look at the above chart; then a good long look at Pratt & Whitney Aircraft—where technical careers offer exciting growth, continuing challenge, and lasting stability—where engineers and scientists are recognized as the major reason for the Company's continued success.

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rocket thrust. Drag forces will be quite small, and the velocity of the ship at any time after initiation of retrofire will be given by Eq. (10). Also, by integrating this equation with respect to time, the distance traveled at any time past the time of retro ignition can be calculated. This distance is given by:

$$y = y_0 - v_0 t - 1/2 g t^2 + \frac{T}{\mu}\left[t\left(1 + \ln \frac{M_0}{M_0 - \mu t}\right) + \frac{M_0}{\mu} \ln (M_0 - \mu t)\right] \qquad (13)$$

Moreover, the time required for the descent may be calculated from Eq. (10) since, for a particular rocket whose parameters are known, v is a function of t alone. The time required cannot be found from an explicit equation, since it is a root of a transcendental equation. It can be found, however, by trial-and-error.

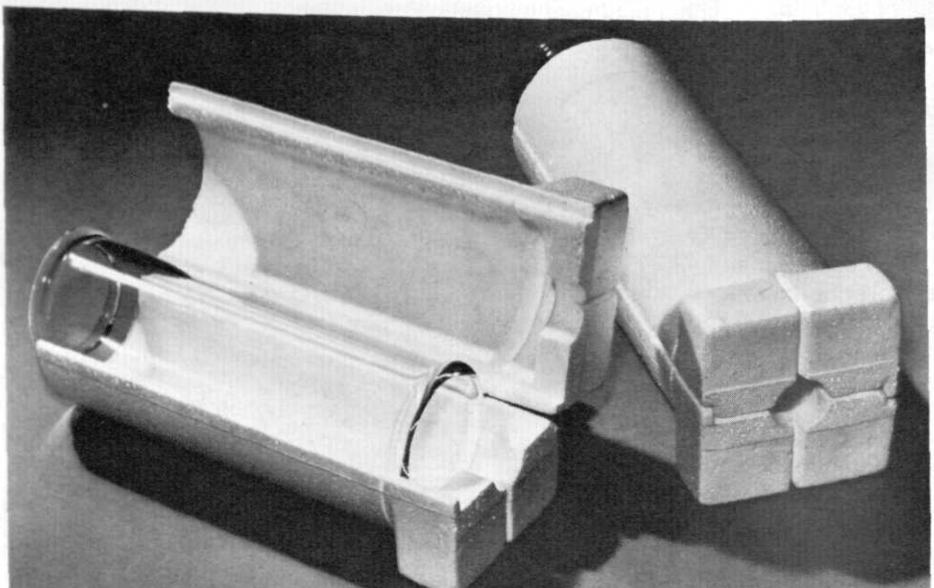
Thus Eqs. (10) and (13) will provide all necessary information about the descent under rocket thrust. If the parameters of the retrorockets are known then these equations will give the time required for descent and the altitude at which retrofire must begin; if a retro system is under design then these equations can be used to determine if a particular design will be adequate.

Of course, there is the possibility that the ship can make the descent entriely by parachute. This is the case for all Mercury and Gemini flights to date; it will also be the case for a landing on Venus, which has exceptionally dense atmosphere. Thus, a soft landing on Venus is a great deal easier than a soft landing on the moon, and this fact is borne out by recent history: the Russians had to try several times before they succeeded in a soft landing on the moon, but seem to have very nearly succeeded on the first attempt at a soft landing on Venus.

So it is that the ship touches down and comes to rest on the surface. Now that it has landed, it is well to look back and note that this analysis is not exact, but is only a resonable approximation. It is relevant to note that an exact analysis was worked out by Stuhlinger about ten years ago; a key feature of his work is an exact description of the motion of spacecraft in an atmosphere. Our analysis is limited in its accuracy by two assumptions, namely, that the velocity can be treated as dropping off very rapidly in the vicinity of the perigee, remaining essentially constant elsewhere, and that the increase in velocity which results from falling inward to lower altitudes, an increase predicted by Kepler's Second Law, is negligible. However, in cases where these assumptions are justified, our analysis can serve as a first approximation.

EAST LANSING, MICHIGAN
March 6, 1966

Industrial News



A new expanded polystyrene casing for glass underground piping offers greater safety and extreme impact strength, according to the manufacturer, Corning Glass Works. The casing, trademarked Bury-Pac, compresses and absorbs loads to compensate for changing soil conditions. Actual earth-loading tests have demonstrated that resistance to soil loadings of Pyrex brand underground piping with the new casing exceeds maximum load conditions in any field application, Corning said. The casing is installed by Corning on Pyrex brand underground pipe at no increase in cost.

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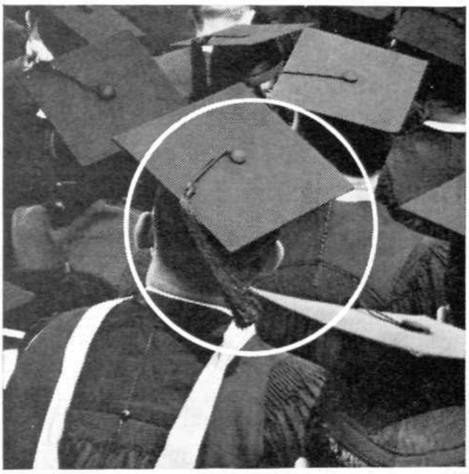

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Eric Mangelsen
B.S., Univ. of Kentucky

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Industrial News

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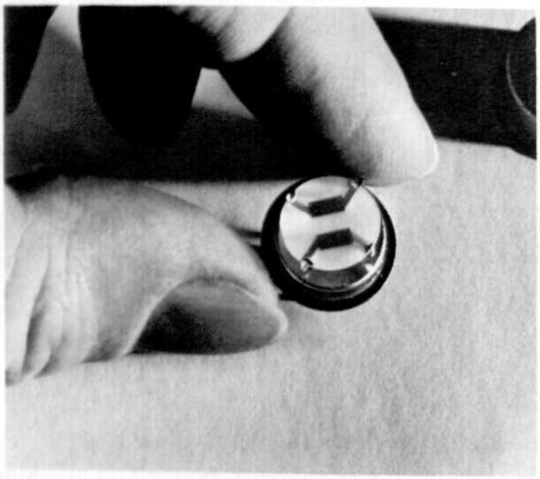
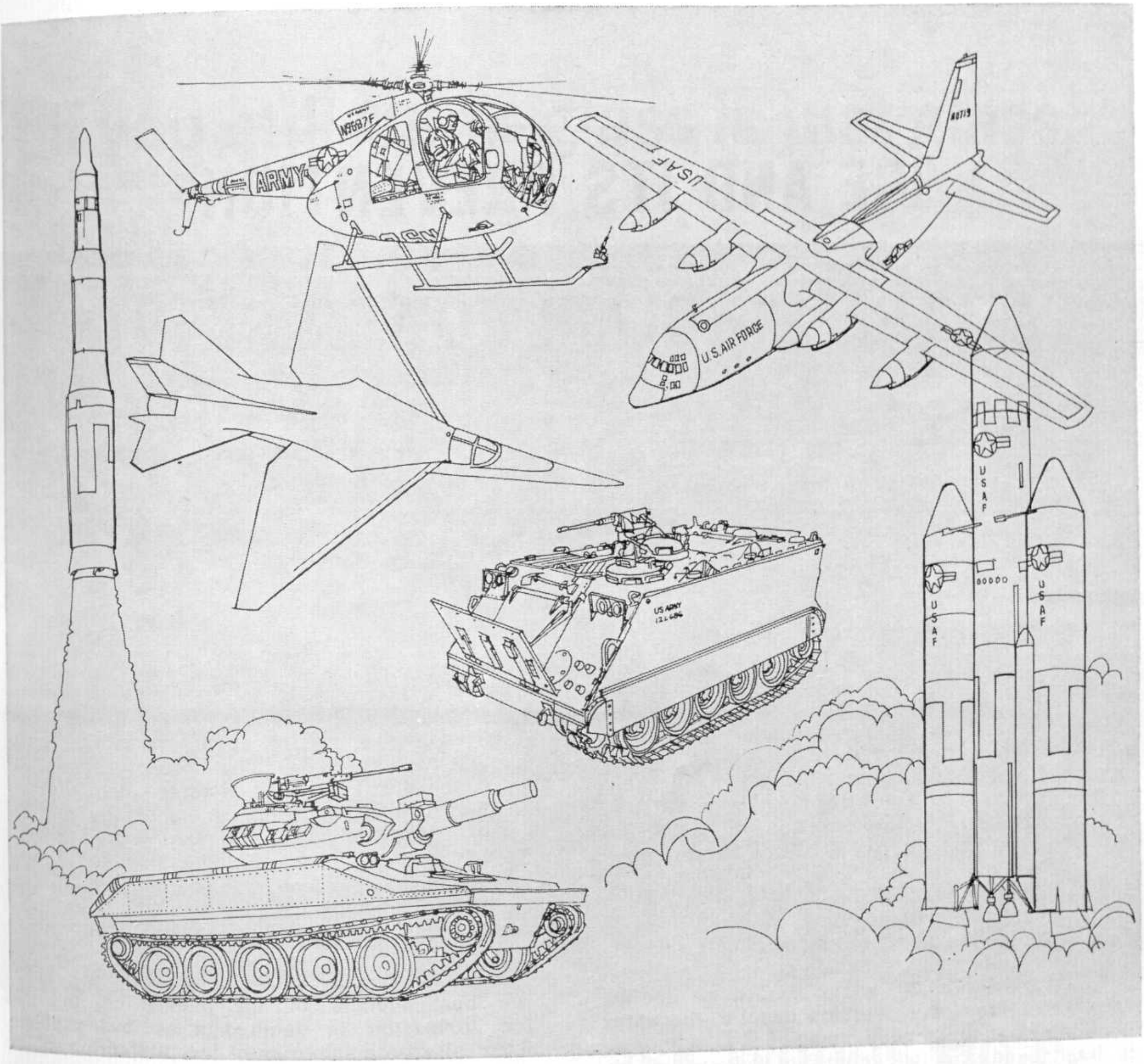


This simple device – a single wafer of quartz with two electrodes – is now performing a complex function of frequency selection which previously required eight individual electronic components. Developed by Roger A. Sykes and William D. Beaver of Bell Telephone Laboratories, the new device is a bandpass filter which can be used in radio, narrow-band FM, and telephone communication systems.

New plastic covering system for suspension-bridge cables – a joint development by Bethlehem Steel Corporation and E. I. du Pont de Nemours & Co., Inc. – has many advantages: speedy installation, handsome appearance, true watertight protection of the cables, and no maintenance.

Iniating a unique "first" in the annals of bowling are (left to right) Ray R. Eppert, president Burroughs Corporation; Joe Joseph, professional bowler, Manny Levy, president of the Bowling Proprietors' Association of America, and Howard C. Seehausen, executive director of the association. Burroughs and BPAA joined forces to automate the reporting of results at the 1966 Silver Anniversary All-Star Bowling Tournament at Joe Joseph's Pro Bowl in Lansing, Michigan, which was held January 18-30. A Burroughs B300 computer was on site to process results and to provide biographical and statistical information to the sports press on individual bowlers or the entire field.



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ICE AND ITS PREVENTION AT THE ST. CLAIR POWER PLANT

by Richard F. Luxton

SUMMARY

Experiences by the Company, and other organizations, with icing difficulties were investigated. One instance of ice difficulty was located which was sufficient to show possibility of difficulties, but not frequency. Hence, it was assumed that frazil ice presented a significant problem for further consideration.

Methods of ice prevention were investigated; they consist of:

1. Electric heating of the intake flow,
2. Steam heating of the intake flow,
3. Compressed air bubbling through the intake flow, and
4. Recirculation of warm condenser cooling water from the overflow canal to the water intake at the screen house.

It was found that the costs of preventing ice formation using steam or electric heat would be large and, within a short time, would exceed the cost of installing a recirculation system. Furthermore, for the air bubbling scheme to be effective there must be temperature stratification of the water ahead of the screens, and information from the U.S. Army Corps of Engineers indicated that there is no significant temperature gradient in the St. Clair River. Hence, air bubbling would be ineffectual. It is, therefore, recommended that recirculation be continued on the basis of its low cost, effectiveness, and reliability.

To prove the absolute necessity for ice prevention, it is recommended that preventive measures be ceased at St. Clair until either ice difficulty is encountered, or sever winter conditions are experienced with no difficulty.

Richard F. Luxton, senior in electrical engineering, served as a student engineer this past summer for the Detroit Edison Company. This article represents his findings compiled as a part of his summer work experience.

INTRODUCTION

Universally, industry requires a vast supply of water, and a steam generating power plant is not unique in its great thirst for water. Not the least of the power plant's water requirements is for condenser cooling. For the new units, such as the main turbo-generator to be added at St. Clair, this need runs to nearly a quarter of a million gallons per minute. As with smaller systems there arises vulnerability of the operation to ice formation that could seriously curtail the intake water flow and with that the efficient operation of the condenser.

The purpose of this report is to determine the best method of ice prevention, provided ice formation is deemed a serious problem. Four alternate schemes of ice prevention will be investigated.

The consequences of ice formation arise from plugging of the intake water screens due to the phenomenon of frazil ice formation in water near thirty-two degrees Fahrenheit (32° F). Frazil ice, commonly called "needle" ice, consists of minute particles of ice that nucleate about impurities and microscopic eddies in the water. The particles, then grow at such a rate that they can accumulate on the screens and result in a substantial decrease in flow within a few minutes. 1

The methods of elimination of frazil ice to be considered here are steam or electrical heating of the intake flow, compressed air bubbling in the ice formation region, and recirculation of condenser cooling water from the overflow canal into the intake flow. (See Figure 1) At the conclusion of the report a recommendation will be made in favor of the most advantageous and reliable scheme, if one is necessary.

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1. G. P. Williams, "Frazil Ice," Engineering Journal, XLII, 55-60, November, 1959

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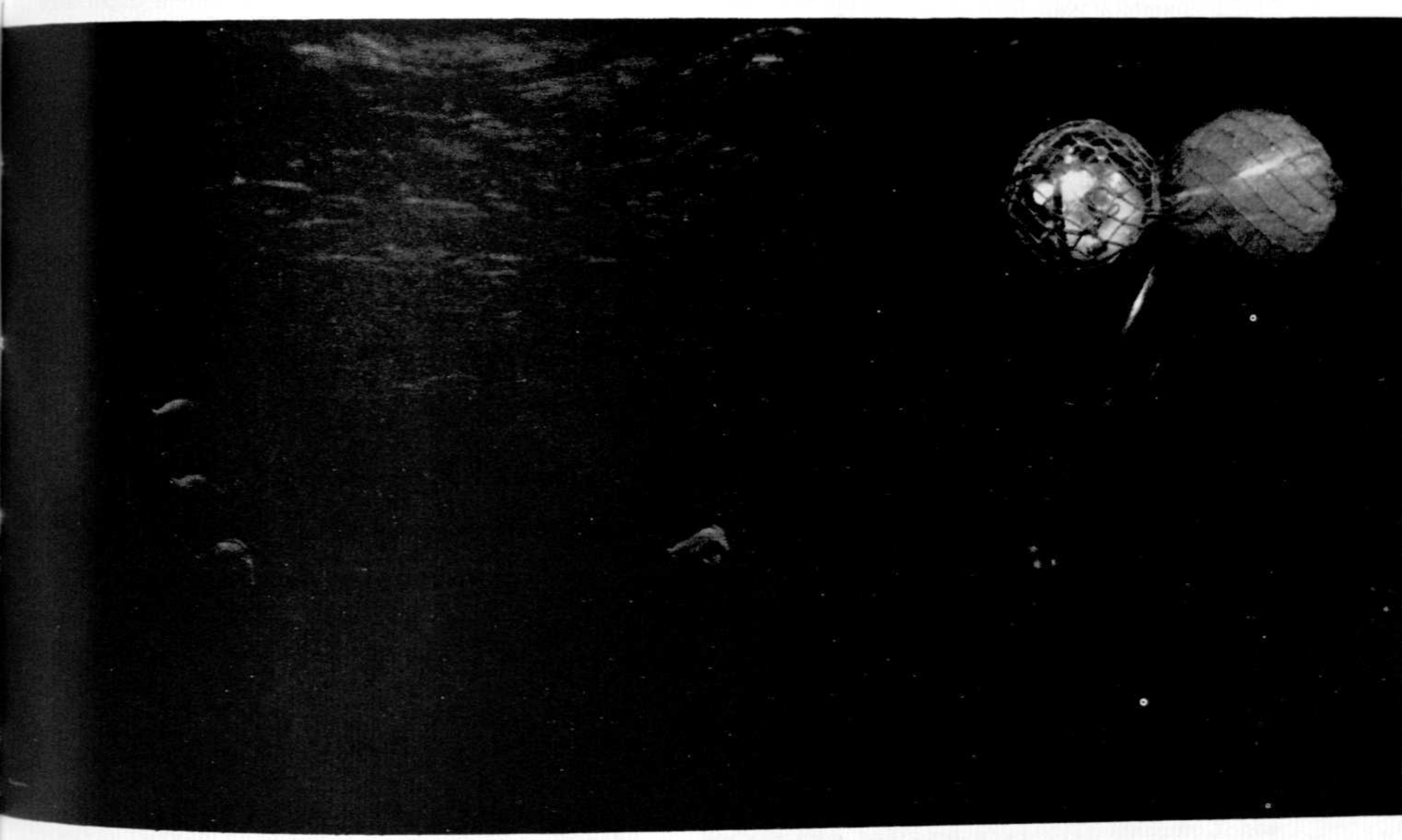
If you still think glass is just glass,



ask Woods Hole.

Now glass can do what metals can—and more. Much more. Ask Woods Hole Oceanographic Institution.

For years, the only way to get a sample of the ocean's floor was to lower what they call a "bottom corer" on a wire and then laboriously haul it up again. Not any more. Now a "Boomerang Corer"—made for Woods Hole by Benthos, Inc.—is simply tossed overboard and allowed to plunge freely. When it slams into the ocean floor, it drives a sample of sediment into a hollow tube inside the corer.

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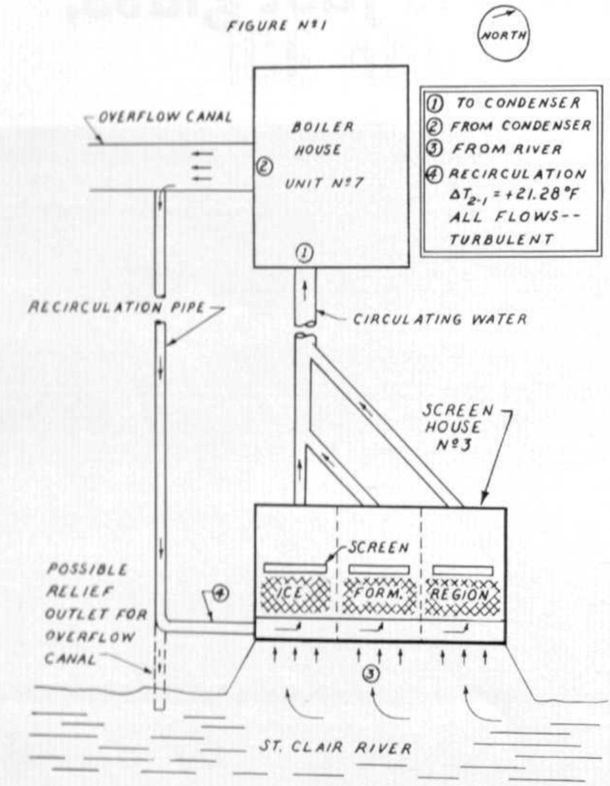
CIRCULATION AND RECIRCULATION FLOW SCHEMATIC

FIGURE Nº1



ICE FORMATION

To determine the extent of ice problems, inquiries were made within the Company and directed to the U.S. Army Corps of Engineers. Because all power plants of The Detroit Edison Company have been provided with the recirculation scheme, no history of ice difficulty at Company plants was evidenced. One instance was indicated of a frazil ice problem at the City of Port Huron Water Supply Station during the winter of 1962-63 for a period of a few days. This instance, though not conclusive in respect to frequency, did establish the definite possibility of occurance; therefore, it is assumed that frazil ice problems are of sufficient importance to necessitate measures for its prevention.

ELECTRIC HEAT

One system for prevention of frazil ice formation would be the installation of submerged electric heaters.

In considering this scheme it was thought that the heating devices could be located at a preferred depth where frazil ice would be prone to originate due to water temperature stratification. It was found, though, that such stratification does not occur in the St. Clair River[2]; hence, it is reasonable to assume that it does not occur in the turbulent flow of the intake canal. Therefore, the entire flow must be heated for the heating system to be effective.

The following calculations indicate the power requirements and expense to bring about specific temperature increments to the total intake water flow based on the assumption that complete energy transfer will occur between the heaters and the water.

From the tabulation, the requirements for a five degree Fahrenheit (5° F) increase (this value to be used for effect and cost comparison) in the intake water temperature are 161,336 kilowatts at an operating cost of \$363.01 per hour. Furthermore, there is a \$10,486,840 capacity reduction cost that would be charged to the system if it operated during a peak period for the total power system.[3]

TABLE I

TERMS

$\dot{m}$ - Mass rate of flow = 30,588 lbm/sec

c - Specific heat of water = 1 Btu/°F - lbm

ΔT_i - Temperature in c r e ment (° F) to intake water at maximum flow of 220,000 gpm (30,588 lbm/sec)

$\dot{Q}$ - Heat transfer

P - Power (kw)

EQUATIONS

A. $P = (0.948 \frac{\text{kw-sec}}{\text{Btu}}) \dot{Q}$; conversion factor

B. From the thermodynamic definition of specific heat $c \equiv \partial u/\partial T)v$, which for an incompressible fluid becomes $c = du/dT$, where u denotes the internal energy per unit mass; it follows that

$$\dot{Q} = \dot{m}\, c\, \Delta T_i$$

for finite increments.

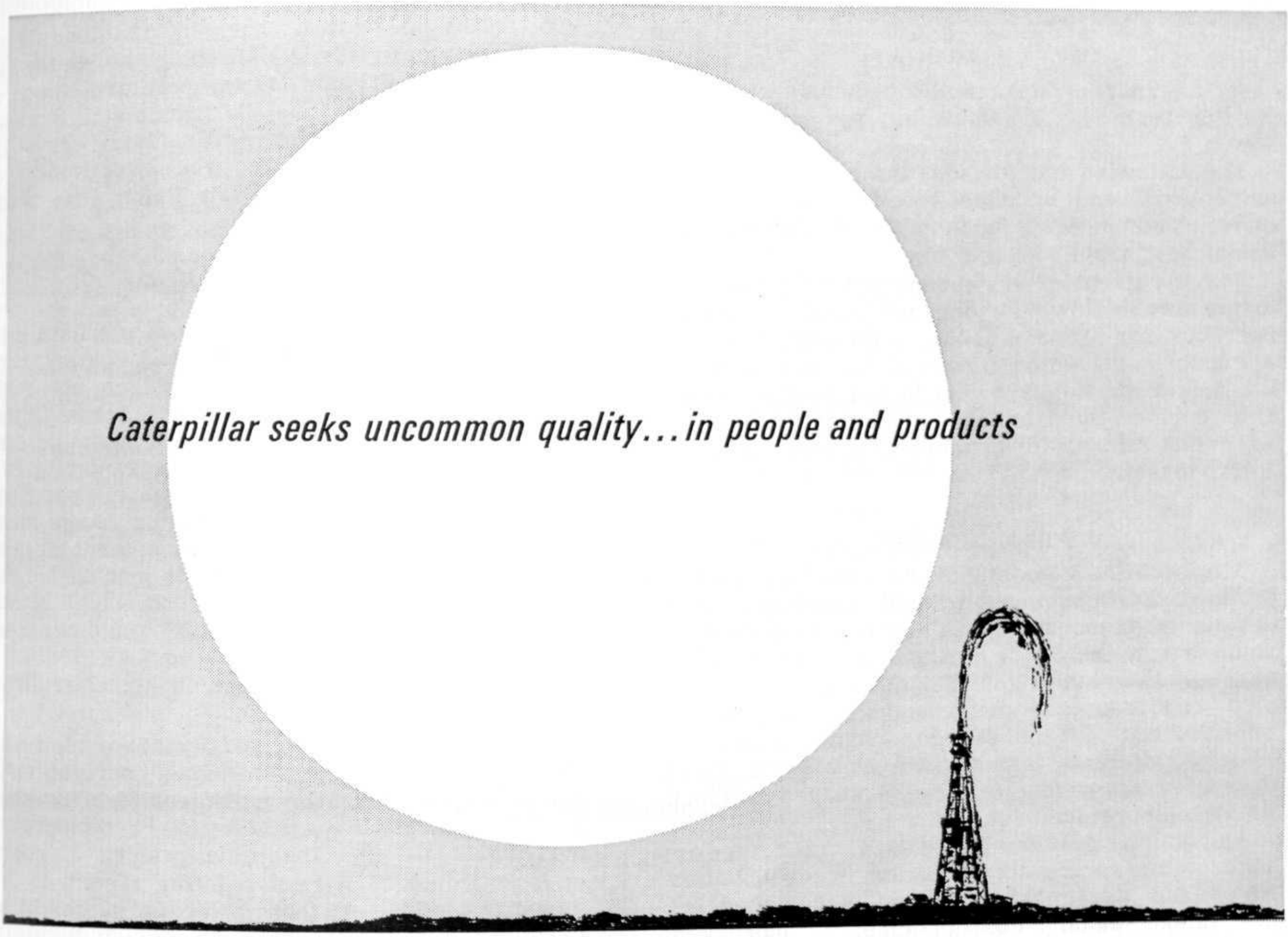
TABULATION

ΔT_i (%F)	1	2	3	4	5	6
$\dot{Q}$ (Btu/sec)	30,588	61,764	91,764	122,352	152,940	183,528
P (kw)	32.267	64,535	96,802	129,069	161,336	193,604
Capacity Reduction Cost	2,097,355	4,194,775	6,292,130	8,389,485	10,486,840	12,584,260
Operating Cost (\$/hr)	72.60	145.20	217.80	290.41	363.01	435.61

2. Mr. Leonard; U.S. Army Corps of Engineers, Lake Survey; Verbal, 7-12-65

3. Mr. R. Stanley; General Engineering Department, The Detroit Edison Company; Verbal, 7-13-65

CONTINUED ON PAGE 34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STEAM HEAT

An alternate system would provide the energy in the form of available energy in throttle steam.[4]

The following calculations indicate energy requirements and expense based on the use of throttle steam at a cost of $0.30 per million Btu of heat supplied.[5]

For the five degree Fahrenheit (5°) increment to the intake flow, the operating cost is $165.18 per hour for steam heating. Similarly, there is a capacity reduction incurred for this system, but a specific dollar value is not readily available nor easily calculated. It would be safe to say, that it would represent a multi-million dollar investment.

AIR BUBBLING

Air bubbling consists of releasing compressed air from a submerged location so that the air may ascend to the surface of the water. Since water density is a maximum at thirty-nine degrees Fahrenheit (39° F), the ascending bubbles will transport dense and relatively warm water to the surface where the water is coldest. The net effect is that surface and frazil ice is prevented from forming by increased surface water temperature.

As stated previously, though, there is no water temperature stratification in the St. Clair River and a similar condition is assumed for the intake canal. Furthermore, it has been established from technical publications that frazil ice forms either about foreign particles or minute eddies in the water. It can be seen, then, that turbulence caused by bubbling could unbalance, with an adverse effect, any equilibrium conditions that may exist. Of course the air could be heated, but any resultant effect would be negligible considering the energy required for a substantial temperature change of the flow. And, without a substantial increase of temperature of the water, frazil ice would continue to form. Hence, the effectiveness of this system at the St. Clair Power Plant would be, at best, nil.

RECIRCULATION

Recirculation, which has been standard procedure for The Detroit Edison Company, consists of diverting warm water from the overflow canal to the intake at the Screen House where it is allowed to mix with incoming condenser cooling water (See Figure 1) during cold water periods. By following this procedure, no Company plant has suffered an outage due to frazil ice accumulation and consequent plugging of the screens, in which event manual ice removal would be required and an outage of that section of the circulating system would be necessitated.

The thermal effectiveness of the recirculation is indicated by table 3.

From the definition of percentage or recirculation, "Equation (6)", nineteen percent (19%) recirculation would be sufficient to bring about a five degree Fahrenheit (5° F) temperature increment to the incoming water. A recent cost estimate of a recirculating system of 25% capacity indicates that it would represent an installation cost of $100,000.[6]

Furthermore, this sytem with minor modification would offer a simple and inexpensive alternate outlet to the river for the overflow canal, which would be operating at near-capacity with the addition of Main Turbo-Generator Unit Number Seven at St. Clair.

TABLE II

TERMS

$\dot{m}$ = Mass rate of flow - 30,588 lbm/sec

c = Specific heat of water - 1 Btu/° F - lbm

ΔT_i = Temperature increment (° F) to intake water at maximum flow of 220,000 gpm

$\dot{Q}$ = Heat transfer (Btu/sec)

c_v

EQUATIONS

As in Table I, from the definition of specific heat - $c = \partial u/\partial T)_v$, it follows that -

$$\dot{Q} = \dot{m}\, c\, \Delta T_i$$

TABULATION

ΔT_i (° F)	1	2	3	4	5	6
$\dot{Q}$ (Btu/sec)	30,588	61,176	91,764	122,352	152,940	183,528
(Btu/hr)	110,116,800	220,233,600	330,350,400	440,467,200	550,584,000	660,700,800
Operating Cost ($/hr)	33.04	66.07	99.11	132.14	165.18	198.21

4. This is steam that has a high (original) enthalpy but a lowered pressure due to having passed through a pressure reducing valve

5. Mr. R. Stanley; General Engineering Department, The Detroit Edison Company; Verbal, 7-13-65

6. J. M. Geisinger, Construction and Maintenance Department, The Detroit Edison Company; letter to C. A. Kus, 6-29-65

TABLE III

TERMS

(Refer to Figure No. 1)

Subscripts denote location of value

ΔT_{2-1} = Temperature increment through condenser (° F) = 21.28° F[7]

ΔT_{1-3} = Temperature increment due to recirculation (° F)

$\dot{m}$ = Mass rate of flow (lbm/sec)

q = Volume rate of flow (gpm)

Q = Heat transfer of mass denoted (Btu/sec)

γ = Density of water - 62.4 lbm/ft 3

u = Internal energy

c = Specific heat of water - 1 Btu/° F - lbm

EQUATIONS

$$\dot{m}_1 = q_1 \gamma \left(\frac{1}{7.48}\right) \left(\frac{1}{60}\right) = 30{,}588 \text{ lbm/sec}$$

$$T_1 = T_3 + \Delta T_{1-3} \qquad (1)$$
$$T_2 = T_1 + \Delta T_{2-1} = T_4 \qquad (2)$$
$$\dot{m}_1 = \dot{m}_4 + \dot{m}_3 \qquad (3)$$
$$u = 0$$
$$u_4 - u_3 = u_1$$
$$c(\dot{m}_4 T_4 - \dot{m}_3 T_3) - (\dot{m}_1 T_1)c \qquad (4)$$

Substituting from Equation (1), (2), and (3) into equation (4) and rearranging terms, as shown

$$\dot{m}_4 (T_1 + \Delta T_{2-1}) + (\dot{m}_1 - \dot{m}_4)(T_1 - \Delta T_{1-3}) = \dot{m}_1 T_1$$

$$\dot{m}_4 \Delta T_{2-1} - \dot{m}_1 \Delta T_{1-3} + m_4 \Delta T_{1-3} = 0$$
$$\dot{m}_4 (\Delta T_{2-1} + \Delta T_{1-3}) = m_1 \Delta T_{1-3}$$
$$\frac{\dot{m}_4}{\dot{m}_1} = \frac{\Delta T_{1-3}}{\Delta T_{2-1} + \Delta T_{1-3}} = \frac{q_4}{q_1}$$

Ratio of Recirculation (5)

q_4/q_1 100% ≡ Percentage of recirculation (definition) (6)

TABULATION

ΔT_{1-3} (° F)	1	2	3	4	5	6
q_4 (gpm)	9,876	18,899	27,183	34,810	41,857	48,386
Percentage Recirculation	4.49	8.59	12.36	15.82	19.03	21.99
Heat Added (Btu/sec) to Incoming River Water	29,215	55,920	80,427	102,992	123,840	143,160

CONCLUSION AND RECOMMENDATIONS

In comparison, both electric and steam heating preclude themselves from practical consideration because of their huge energy requirements and cost. Based on a thirty year estimated life span of a main turbo-generator plant, 200 hours per year of operation, and a five degree temperature increment to the water, the cost for these two systems would be:

1. Electric -- $2,178,060, and
2. Steam -- $991,070,

compared to $100,000 initial investment, only, for recirculation.

Air bubbling, by its theory of operation and the lack of conditions that would make it effective, is rendered ineffectual at the St. Clair Power Plant. Therefore, the final alternative of recirculation is open for approval and it is, indeed, favorable in perspective of initial cost with no operating expense and proven reliability.

The one inconclusively founded premise is whether ice preventive measures are necessary, or not. This could be answered to satisfaction by discontinuing recirculation where it is now being used and allowing the operation of the plant to continue without recirculation. The instant availability of recirculation would serve as a "back up" in case of icing difficulties, but, if no difficulties were encountered, ice prevention measures could be totally discontinued at that and similar locations. Of course, it must be remembered that such operation should be proven through both sever and mild winter conditions to be conclusive. In the case of the Main Turbo-Generator Unit Number Seven at the St. Clair Power Plant, it would be advisable to provide for ready availablility of recirculation because of the few test seasons available before the 1968 operational date.

7. ΔT_{2-1} value taken from Ingersoll-Rand Company Condenser Specifications for Trenton Channel MTG #9, 500 MW

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ACKNOWLEDGMENT

Acknowledgment of specific data and general background information is due to the following individuals:

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Stanley, R. J., General Engineering Department, The Detroit Edison Company;

Mr. Townsend, Lake Survey, U.S. Army Corps of Engineers.

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Williams, G. P., "Frazil Ice", Engineering Journal, XLII, (November, 1959), 55-60

ENGRINEERS

You might be able to avoid usual winter colds if you would avoid fatigue, loss of sleep and overcreating.

Headline of an ad in a New York bus: "For a successful affair, it's the Empire Hotel."

One of the boys in the Mech. Engineering Dept. has invented what he calls the "Square Dance Bra"-it stays up beautifully no matter how frantic the fiddling gets.

Definition of ecstacy-A feeling when you feel you are going to feel a feeling you have never felt before.

"Daughter," said the mother, "didn't I tell you not to let strange men come into your apartment? You know things like that worry me."

"Don't be ridiculous, Mother!" laughed the girl. "I went to his apartment this time. Now, let his mother worry!"

To quote an old-time Quaker, "Everyone is queer except me and thee, and even thee's a little queer."

The engineer and his new bride were anxious not to be recognized as newlyweds, so the bride removed her corsage and they both shook off the last bits a rice before entering the hotel. Then, sure that no one would ever suspect that they had just been married that afternoon, the engineer walked up to the desk and said with studied casualness: "I'd like a double bed with room, please."

The State Theatre is having a special double feature this term: "In the French Style" & "The Fighting 69th".

The trouble with being a parent is that by the time your experienced, you're unemployable.

The original mistake was inventing the calendar. This led, in due course, to having Mondays.

Crossing Grand River Avenue one morning, I was mearly run down by an antiquated car literally overflowing with about a dozen children. Since the red light has been against the woman driver, as she came to a halt I shouted: "Lady, don't you know when to stop?"

Glancing back at the moppets, she answered icily: "They aren't all mine."

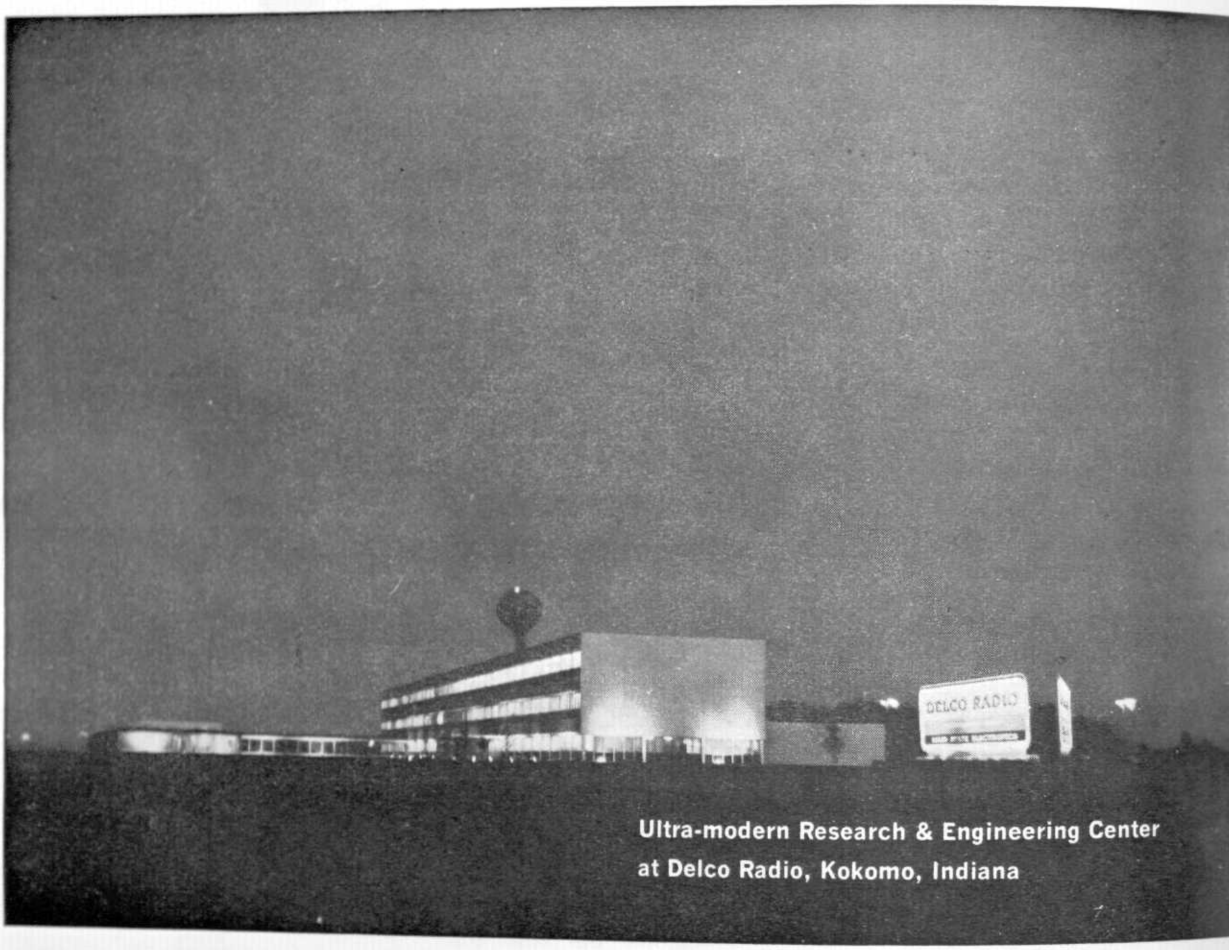
A car came to a stop in the middle of East Lansing, and the driver asked a student on the curb, "Say where's MAC Avenue?"

"I'm a pedestrian," the student replied. "I don't help automobiles."

The coed called up Coral Gables and asked imperiously if her pinmate was there. When the proprietor asked how he'd recognize him, she huffed: "He's the one that looks guilty."

Thought for the Day: "You're only young once. After that, you have to find other excuses for your indiscretions."

THE ADVERTISER'S INDEX



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PAUL HENRY is assigned to design and analysis of compressor components for G.E.'s Large Jet Engine Dept. He holds a BSME from the University of Cincinnati, 1964.

ANDY O'KEEFE, Villanova U., BSEE, 1965, Manufacturing Training Program, works on fabrications for large jet engines at LJED, Evendale, Ohio.

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